Advanced Digital Signal Processing and Noise Reduction, Second Edition.
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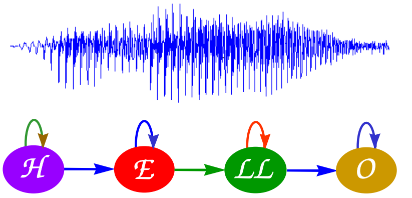

INTRODUCTION

Signal processing is concerned with the modelling, detection, identification and utilisation of patterns and structures in a signal process. Applications of signal processing methods include audio hi-fi, digital TV and radio, cellular mobile phones, voice recognition, vision, radar, sonar, geophysical exploration, medical electronics, and in general any system that is concerned with the communication or processing of information. Signal processing theory plays a central role in the development of digital telecommunication and automation systems, and in efficient and optimal transmission, reception and decoding of information. Statistical signal processing theory provides the foundations for modelling the distribution of random signals and the environments in which the signals propagate. Statistical models are applied in signal processing, and in decision-making systems, for extracting information from a signal that may be noisy, distorted or incomplete. This chapter begins with a definition of signals, and a brief introduction to various signal processing methodologies. We consider several key applications of digital signal processing in adaptive noise reduction, channel equalisation, pattern classification/recognition, audio signal coding, signal detection, spatial processing for directional reception of signals, Dolby noise reduction and radar. The chapter concludes with an introduction to sampling and conversion of continuous-time signals to digital signals.

1.1 Signals and Information

A signal can be defined as the variation of a quantity by which information is conveyed regarding the state, the characteristics, the composition, the trajectory, the course of action or the intention of the signal source. *A signal is a means to convey information.* The information conveyed in a signal may be used by humans or machines for communication, forecasting, decision-making, control, exploration etc. Figure 1.1 illustrates an information source followed by a system for signalling the information, a communication channel for propagation of the signal from the transmitter to the receiver, and a signal processing unit at the receiver for extraction of the information from the signal. In general, there is a mapping operation that maps the information $I(t)$ to the signal $x(t)$ that carries the information, this mapping function may be denoted as $T[\cdot]$ and expressed as

$$x(t)=T[I(t)] \tag{1.1}$$

For example, in human speech communication, the voice-generating mechanism provides a means for the talker to map each word into a distinct acoustic speech signal that can propagate to the listener. To communicate a word w, the talker generates an acoustic signal realisation of the word; this acoustic signal x(t) may be contaminated by ambient noise and/or distorted by a communication channel, or impaired by the speaking abnormalities of the talker, and received as the noisy and distorted signal $y(t)$. In addition to conveying the spoken word, the acoustic speech signal has the capacity to convey information on the speaking characteristic, accent and the emotional state of the talker. The listener extracts these information by processing the signal $y(t)$.

In the past few decades, the theory and applications of digital signal processing have evolved to play a central role in the development of modern telecommunication and information technology systems.

Signal processing methods are central to efficient communication, and to the development of intelligent man/machine interfaces in such areas as

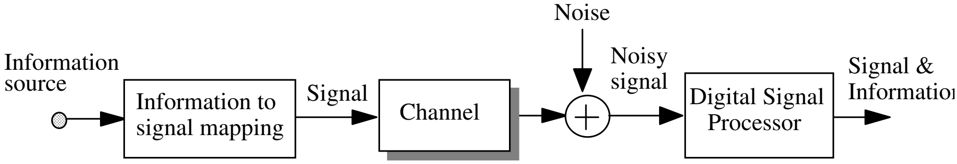

Figure 1.1 Illustration of a communication and signal processing system.

speech and visual pattern recognition for multimedia systems. In general, digital signal processing is concerned with two broad areas of information theory:

(a) efficient and reliable coding, transmission, reception, storage and representation of signals in communication systems, and
(b) the extraction of information from noisy signals for pattern recognition, detection, forecasting, decision-making, signal enhancement, control, automation etc.

In the next section we consider four broad approaches to signal processing problems.

1.2 Signal Processing Methods

Signal processing methods have evolved in algorithmic complexity aiming for optimal utilisation of the information in order to achieve the best performance. In general the computational requirement of signal processing methods increases, often exponentially, with the algorithmic complexity. However, the implementation cost of advanced signal processing methods has been offset and made affordable by the consistent trend in recent years of a continuing increase in the performance, coupled with a simultaneous decrease in the cost, of signal processing hardware.

Depending on the method used, digital signal processing algorithms can be categorised into one or a combination of four broad categories. These are non–parametric signal processing, model-based signal processing, Bayesian statistical signal processing and neural networks. These methods are briefly described in the following.

1.2.1 Non–parametric Signal Processing

Non–parametric methods, as the name implies, do *not* utilise a parametric model of the signal generation or a model of the statistical distribution of the signal. The signal is processed as a waveform or a sequence of digits. Non–parametric methods are not specialised to any particular class of signals, they are broadly applicable methods that can be applied to any signal regardless of the characteristics or the source of the signal. The drawback of these methods is that they do not utilise the distinct characteristics of the signal process that may lead to substantial improvement in performance. Some examples of non–parametric methods include digital filtering and transform-based signal processing methods such as the Fourier analysis/synthesis relations and the discrete cosine transform. Some non–parametric methods of power spectrum estimation, interpolation and signal restoration are described in Chapters 9, 10 and 11.

1.2.2 Model-Based Signal Processing

Model-based signal processing methods utilise a parametric model of the signal generation process. The parametric model normally describes the predictable structures and the expected patterns in the signal process, and can be used to forecast the future values of a signal from its past trajectory. Model-based methods normally outperform non–parametric methods, since they utilise more information in the form of a model of the signal process. However, they can be sensitive to the deviations of a signal from the class of signals characterised by the model. The most widely used parametric model is the linear prediction model, described in Chapter 8. Linear prediction models have facilitated the development of advanced signal processing methods for a wide range of applications such as low–bit–rate speech coding in cellular mobile telephony, digital video coding, high–resolution spectral analysis, radar signal processing and speech recognition.

1.2.3 Bayesian Statistical Signal Processing

The fluctuations of a purely random signal, or the distribution of a class of random signals in the signal space, cannot be modelled by a predictive equation, but can be described in terms of the statistical average values, and modelled by a probability distribution function in a multidimensional signal space. For example, as described in Chapter 8, a linear prediction model driven by a random signal can model the acoustic realisation of a spoken word. However, the random input signal of the linear prediction model, or the variations in the characteristics of different acoustic realisations of the same word across the speaking population, can only be described in statistical terms and in terms of probability functions. Bayesian inference theory provides a generalised framework for statistical processing of random signals, and for formulating and solving estimation and decision-making problems. Chapter 4 describes the Bayesian inference methodology and the estimation of random processes observed in noise.

1.2.4 Neural Networks

Neural networks are combinations of relatively simple non-linear adaptive processing units, arranged to have a structural resemblance to the transmission and processing of signals in biological neurons. In a neural network several layers of parallel processing elements are interconnected with a hierarchically structured connection network. The connection weights are trained to perform a signal processing function such as prediction or classification. Neural networks are particularly useful in non-linear partitioning of a signal space, in feature extraction and pattern recognition, and in decision-making systems. In some hybrid pattern recognition systems neural networks are used to complement Bayesian inference methods. Since the main objective of this book is to provide a coherent presentation of the theory and applications of statistical signal processing, neural networks are not discussed in this book.

1.3 Applications of Digital Signal Processing

In recent years, the development and commercial availability of increasingly powerful and affordable digital computers has been accompanied by the development of advanced digital signal processing algorithms for a wide variety of applications such as noise reduction, telecommunication, radar, sonar, video and audio signal processing, pattern recognition, geophysics explorations, data forecasting, and the processing of large databases for the identification extraction and organisation of unknown underlying structures and patterns. Figure 1.2 shows a broad categorisation of some DSP applications. This section provides a review of several key applications of digital signal processing methods.

