Advanced Digital Signal Processing and Noise Reduction, Second Edition.
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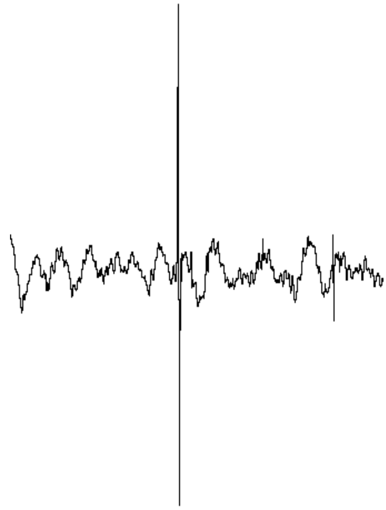

IMPULSIVE NOISE

Impulsive noise consists of relatively short duration "on/off" noise pulses, caused by a variety of sources, such as switching noise, adverse channel environments in a communication system, dropouts or surface degradation of audio recordings, clicks from computer keyboards, etc. An impulsive noise filter can be used for enhancing the quality and intelligibility of noisy signals, and for achieving robustness in pattern recognition and adaptive control systems. This chapter begins with a study of the frequency/time characteristics of impulsive noise, and then proceeds to consider several methods for statistical modelling of an impulsive noise process. The classical method for removal of impulsive noise is the median filter. However, the median filter often results in some signal degradation. For optimal performance, an impulsive noise removal system should utilise (a) the distinct features of the noise and the signal in the time and/or frequency domains, (b) the statistics of the signal and the noise processes, and (c) a model of the physiology of the signal and noise generation. We describe a model-based system that detects each impulsive noise, and then proceeds to replace the samples obliterated by an impulse. We also consider some methods for introducing robustness to impulsive noise in parameter estimation.

12.1 Impulsive Noise

In this section, first the mathematical concepts of an analog and a digital impulse are introduced, and then the various forms of real impulsive noise in communication systems are considered.

The mathematical concept of an analog impulse is illustrated in Figure 12.1. Consider the unit-area pulse $p(t)$ shown in Figure 12.1(a). As the pulse width Δ tends to zero, the pulse tends to an impulse. The impulse function shown in Figure 12.1(b) is defined as a pulse with an infinitesimal time width as

$$\delta(t) = \underset{\Delta \to 0}{\text{limit}}\ p(t) = \begin{cases} 1/\Delta, & |t| \le \Delta/2 \\ 0, & |t| > \Delta/2 \end{cases} \tag{12.1}$$

The integral of the impulse function is given by

$$\int_{-\infty}^{\infty} \delta(t)\, dt = \Delta \times \frac{1}{\Delta} = 1 \tag{12.2}$$

The Fourier transform of the impulse function is obtained as

$$\Delta(f) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt = e^{0} = 1 \tag{12.3}$$

where f is the frequency variable. The impulse function is used as a *test function* to obtain the impulse response of a system. This is because as

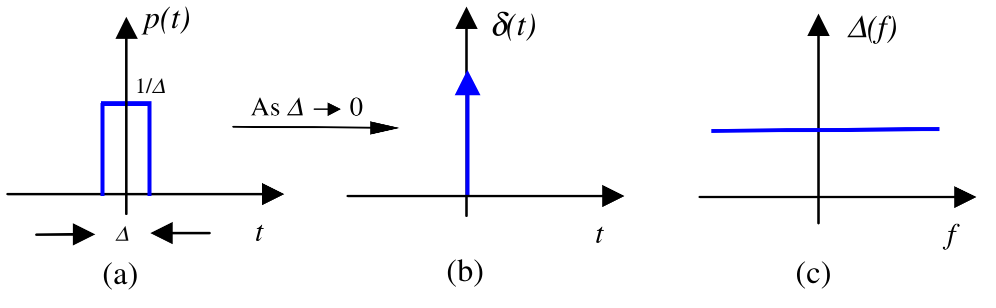

Figure 12.1 (a) A unit-area pulse, (b) The pulse becomes an impulse as $\Delta \to 0$, (c) The spectrum of the impulse function.

shown in Figure 12.1(c), *an impulse is a spectrally rich signal containing all frequencies in equal amounts*.