1.3.1 Adaptive Noise Cancellation and Noise Reduction

In speech communication from a noisy acoustic environment such as a moving car or train, or over a noisy telephone channel, the speech signal is observed in an additive random noise. In signal measurement systems the information-bearing signal is often contaminated by noise from its surrounding environment. The noisy observation $y(m)$ can be modelled as

$$y(m) = x(m) + n(m) \tag{1.2}$$

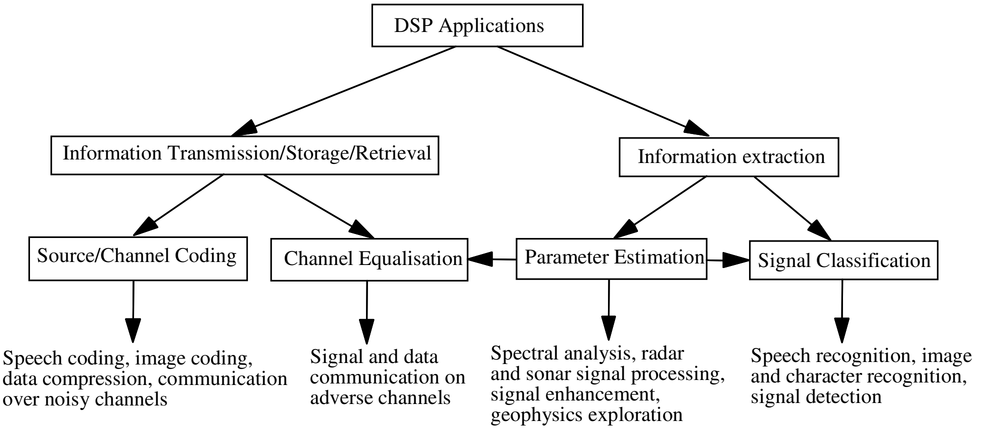

Figure 1.2 A classification of the applications of digital signal processing.

where $x(m)$ and $n(m)$ are the signal and the noise, and m is the discrete-time index. In some situations, for example when using a mobile telephone in a moving car, or when using a radio communication device in an aircraft cockpit, it may be possible to measure and estimate the instantaneous amplitude of the ambient noise using a directional microphone. The signal $x(m)$ may then be recovered by subtraction of an estimate of the noise from the noisy signal.

Figure 1.3 shows a two-input adaptive noise cancellation system for enhancement of noisy speech. In this system a directional microphone takes

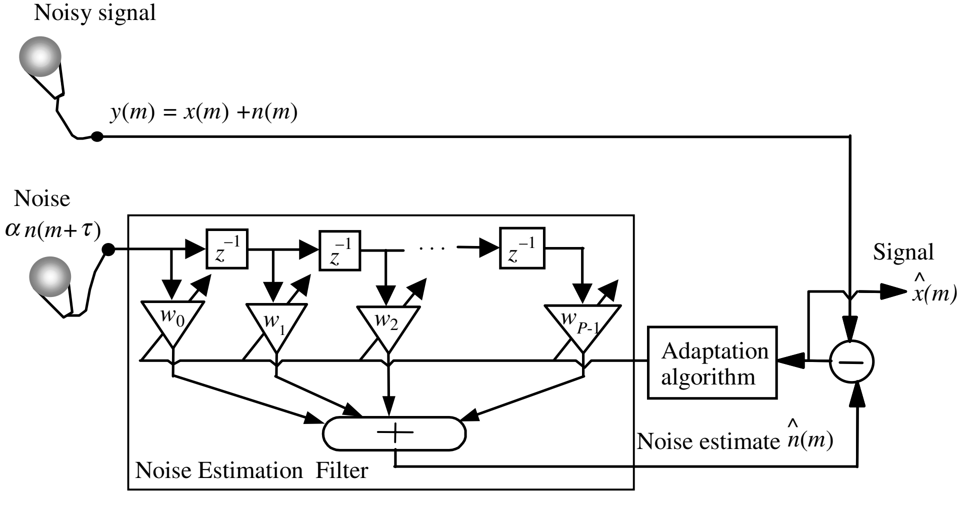

Figure 1.3 Configuration of a two-microphone adaptive noise canceller.

as input the noisy signal $x(m)+n(m)$, and a second directional microphone, positioned some distance away, measures the noise $\alpha n(m+\tau)$. The attenuation factor α and the time delay τ provide a rather over-simplified model of the effects of propagation of the noise to different positions in the space where the microphones are placed. The noise from the second microphone is processed by an adaptive digital filter to make it equal to the noise contaminating the speech signal, and then subtracted from the noisy signal to cancel out the noise. The adaptive noise canceller is more effective in cancelling out the low-frequency part of the noise, but generally suffers from the non-stationary character of the signals, and from the over-simplified assumption that a linear filter can model the diffusion and propagation of the noise sound in the space.

In many applications, for example at the receiver of a telecommunication system, there is no access to the instantaneous value of the contaminating noise, and only the noisy signal is available. In such cases the noise cannot be cancelled out, but it may be reduced, in an average sense, using the statistics of the signal and the noise process. Figure 1.4 shows a bank of Wiener filters for reducing additive noise when only the

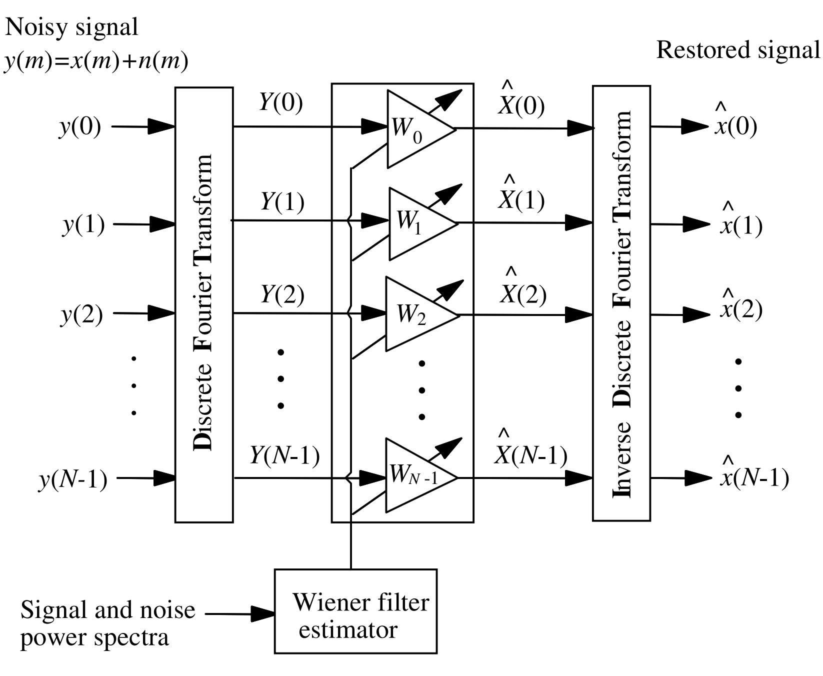

Figure 1.4 A frequency–domain Wiener filter for reducing additive noise.

noisy signal is available. The filter bank coefficients attenuate each noisy signal frequency in inverse proportion to the signal–to–noise ratio at that frequency. The Wiener filter bank coefficients, derived in Chapter 6, are calculated from estimates of the power spectra of the signal and the noise processes.

1.3.2 Blind Channel Equalisation

Channel equalisation is the recovery of a signal distorted in transmission through a communication channel with a non-flat magnitude or a non-linear phase response. When the channel response is unknown the process of signal recovery is called blind equalisation. Blind equalisation has a wide range of applications, for example in digital telecommunications for removal of inter-symbol interference due to non-ideal channel and multi-path propagation, in speech recognition for removal of the effects of the microphones and the communication channels, in correction of distorted images, analysis of seismic data, de-reverberation of acoustic gramophone recordings etc.