A digital impulse $\delta(m)$, shown Figure 12.2(a), is defined as a signal with an "on" duration of one sample, and is expressed as:

$$\delta(m)=\begin{cases} 1, & m=0 \\ 0, & m\neq 0 \end{cases} \tag{12.4}$$

where the variable *m* designates the discrete-time index. Using the Fourier transform relation, the frequency spectrum of a digital impulse is given by

$$\Delta(f)=\sum_{m=-\infty}^{\infty}\delta(m)e^{-j2\pi fm}=1.0,\quad -\infty<f<\infty \tag{12.5}$$

In communication systems, real impulsive-type noise has a duration that is normally more than one sample long. For example, in the context of audio signals, short-duration, sharp pulses, of up to 3 milliseconds (60 samples at a 20 kHz sampling rate) may be considered as impulsive-type noise. Figures 12.1(b) and 12.1(c) illustrate two examples of short-duration pulses and their respective spectra.

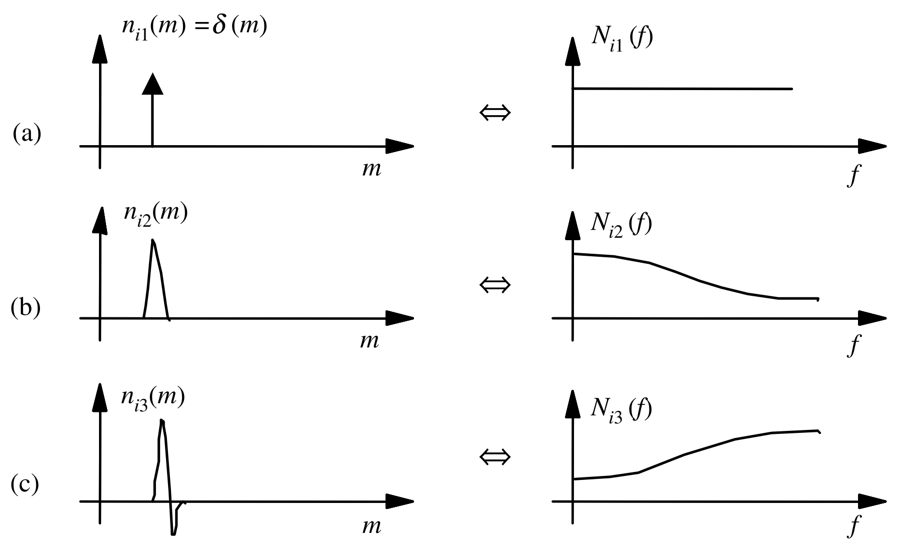

Figure 12.2 Time and frequency sketches of (a) an ideal impulse, and (b) and (c) short-duration pulses.

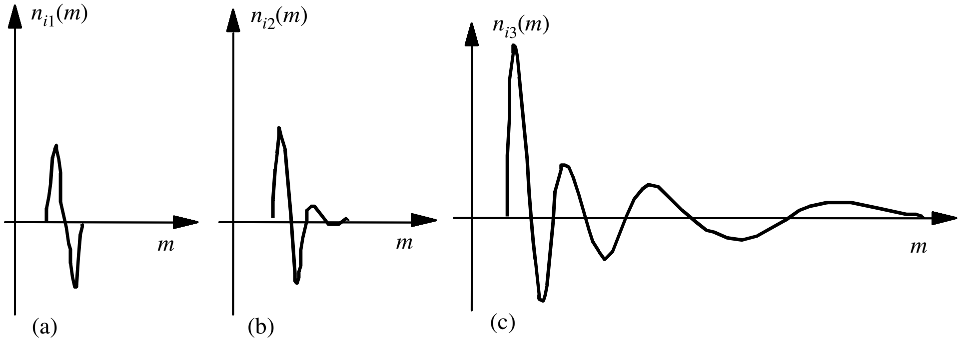

Figure 12.3 Illustration of variations of the impulse response of a non-linear system with increasing amplitude of the impulse.

In a communication system, an impulsive noise originates at some point in time and space, and then propagates through the channel to the receiver. The received noise is shaped by the channel, and can be considered as the channel impulse response. In general, the characteristics of a communication channel may be linear or non-linear, stationary or time varying. Furthermore, many communication systems, in response to a large-amplitude impulse, exhibit a nonlinear characteristic.

Figure 12.3 illustrates some examples of impulsive noise, typical of those observed on an old gramophone recording. In this case, the communication channel is the playback system, and may be assumed time-invariant. The figure also shows some variations of the channel characteristics with the amplitude of impulsive noise. These variations may be attributed to the non-linear characteristics of the playback mechanism.

An important consideration in the development of a noise processing system is the choice of an appropriate domain (time or the frequency) for signal representation. The choice should depend on the specific objective of the system. In signal restoration, the objective is to separate the noise from the signal, and the representation domain must be the one that emphasises the distinguishing features of the signal and the noise. Impulsive noise is normally more distinct and detectable in the time domain than in the frequency domain, and it is appropriate to use time-domain signal processing for noise detection and removal. In signal classification and parameter estimation, the objective may be to compensate for the average effects of the noise over a number of samples, and in some cases, it may be more appropriate to process the impulsive noise in the frequency domain where the effect of noise is a change in the mean of the power spectrum of the signal.

12.1.1 Autocorrelation and Power Spectrum of Impulsive Noise

Impulsive noise is a non-stationary, binary-state sequence of impulses with random amplitudes and random positions of occurrence. The non-stationary nature of impulsive noise can be seen by considering the power spectrum of a noise process with a few impulses per second: when the noise is absent the process has zero power, and when an impulse is present the noise power is the power of the impulse. Therefore the power spectrum and hence the autocorrelation of an impulsive noise is a binary state, time-varying process. An impulsive noise sequence can be modelled as an amplitude-modulated binary-state sequence, and expressed as

$$n_i(m) = n(m)\, b(m) \tag{12.6}$$

where $b(m)$ is a binary-state random sequence of ones and zeros, and $n(m)$ is a random noise process. Assuming that impulsive noise is an uncorrelated random process, the autocorrelation of impulsive noise may be defined as a binary-state process:

$$r_{nn}(k,m) = \mathcal{E}[n_i(m) n_i(m+k)] = \sigma_n^2\, \delta(k) b(m) \tag{12.7}$$

where $\delta(k)$ is the Kronecker delta function. Since it is assumed that the noise is an uncorrelated process, the autocorrelation is zero for $k \neq 0$, therefore Equation (12.7) may be written as

$$r_{nn}(0,m) = \sigma_n^2\, b(m) \tag{12.8}$$

Note that for a zero-mean noise process, $r_{nn}(0,m)$ is the time-varying binary-state noise power. The power spectrum of an impulsive noise sequence is obtained, by taking the Fourier transform of the autocorrelation function Equation (12.8), as

$$P_{N_I N_I}(f,m) = \sigma_n^2\, b(m) \tag{12.9}$$

In Equation (12.8) and (12.9) the autocorrelation and power spectrum are expressed as binary state functions that depend on the "on/off" state of impulsive noise at time m.

12.2 Statistical Models for Impulsive Noise

In this section, we study a number of statistical models for the characterisation of an impulsive noise process. An impulsive noise sequence $n_i(m)$ consists of short duration pulses of a random amplitude, duration, and time of occurrence, and may be modelled as the output of a filter excited by an amplitude-modulated random binary sequence as

$$n_i(m) = \sum_{k=0}^{P-1} h_k n(m-k) b(m-k) \tag{12.10}$$

Figure 12.4 illustrates the impulsive noise model of Equation (12.10). In Equation (12.10) $b(m)$ is a binary-valued random sequence model of the time of occurrence of impulsive noise, $n(m)$ is a continuous-valued random process model of impulse amplitude, and $h(m)$ is the impulse response of a filter that models the duration and shape of each impulse. Two important statistical processes for modelling impulsive noise as an amplitude-modulated binary sequence are the Bernoulli-Gaussian process and the Poisson–Gaussian process, which are discussed next.

12.2.1 Bernoulli–Gaussian Model of Impulsive Noise

In a Bernoulli-Gaussian model of an impulsive noise process, the random time of occurrence of the impulses is modelled by a binary Bernoulli process $b(m)$ and the amplitude of the impulses is modelled by a Gaussian

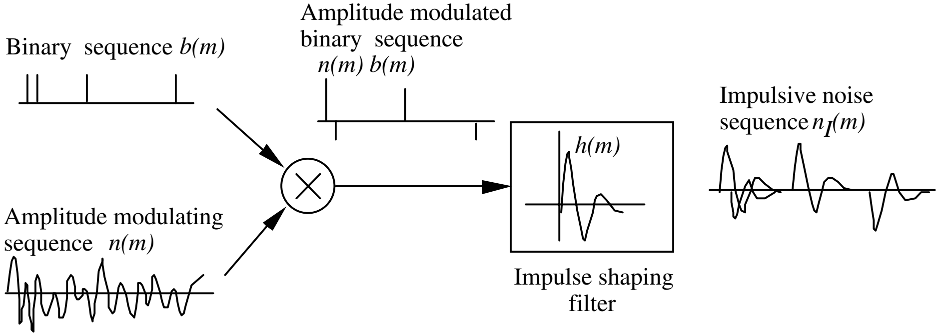

Figure 12.4 Illustration of an impulsive noise model as the output of a filter excited by an amplitude-modulated binary sequence.