In practice, blind equalisation is feasible only if some useful statistics of the channel input are available. The success of a blind equalisation method depends on how much is known about the characteristics of the input signal and how useful this knowledge can be in the channel identification and equalisation process. Figure 1.5 illustrates the configuration of a decision-directed equaliser. This blind channel equaliser is composed of two distinct sections: an adaptive equaliser that removes a large part of the channel distortion, followed by a non-linear decision device for an improved estimate of the channel input. The output of the decision device is the final

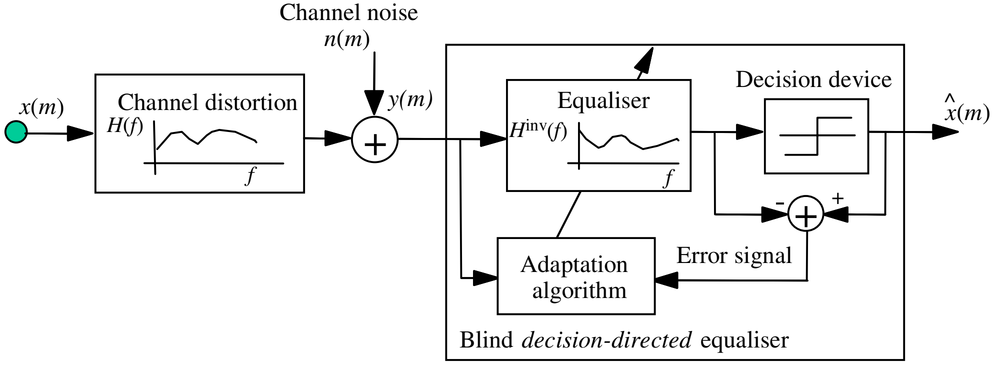

Figure 1.5 Configuration of a decision-directed blind channel equaliser.

estimate of the channel input, and it is used as the desired signal *to direct* the equaliser adaptation process. Blind equalisation is covered in detail in Chapter 15.

1.3.3 Signal Classification and Pattern Recognition

Signal classification is used in detection, pattern recognition and decision-making systems. For example, a simple binary-state classifier can act as the detector of the presence, or the absence, of a known waveform in noise. In signal classification, the aim is to design a minimum-error system for *labelling* a signal with one of a number of likely classes of signal.

To design a classifier; a set of models are trained for the classes of signals that are of interest in the application. The simplest form that the models can assume is a bank, or code book, of waveforms, each representing the prototype for one class of signals. A more complete model for each class of signals takes the form of a probability distribution function. In the classification phase, a signal is labelled with the nearest or the most likely class. For example, in communication of a binary bit stream over a band-pass channel, the binary phase–shift keying (BPSK) scheme signals the bit “1” using the waveform $A_c \sin\omega_c t$ and the bit “0” using $-A_c \sin\omega_c t$. At the receiver, the decoder has the task of classifying and labelling the received noisy signal as a “1” or a “0”. Figure 1.6 illustrates a correlation receiver for a BPSK signalling scheme. The receiver has two correlators, each programmed with one of the two symbols representing the binary

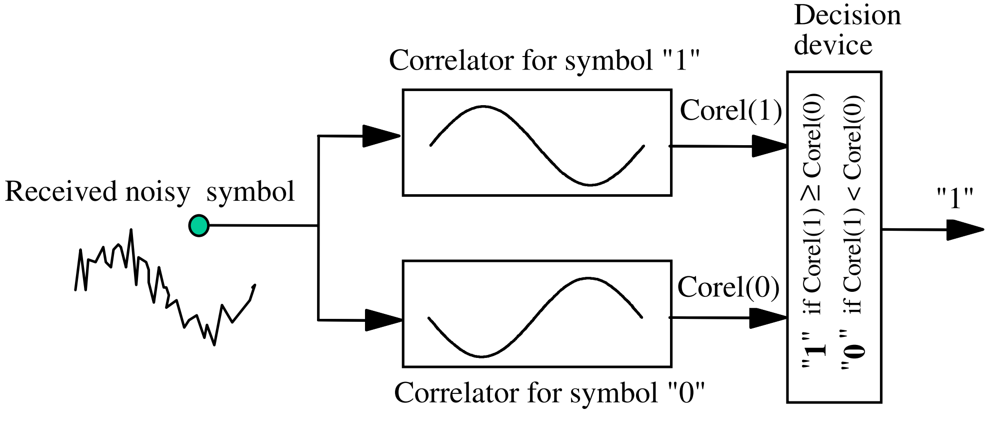

Figure 1.6 A block diagram illustration of the classifier in a binary phase-shift keying demodulation.

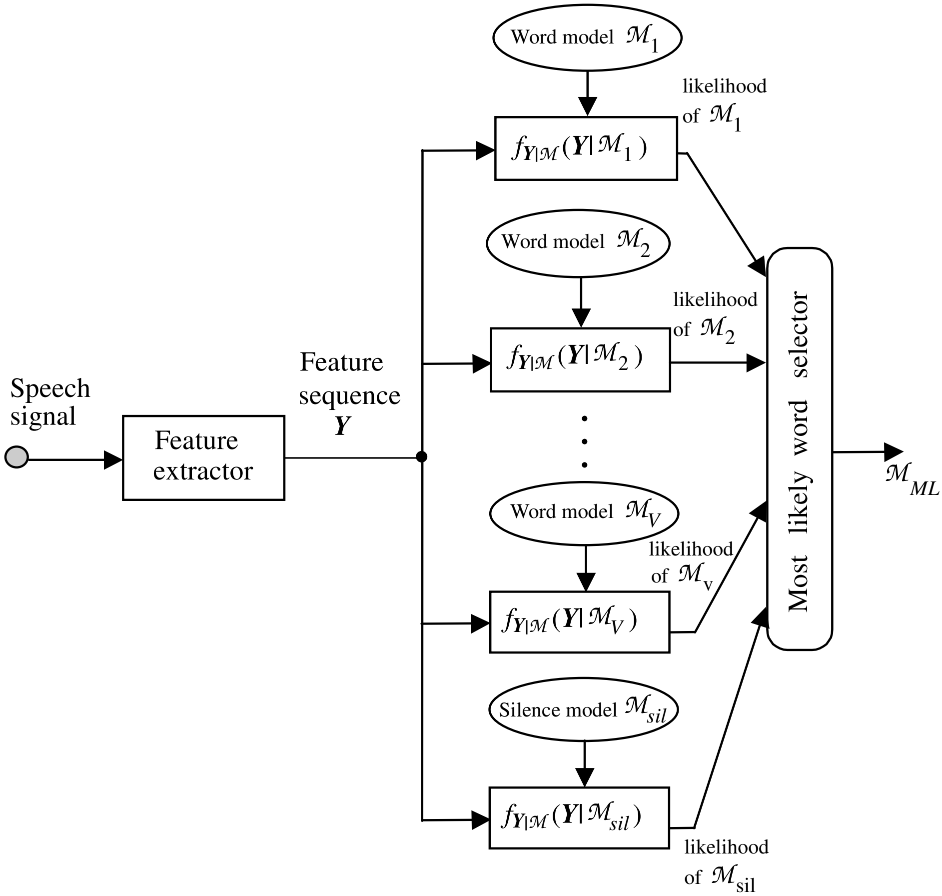

Figure 1.7 Configuration of speech recognition system, $f(Y|\mathcal{M}_i)$ is the likelihood of the model $\mathcal{M}_i$ given an observation sequence Y.

states for the bit "1" and the bit "0". The decoder correlates the unlabelled input signal with each of the two candidate symbols and selects the candidate that has a higher correlation with the input.

Figure 1.7 illustrates the use of a classifier in a limited–vocabulary, isolated-word speech recognition system. Assume there are V words in the vocabulary. For each word a model is trained, on many different examples of the spoken word, to capture the average characteristics and the statistical variations of the word. The classifier has access to a bank of V+1 models, one for each word in the vocabulary and an additional model for the silence periods. In the speech recognition phase, the task is to decode and label an acoustic speech feature sequence, representing an unlabelled spoken word, as one of the V likely words or silence. For each candidate word the classifier calculates a probability score and selects the word with the highest score.