process $n(m)$. A Bernoulli process $b(m)$ is a binary-valued process that takes a value of "1" with a probability of α and a value of "0" with a probability of $1-\alpha$. The probability mass function of a Bernoulli process is given by

$$P_B\big(b(m)\big)=\begin{cases}\alpha & \text{for} \quad b(m)=1\\ 1-\alpha & \text{for} \quad b(m)=0.\end{cases} \tag{12.11}$$

A Bernoulli process has a mean

$$\mu_b = \mathcal{E}\big[(b(m))\big]=\alpha \tag{12.12}$$

and a variance

$$\sigma_b^2 = \mathcal{E}\Big[\big(b(m)-\mu_b\big)^2\Big]=\alpha(1-\alpha) \tag{12.13}$$

A zero-mean Gaussian pdf model of the random amplitudes of impulsive noise is given by

$$f_N\big(n(m)\big)=\frac{1}{\sqrt{2\pi}\,\sigma_n}\exp\left[-\frac{n^2(m)}{2\sigma_n^2}\right] \tag{12.14}$$

where σ_n^2 is the variance of the noise amplitude. In a Bernoulli–Gaussian model the probability density function of an impulsive noise $n_i(m)$ is given by

$$f_N^{BG}\big(n_i(m)\big)=(1-\alpha)\delta\big(n_i(m)\big)+\alpha f_N\big(n_i(m)\big) \tag{12.15}$$

where $\delta\big(n_i(m)\big)$ is the Kronecker delta function. Note that the function $f_N^{BG}\big(n_i(m)\big)$ is a mixture of a discrete probability mass function $\delta\big(n_i(m)\big)$ and a continuous probability density function $f_N\big(n_i(m)\big)$.

An alternative model for impulsive noise is a binary-state Gaussian process (Section 2.5.4), with a low-variance state modelling the absence of impulses and a relatively high-variance state modelling the amplitude of impulsive noise.

12.2.2 Poisson–Gaussian Model of Impulsive Noise

In a Poisson–Gaussian model the probability of occurrence of an impulsive noise event is modelled by a Poisson process, and the distribution of the random amplitude of impulsive noise is modelled by a Gaussian process. The Poisson process, described in Chapter 2, is a random event-counting process. In a Poisson model, the probability of occurrence of k impulsive noise in a time interval of T is given by

$$P(k,T)=\frac{(\lambda T)^k}{k!}e^{-\lambda T} \tag{12.16}$$

where λ is a rate function with the following properties:

$$\begin{aligned} &Prob(\text{one impulse in a small time interval } \Delta t)=\lambda\Delta t \\ &Prob(\text{zero impulse in a small time interval } \Delta t)=1-\lambda\Delta t \end{aligned} \tag{12.17}$$

It is assumed that no more than one impulsive noise can occur in a time interval Δt. In a Poisson–Gaussian model, the pdf of an impulsive noise $n_i(m)$ in a small time interval of Δt is given by

$$f_{N_I}^{PG}\big(n_i(m)\big)=(1-\lambda\Delta t)\,\delta\big(n_i(m)\big)+\lambda\Delta t\, f_N\big(n_i(m)\big) \tag{12.18}$$

where $f_N\big(n_i(m)\big)$ is the Gaussian pdf of Equation (12.14).

12.2.3 A Binary-State Model of Impulsive Noise

An impulsive noise process may be modelled by a binary-state model as shown in Figure 12.4. In this binary model, the state S_0 corresponds to the "off" condition when impulsive noise is absent; in this state, the model emits zero-valued samples. The state S_1 corresponds to the "on" condition; in this state the model emits short-duration pulses of random amplitude and duration. The probability of a transition from state S_i to state S_j is denoted by a_{ij}. In its simplest form, as shown in Figure 12.5, the model is memoryless, and the probability of a transition to state S_i is independent of the current state of the model. In this case, the probability that at time $t+1$

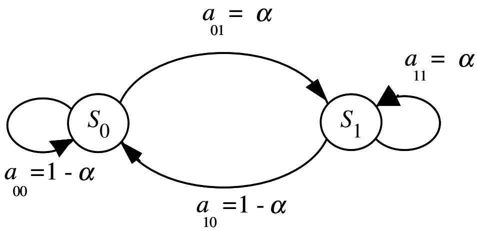

Figure 12.5 A binary-state model of an impulsive noise generator.

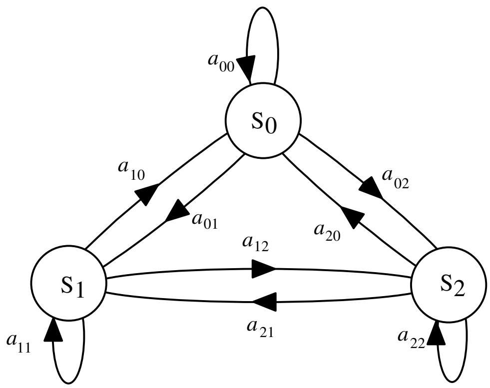

Figure 12.6 A 3-state model of impulsive noise and the decaying oscillations that often follow the impulses.

the signal is in the state S_0 is independent of the state at time t, and is given by

$$P\big(s(t+1) = S_0 \big| s(t) = S_0\big) = P\big(s(t+1) = S_0 \big| s(t) = S_1\big) = 1 - \alpha \qquad (12.19)$$

where s_t denotes the state at time t. Likewise, the probability that at time t+1 the model is in state S_1 is given by

$$P\big(s(t+1) = S_1 \big| s(t) = S_0\big) = P\big(s(t+1) = S_1 \big| s(t) = S_1\big) = \alpha \qquad (12.20)$$

In a more general form of the binary-state model, a Markovian state-transition can model the dependencies in the noise process. The model then becomes a 2-state hidden Markov model considered in Chapter 5.
In one of its simplest forms, the state S_1 emits samples from a zero-mean Gaussian random process. The impulsive noise model in state S_1 can be configured to accommodate a variety of impulsive noise of different shapes,

durations and pdfs. A practical method for modelling a variety of impulsive noise is to use a code book of M prototype impulsive noises, and their associated probabilities $[(n_{i1}, p_{i1}), (n_{i2}, p_{i2}), ..., (n_{iM}, p_{iM})]$, where p_j denotes the probability of impulsive noise of the type n_j. The impulsive noise code book may be designed by classification of a large number of "training" impulsive noises into a relatively small number of clusters. For each cluster, the average impulsive noise is chosen as the representative of the cluster. The number of impulses in the cluster of type j divided by the total number of impulses in all clusters gives p_j, the probability of an impulse of type j.

Figure 12.6 shows a three-state model of the impulsive noise and the decaying oscillations that might follow the noise. In this model, the state S_0 models the absence of impulsive noise, the state S_1 models the impulsive noise and the state S_2 models any oscillations that may follow a noise pulse.

12.2.4 Signal to Impulsive Noise Ratio

For impulsive noise the average signal to impulsive noise ratio, averaged over an entire noise sequence including the time instances when the impulses are absent, depends on two parameters: (a) the average power of each impulsive noise, and (b) the rate of occurrence of impulsive noise. Let P_{impulse} denote the average power of each impulse, and P_{signal} the signal power. We may define a "local" time-varying signal to impulsive noise ratio as

$$SINR(m) = \frac{P_{\text{signal}}(m)}{P_{\text{impulse}}\, b(m)} \tag{12.21}$$

The average signal to impulsive noise ratio, assuming that the parameter α is the fraction of signal samples contaminated by impulsive noise, can be defined as

$$SINR = \frac{P_{\text{signal}}}{\alpha\, P_{\text{impulse}}} \tag{12.22}$$

Note that from Equation (12.22), for a given signal power, there are many pair of values of α and P_{Impulse} that can yield the same average SINR.