1.3.4 Linear Prediction Modelling of Speech

Linear predictive models are widely used in speech processing applications such as low–bit–rate speech coding in cellular telephony, speech enhancement and speech recognition. Speech is generated by inhaling air into the lungs, and then exhaling it through the vibrating glottis cords and the vocal tract. The random, noise-like, air flow from the lungs is spectrally shaped and amplified by the vibrations of the glottal cords and the resonance of the vocal tract. The effect of the vibrations of the glottal cords and the vocal tract is to introduce a measure of correlation and predictability on the random variations of the air from the lungs. Figure 1.8 illustrates a model for speech production. The source models the lung and emits a random excitation signal which is filtered, first by a pitch filter model of the glottal cords and then by a model of the vocal tract.

The main source of correlation in speech is the vocal tract modelled by a linear predictor. A linear predictor forecasts the amplitude of the signal at time m, $x(m)$, using a linear combination of P previous samples $[x(m-1),\cdots,x(m-P)]$ as

$$\hat{x}(m)=\sum_{k=1}^{P} a_k x(m-k) \tag{1.3}$$

where $\hat{x}(m)$ is the prediction of the signal $x(m)$, and the vector $\boldsymbol{a}^{\mathrm{T}}=[a_1,\ldots,a_P]$ is the coefficients vector of a predictor of order P. The

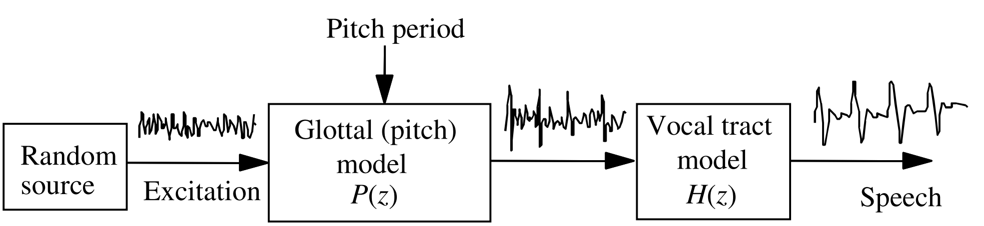

Figure 1.8 Linear predictive model of speech.

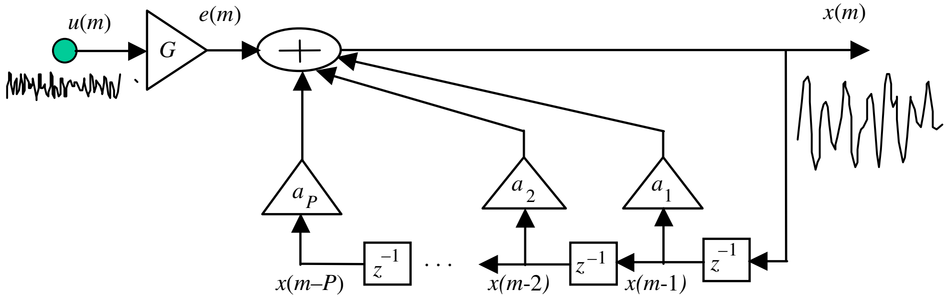

Figure 1.9 Illustration of a signal generated by an all-pole, linear prediction model.

prediction error $e(m)$, i.e. the difference between the actual sample $x(m)$ and its predicted value $\hat{x}(m)$, is defined as

$$e(m) = x(m) - \sum_{k=1}^{P} a_k x(m-k) \tag{1.4}$$

The prediction error $e(m)$ may also be interpreted as the random excitation or the so-called innovation content of $x(m)$. From Equation (1.4) a signal generated by a linear predictor can be synthesised as

$$x(m) = \sum_{k=1}^{P} a_k x(m-k) + e(m) \tag{1.5}$$

Equation (1.5) describes a speech synthesis model illustrated in Figure 1.9.

1.3.5 Digital Coding of Audio Signals

In digital audio, the memory required to record a signal, the bandwidth required for signal transmission and the signal–to–quantisation–noise ratio are all directly proportional to the number of bits per sample. The objective in the design of a coder is to achieve high fidelity with as few bits per sample as possible, at an affordable implementation cost. Audio signal coding schemes utilise the statistical structures of the signal, and a model of the signal generation, together with information on the psychoacoustics and the masking effects of hearing. In general, there are two main categories of audio coders: model-based coders, used for low–bit–rate speech coding in

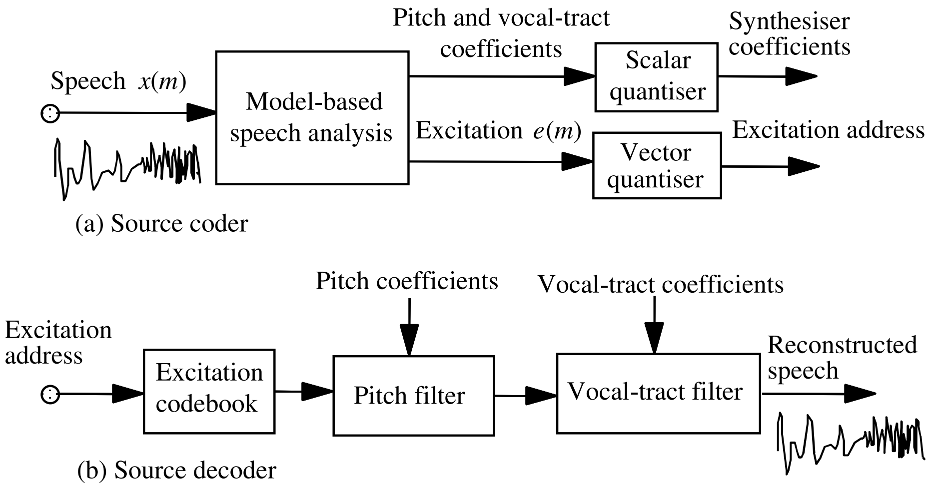

Figure 1.10 Block diagram configuration of a model-based speech coder.

applications such as cellular telephony; and transform-based coders used in high–quality coding of speech and digital hi-fi audio.

Figure 1.10 shows a simplified block diagram configuration of a speech coder–synthesiser of the type used in digital cellular telephone. The speech signal is modelled as the output of a filter excited by a random signal. The random excitation models the air exhaled through the lung, and the filter models the vibrations of the glottal cords and the vocal tract. At the transmitter, speech is segmented into blocks of about 30 ms long during which speech parameters can be assumed to be stationary. Each block of speech samples is analysed to extract and transmit a set of excitation and filter parameters that can be used to synthesis the speech. At the receiver, the model parameters and the excitation are used to reconstruct the speech.

A transform-based coder is shown in Figure 1.11. The aim of transformation is to convert the signal into a form where it lends itself to a more convenient and useful interpretation and manipulation. In Figure 1.11 the input signal is transformed to the frequency domain using a filter bank, or a discrete Fourier transform, or a discrete cosine transform. Three main advantages of coding a signal in the frequency domain are:

(a) The frequency spectrum of a signal has a relatively well–defined structure, for example most of the signal power is usually concentrated in the lower regions of the spectrum.

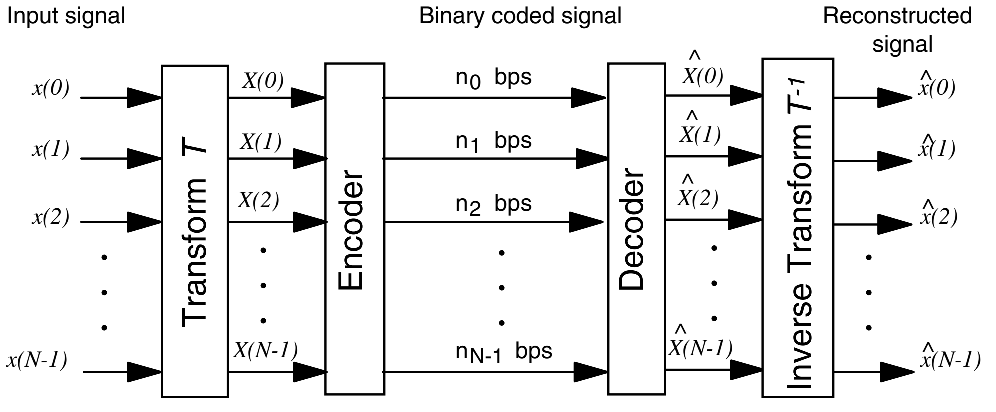

Figure 1.11 Illustration of a transform-based coder.

(b) A relatively low–amplitude frequency would be masked in the near vicinity of a large–amplitude frequency and can therefore be coarsely encoded without any audible degradation.
(c) The frequency samples are orthogonal and can be coded independently with different precisions.

The number of bits assigned to each frequency of a signal is a variable that reflects the contribution of that frequency to the reproduction of a perceptually high quality signal. In an adaptive coder, the allocation of bits to different frequencies is made to vary with the time variations of the power spectrum of the signal.

1.3.6 Detection of Signals in Noise

In the detection of signals in noise, the aim is to determine if the observation consists of noise alone, or if it contains a signal. The noisy observation $y(m)$ can be modelled as

$$y(m) = b(m)x(m) + n(m) \tag{1.6}$$

where $x(m)$ is the signal to be detected, $n(m)$ is the noise and $b(m)$ is a binary-valued state indicator sequence such that $b(m) = 1$ indicates the presence of the signal $x(m)$ and $b(m) = 0$ indicates that the signal is absent. If the signal $x(m)$ has a known shape, then a correlator or a matched filter

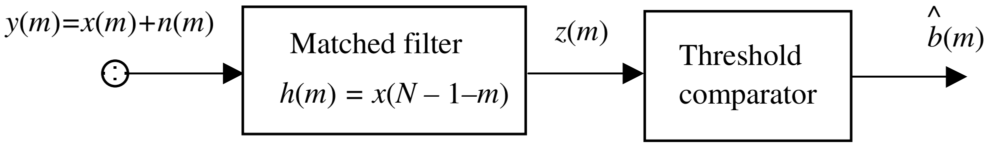

Figure 1.12 Configuration of a matched filter followed by a threshold comparator for detection of signals in noise.

can be used to detect the signal as shown in Figure 1.12. The impulse response $h(m)$ of the matched filter for detection of a signal $x(m)$ is the time-reversed version of $x(m)$ given by

$$h(m)=x(N-1-m) \qquad 0 \leq m \leq N-1 \tag{1.7}$$

where N is the length of $x(m)$. The output of the matched filter is given by

$$z(m)=\sum_{m=0}^{N-1} h(m-k)\,y(m) \tag{1.8}$$

The matched filter output is compared with a threshold and a binary decision is made as

$$\hat{b}(m)=\begin{cases}1 & \text{if } z(m) \geq \text{threshold} \\ 0 & \text{otherwise}\end{cases} \tag{1.9}$$

where $\hat{b}(m)$ is an estimate of the binary state indicator sequence $b(m)$, and it may be erroneous in particular if the signal–to–noise ratio is low. Table1.1 lists four possible outcomes that together $b(m)$ and its estimate $\hat{b}(m)$ can assume. The choice of the threshold level affects the sensitivity of the

$\hat{b}(m)$	$b(m)$	Detector decision	
0	0	Signal absent	*Correct*
0	1	Signal absent	(*Missed*)
1	0	Signal present	(*False alarm*)
1	1	Signal present	*Correct*

Table 1.1 Four possible outcomes in a signal detection problem.

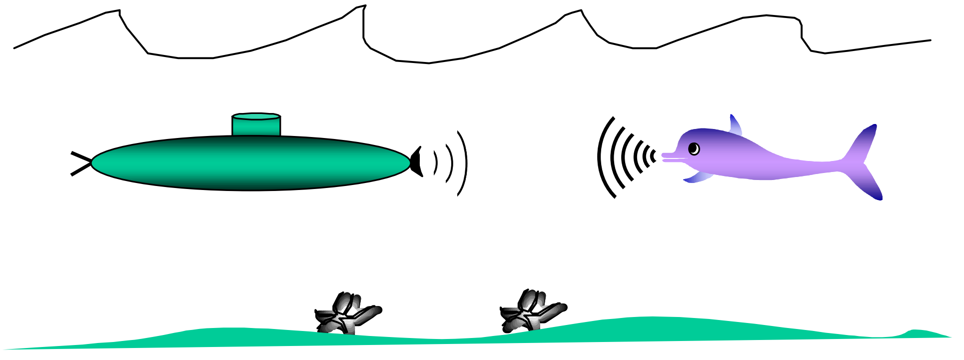

Figure 1.13 Sonar: detection of objects using the intensity and time delay of reflected sound waves.

detector. The higher the threshold, the less the likelihood that noise would be classified as signal, so the false alarm rate falls, but the probability of misclassification of signal as noise increases. The risk in choosing a threshold value θ can be expressed as

$$\mathcal{R}(\text{Threshold} = \theta) = P_{\text{False Alarm}}(\theta) + P_{\text{Miss}}(\theta) \tag{1.10}$$

The choice of the threshold reflects a trade-off between the misclassification rate $P_{\text{Miss}}(\theta)$ and the false alarm rate $P_{\text{False Alarm}}(\theta)$.

1.3.7 Directional Reception of Waves: Beam-forming

Beam-forming is the spatial processing of plane waves received by an array of sensors such that the waves incident at a particular spatial angle are passed through, whereas those arriving from other directions are attenuated. Beam-forming is used in radar and sonar signal processing (Figure 1.13) to steer the reception of signals towards a desired direction, and in speech processing for reducing the effects of ambient noise.

To explain the process of beam-forming consider a uniform linear array of sensors as illustrated in Figure 1.14. The term *linear array* implies that the array of sensors is spatially arranged in a straight line and with equal spacing d between the sensors. Consider a sinusoidal far–field plane wave with a frequency F_0 propagating towards the sensors at an incidence angle of θ as illustrated in Figure 1.14. The array of sensors samples the incoming wave as it propagates in space. The time delay for the wave to travel a distance of d between two adjacent sensors is given by

$$\tau = \frac{d \sin\theta}{c} \tag{1.11}$$

where c is the speed of propagation of the wave in the medium. The phase difference corresponding to a delay of τ is given by

$$\varphi = 2\pi \frac{\tau}{T_0} = 2\pi F_0 \frac{d \sin\theta}{c} \tag{1.12}$$

where T_0 is the period of the sine wave. By inserting appropriate corrective

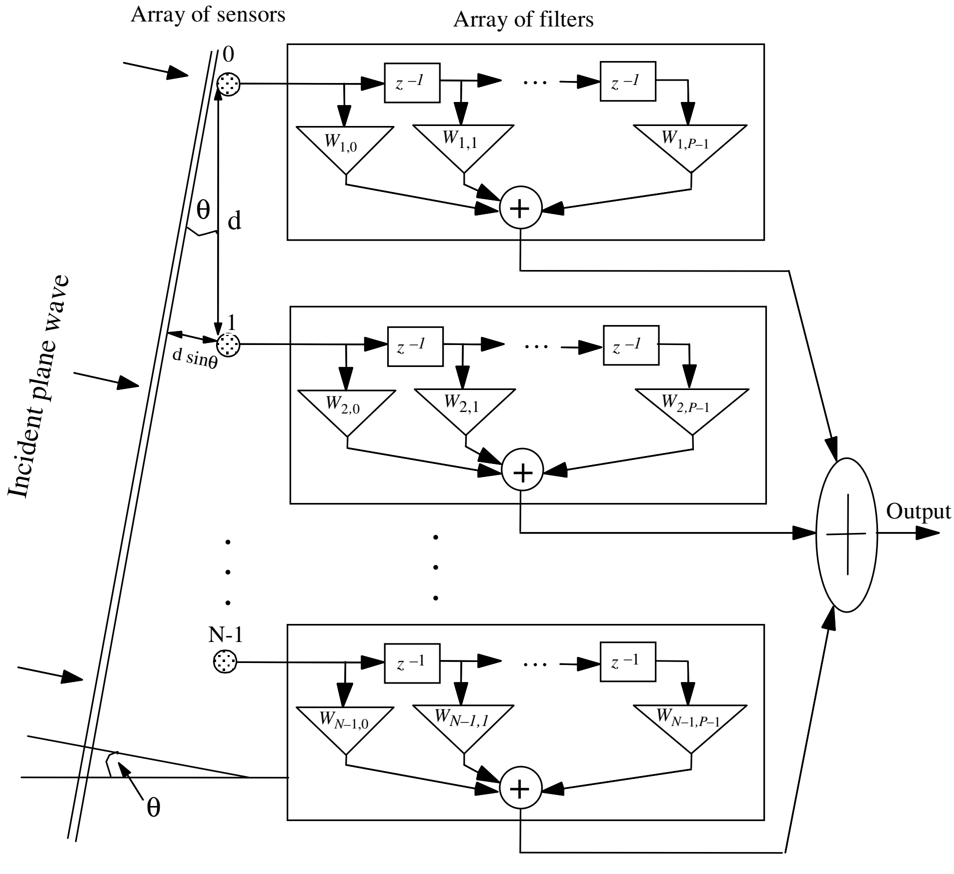

Figure 1.14 Illustration of a beam-former, for directional reception of signals.

time delays in the path of the samples at each sensor, and then averaging the outputs of the sensors, the signals arriving from the direction θ will be time-aligned and coherently combined, whereas those arriving from other directions will suffer cancellations and attenuations. Figure 1.14 illustrates a beam-former as an array of digital filters arranged in space. The filter array acts as a two–dimensional space–time signal processing system. The space filtering allows the beam-former to be steered towards a desired direction, for example towards the direction along which the incoming signal has the maximum intensity. The phase of each filter controls the time delay, and can be adjusted to coherently combine the signals. The magnitude frequency response of each filter can be used to remove the out–of–band noise.

1.3.8 Dolby Noise Reduction

Dolby noise reduction systems work by boosting the energy and the signal to noise ratio of the high–frequency spectrum of audio signals. The energy of audio signals is mostly concentrated in the low–frequency part of the spectrum (below 2 kHz). The higher frequencies that convey quality and sensation have relatively low energy, and can be degraded even by a low amount of noise. For example when a signal is recorded on a magnetic tape, the tape "hiss" noise affects the quality of the recorded signal. On playback, the higher–frequency part of an audio signal recorded on a tape have smaller signal–to–noise ratio than the low–frequency parts. Therefore noise at high frequencies is more audible and less masked by the signal energy. Dolby noise reduction systems broadly work on the principle of emphasising and boosting the low energy of the high–frequency signal components prior to recording the signal. When a signal is recorded it is processed and encoded using a combination of a pre-emphasis filter and dynamic range compression. At playback, the signal is recovered using a decoder based on a combination of a de-emphasis filter and a decompression circuit. The encoder and decoder must be well matched and cancel out each other in order to avoid processing distortion.

Dolby has developed a number of noise reduction systems designated Dolby A, Dolby B and Dolby C. These differ mainly in the number of bands and the pre-emphasis strategy that that they employ. Dolby A, developed for professional use, divides the signal spectrum into four frequency bands: band 1 is low-pass and covers 0 Hz to 80 Hz; band 2 is band-pass and covers 80 Hz to 3 kHz; band 3 is high-pass and covers above 3 kHz; and band 4 is also high-pass and covers above 9 kHz. At the encoder the gain of each band is adaptively adjusted to boost low–energy signal components. Dolby A

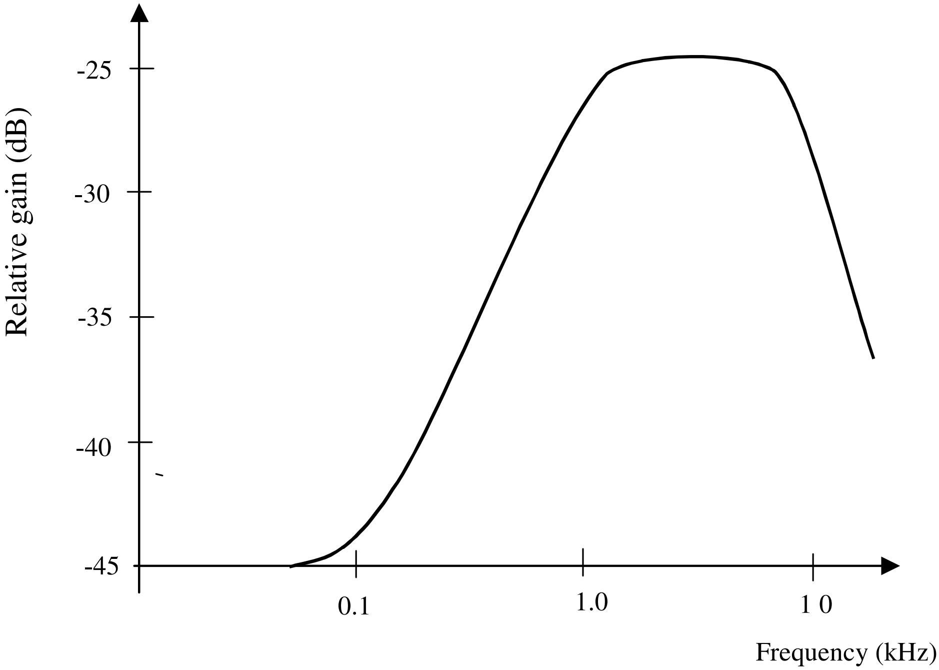

Figure 1.15 Illustration of the pre-emphasis response of Dolby-C: upto 20 dB boost is provided when the signal falls 45 dB below maximum recording level.

provides a maximum gain of 10 to 15 dB in each band if the signal level falls 45 dB below the maximum recording level. The Dolby B and Dolby C systems are designed for consumer audio systems, and use two bands instead of the four bands used in Dolby A. Dolby B provides a boost of up to 10 dB when the signal level is low (less than 45 dB than the maximum reference) and Dolby C provides a boost of up to 20 dB as illustrated in Figure1.15.

1.3.9 Radar Signal Processing: Doppler Frequency Shift

Figure 1.16 shows a simple diagram of a radar system that can be used to estimate the range and speed of an object such as a moving car or a flying aeroplane. A radar system consists of a transceiver (transmitter/receiver) that generates and transmits sinusoidal pulses at microwave frequencies. The signal travels with the speed of light and is reflected back from any object in its path. The analysis of the received echo provides such information as range, speed, and acceleration. The received signal has the form

$$x(t) = A(t)\cos\{\omega_0 [t - 2r(t)/c]\} \tag{1.13}$$

where $A(t)$, the time-varying amplitude of the reflected wave, depends on the position and the characteristics of the target, $r(t)$ is the time-varying distance of the object from the radar and c is the velocity of light. The time-varying distance of the object can be expanded in a Taylor series as

$$r(t) = r_0 + \dot{r}t + \frac{1}{2!}\ddot{r}t^2 + \frac{1}{3!}\dddot{r}t^3 + \cdots \tag{1.14}$$

where r_0 is the distance, $\dot{r}$ is the velocity, $\ddot{r}$ is the acceleration etc. Approximating $r(t)$ with the first two terms of the Taylor series expansion we have

$$r(t) \approx r_0 + \dot{r}t \tag{1.15}$$

Substituting Equation (1.15) in Equation (1.13) yields

$$x(t) = A(t)\cos[(\omega_0 - 2\dot{r}\omega_0 / c)t - 2\omega_0 r_0 / c] \tag{1.16}$$

Note that the frequency of reflected wave is shifted by an amount

$$\omega_d = 2\dot{r}\omega_0 / c \tag{1.17}$$

This shift in frequency is known as the Doppler frequency. If the object is moving towards the radar then the distance $r(t)$ is decreasing with time, $\dot{r}$ is negative, and an increase in the frequency is observed. Conversely if the

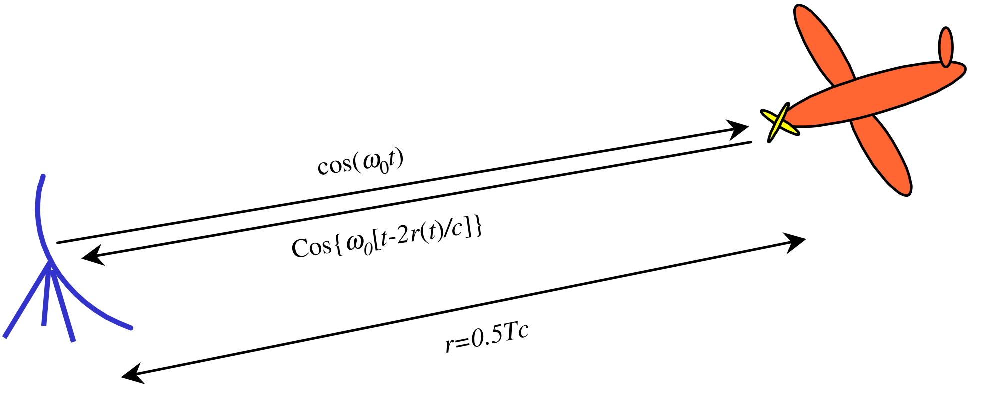

Figure 1.16 Illustration of a radar system.

object is moving away from the radar then the distance $r(t)$ is increasing, $\dot{r}$ is positive, and a decrease in the frequency is observed. Thus the frequency analysis of the reflected signal can reveal information on the direction and speed of the object. The distance r_0 is given by

$$r_0 = 0.5T \times c \qquad (1.18)$$

where T is the round-trip time for the signal to hit the object and arrive back at the radar and c is the velocity of light.

1.4 Sampling and Analog–to–Digital Conversion

A digital signal is a sequence of real–valued or complex–valued numbers, representing the fluctuations of an information bearing quantity with time, space or some other variable. The *basic* elementary discrete-time signal is the unit-sample signal $\delta(m)$ defined as

$$\delta(m) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \qquad (1.19)$$

where m is the discrete time index. A digital signal $x(m)$ can be expressed as the sum of a number of amplitude-scaled and time-shifted unit samples as

$$x(m) = \sum_{k=-\infty}^{\infty} x(k)\delta(m-k) \qquad (1.20)$$

Figure 1.17 illustrates a discrete-time signal. Many random processes, such as speech, music, radar and sonar generate signals that are continuous in

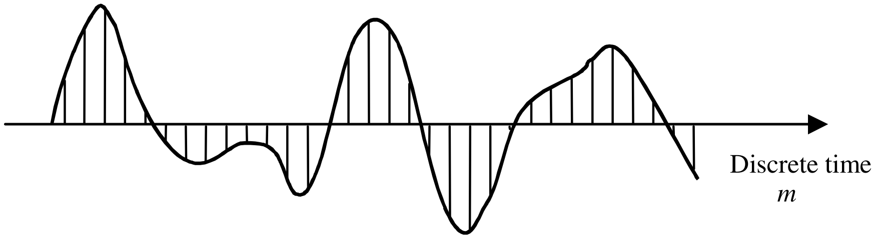

Figure 1.17 A discrete-time signal and its envelope of variation with time.

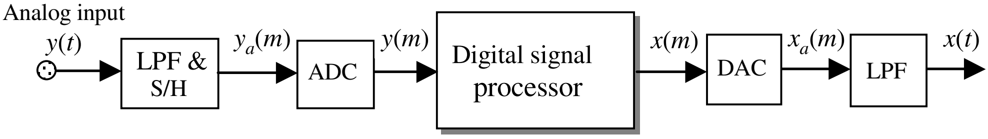

Figure 1.18 Configuration of a digital signal processing system.

time and continuous in amplitude. Continuous signals are termed analog because their fluctuations with time are analogous to the variations of the signal source. For digital processing, analog signals are sampled, and each sample is converted into an *n*-bit digit. The digitisation process should be performed such that the original signal can be recovered from its digital version with no loss of information, and with as high a fidelity as is required in an application. Figure 1.18 illustrates a block diagram configuration of a digital signal processor with an analog input. The low-pass filter removes out–of–band signal frequencies above a pre-selected range. The sample–and–hold (S/H) unit periodically samples the signal to convert the continuous-time signal into a discrete-time signal.

The analog–to–digital converter (ADC) maps each continuous amplitude sample into an *n*-bit digit. After processing, the digital output of the processor can be converted back into an analog signal using a digital–to–analog converter (DAC) and a low-pass filter as illustrated in Figure 1.18.

1.4.1 Time-Domain Sampling and Reconstruction of Analog Signals

The conversion of an analog signal to a sequence of *n*-bit digits consists of two basic steps of sampling and quantisation. The sampling process, when performed with sufficiently high speed, can capture the fastest fluctuations of the signal, and can be a loss-less operation in that the analog signal can be recovered through interpolation of the sampled sequence as described in Chapter 10. The quantisation of each sample into an *n*-bit digit, involves some irrevocable error and possible loss of information. However, in practice the quantisation error can be made negligible by using an appropriately high number of bits as in a digital audio hi-fi. A sampled signal can be modelled as the product of a continuous-time signal $x(t)$ and a periodic impulse train $p(t)$ as

$$
\begin{aligned}
x_{\text{sampled}}(t) &= x(t)\,p(t) \\
&= \sum_{m=-\infty}^{\infty} x(t)\,\delta(t - mT_s)
\end{aligned}
\tag{1.21}
$$

where T_s is the sampling interval and the sampling function $p(t)$ is defined as

$$
p(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_s) \tag{1.22}
$$

The spectrum $P(f)$ of the sampling function $p(t)$ is also a periodic impulse train given by

$$
P(f) = \sum_{k=-\infty}^{\infty} \delta(f - kF_s) \tag{1.23}
$$

where $F_s = 1/T_s$ is the sampling frequency. Since multiplication of two time-domain signals is equivalent to the convolution of their frequency spectra we have

$$
X_{\text{sampled}}(f) = FT[x(t).p(t)] = X(f) * P(f) = \sum_{k=-\infty}^{\infty} \delta(f - kF_s) \tag{1.24}
$$

where the operator FT[.] denotes the Fourier transform. In Equation (1.24) the convolution of a signal spectrum $X(f)$ with each impulse $\delta(f - kF_s)$, shifts $X(f)$ and centres it on kF_s. *Hence, as expressed in Equation (1.24), the sampling of a signal* $x(t)$ *results in a periodic repetition of its spectrum* $X(f)$ *centred on frequencies* $0, \pm F_s, \pm 2F_s, \ldots$. When the sampling frequency is higher than twice the maximum frequency content of the signal, then the repetitions of the signal spectra are separated as shown in Figure 1.19. In this case, the analog signal can be recovered by passing the sampled signal through an analog low-pass filter with a cut-off frequency of F_s. If the sampling frequency is less than $2F_s$, then the adjacent repetitions of the spectrum overlap and the original spectrum cannot be recovered. The distortion, due to an insufficiently high sampling rate, is irrevocable and is known as *aliasing*. This observation is the basis of the *Nyquist sampling theorem* which states: a band-limited continuous-time signal, with a highest

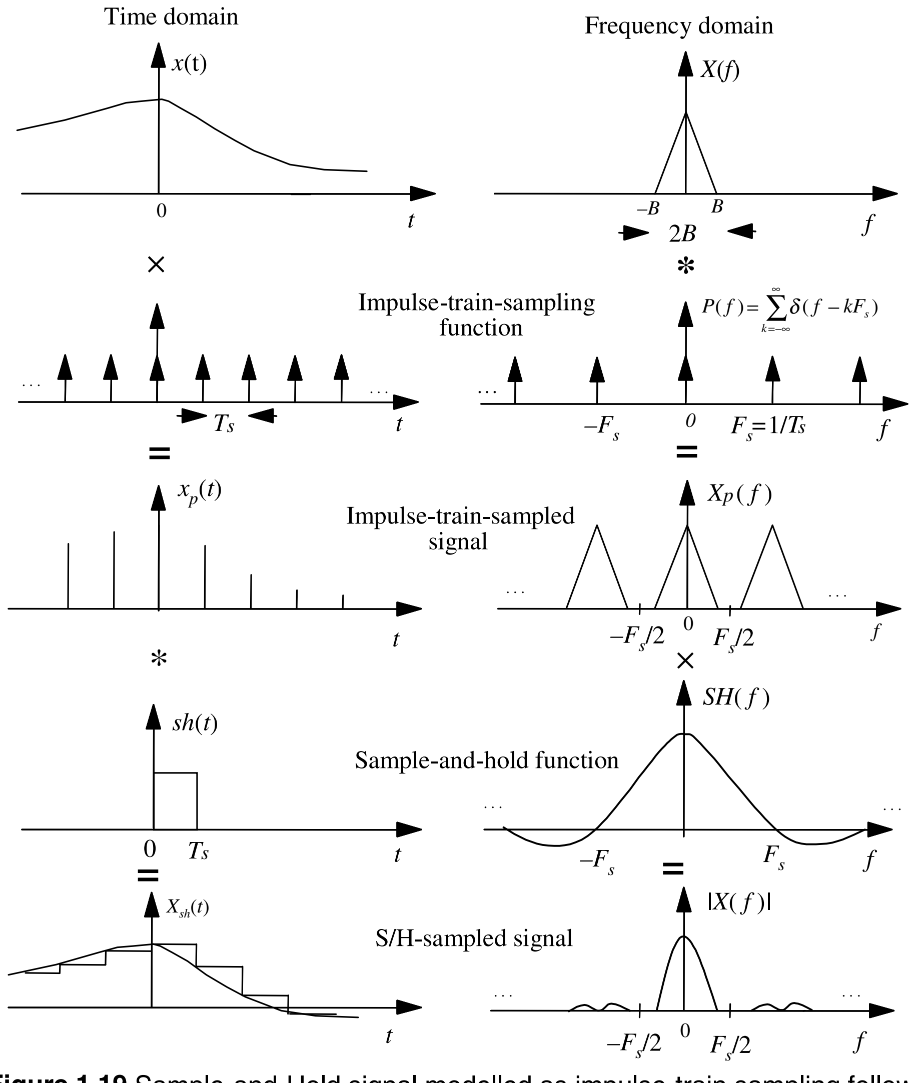

Figure 1.19 Sample-and-Hold signal modelled as impulse-train sampling followed by convolution with a rectangular pulse.

frequency content (bandwidth) of B Hz, can be recovered from its samples provided that the sampling speed $F_s > 2B$ samples per second.

In practice sampling is achieved using an electronic switch that allows a capacitor to charge up or down to the level of the input voltage once every T_s seconds as illustrated in Figure 1.20. The sample-and-hold signal can be modelled as the output of a filter with a rectangular impulse response, and with the impulse–train–sampled signal as the input as illustrated in Figure1.19.

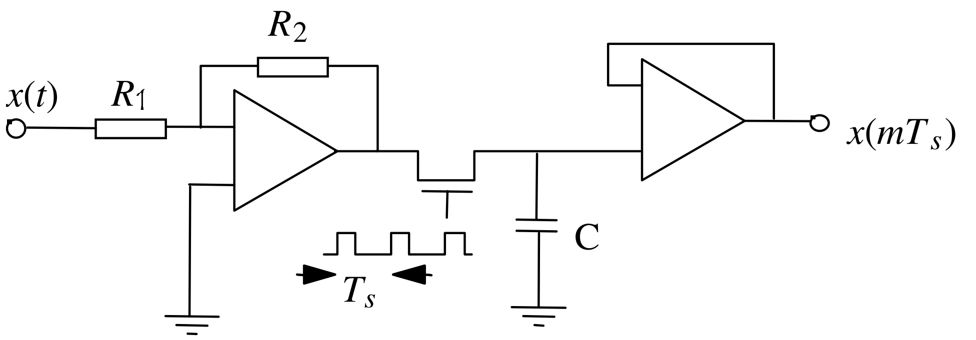

Figure 1.20 A simplified sample-and-hold circuit diagram.

1.4.2 Quantisation

For digital signal processing, continuous-amplitude samples from the sample-and-hold are quantised and mapped into n-bit binary digits. For quantisation to n bits, the amplitude range of the signal is divided into 2^n discrete levels, and each sample is quantised to the nearest quantisation level, and then mapped to the binary code assigned to that level. Figure 1.21 illustrates the quantisation of a signal into 4 discrete levels. Quantisation is a many-to-one mapping, in that all the values that fall within the continuum of a quantisation band are mapped to the centre of the band. The mapping between an analog sample $x_a(m)$ and its quantised value $x(m)$ can be expressed as

$$x(m)=Q[x_a(m)] \tag{1.25}$$

where $Q[\cdot]$ is the quantising function.

The performance of a quantiser is measured by signal–to–quantisation noise ratio SQNR per bit. The quantisation noise is defined as

$$e(m)=x(m)-x_a(m) \tag{1.26}$$

Now consider an n-bit quantiser with an amplitude range of $\pm V$ volts. The quantisation step size is $\Delta=2V/2^n$. Assuming that the quantisation noise is a zero-mean uniform process with an amplitude range of $\pm\Delta/2$ we can express the noise power as

$$\mathcal{E}\left[e^2(m)\right]=\int_{-\Delta/2}^{\Delta/2} f_E(e(m))\,e^2(m)\,de(m)=\frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2(m)\,de(m)$$
$$=\frac{\Delta^2}{12}=\frac{V^2 2^{-2n}}{3} \tag{1.27}$$

where $f_E(e(m))=1/\Delta$ is the uniform probability density function of the noise. Using Equation (1.27) he signal–to–quantisation noise ratio is given by

$$SQNR(n)=10\log_{10}\left(\frac{\mathcal{E}[x^2(m)]}{\mathcal{E}[e^2(m)]}\right)=10\log_{10}\left(\frac{P_{\text{Signal}}}{V^2 2^{-2n}/3}\right)$$
$$=10\log_{10}3-10\log_{10}\left(\frac{V^2}{P_{\text{Signal}}}\right)+10\log_{10}2^{2n} \tag{1.28}$$
$$=4.77-\alpha+6n$$

where P_{signal} is the mean signal power, and α is the ratio in decibels of the peak signal power V^2 to the mean signal power P_{signal}. Therefore, from Equation (1.28) every additional bit in an analog to digital converter results in 6 dB improvement in signal–to–quantisation noise ratio.

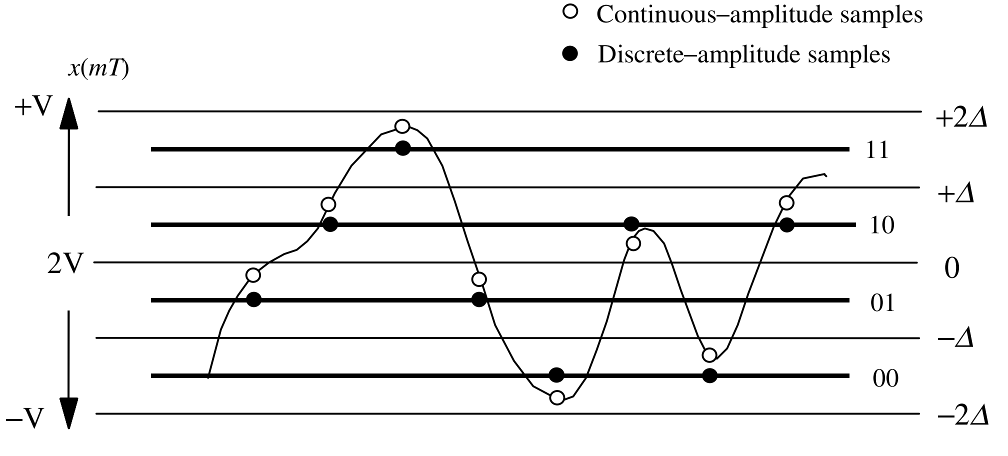

Figure 1.21 Offset-binary scalar quantisation

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