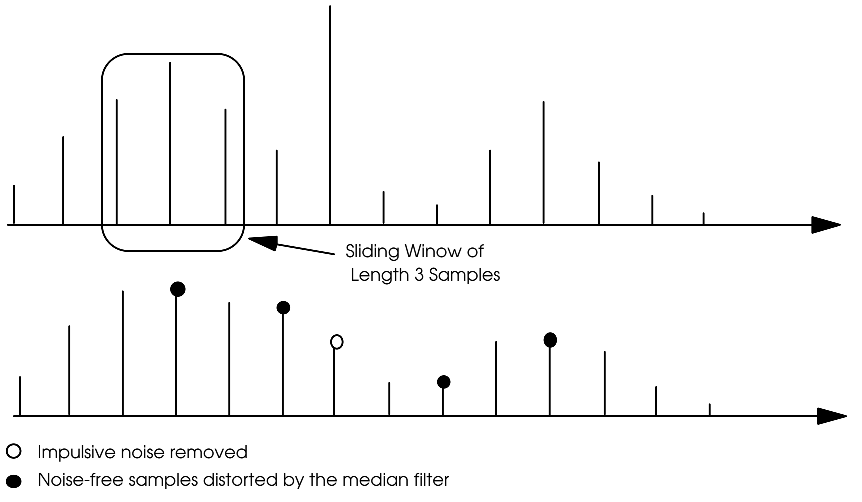

Figure 12.7 Input and output of a median filter. Note that in addition to suppressing the impulsive outlier, the filter also distorts some genuine signal components.

12.3 Median Filters

The classical approach to removal of impulsive noise is the median filter. The median of a set of samples $\{x(m)\}$ is a member of the set $x_{\text{med}}(m)$ such that; half the population of the set are larger than $x_{\text{med}}(m)$ and half are smaller than $x_{\text{med}}(m)$. Hence the median of a set of samples is obtained by sorting the samples in the ascending or descending order, and then selecting the mid-value. In median filtering, a window of predetermined length slides sequentially over the signal, and the mid-sample within the window is replaced by the median of all the samples that are inside the window, as illustrated in Figure 12.7.

The output $\hat{x}(m)$ of a median filter with input $y(m)$ and a median window of length $2K+1$ samples is given by

$$\begin{aligned}\hat{x}(m) &= y_{\text{med}}(m) \\ &= \text{median}\left[y(m-K),\ldots,y(m),\ldots,\,y(m+K)\right]\end{aligned} \tag{12.23}$$

The median of a set of numbers is a non-linear statistics of the set, with the useful property that it is insensitive to the presence of a sample with an unusually large value, a so-called outlier, in the set. In contrast, the mean, and in particular the variance, of a set of numbers are sensitive to the presence of impulsive-type noise. An important property of median filters, particularly useful in image processing, is that they preserves edges or stepwise discontinuities in the signal. Median filters can be used for removing impulses in an image without smearing the edge information; this is of significant importance in image processing. However, experiments with median filters, for removal of impulsive noise from audio signals, demonstrate that median filters are unable to produce high-quality audio restoration. The median filters cannot deal with "real" impulsive noise, which are often more than one or two samples long. Furthermore, median filters introduce a great deal of processing distortion by modifying genuine signal samples that are mistaken for impulsive noise. The performance of median filters may be improved by employing an adaptive threshold, so that a sample is replaced by the median only if the difference between the sample and the median is above the threshold:

$$\hat{x}(m) = \begin{cases} y(m) & \text{if } \left| y(m) - y_{\text{med}}(m) \right| < k\,\theta(m) \\ y_{\text{med}}(m) & \text{otherwise} \end{cases} \tag{12.24}$$

where $\theta(m)$ is an adaptive threshold that may be related to a robust estimate of the average of $\left| y(m) - y_{\text{med}}(m) \right|$, and k is a tuning parameter. Median filters are not optimal, because they do not make efficient use of prior knowledge of the physiology of signal generation, or a model of the signal and noise statistical distributions. In the following section we describe a autoregressive model-based impulsive removal system, capable of producing high-quality audio restoration.

12.4 Impulsive Noise Removal Using Linear Prediction Models

In this section, we study a model-based impulsive noise removal system. Impulsive disturbances usually contaminate a relatively small fraction α of the total samples. Since a large fraction, $1-\alpha$, of samples remain unaffected by impulsive noise, it is advantageous to locate individual noise pulses, and correct *only* those samples that are distorted. This strategy avoids the unnecessary processing and compromise in the quality of the relatively large fraction of samples that are not disturbed by impulsive noise. The impulsive noise removal system shown in Figure 12.8 consists of two subsystems: a detector and an interpolator. The detector locates the position of each noise pulse, and the interpolator replaces the distorted samples

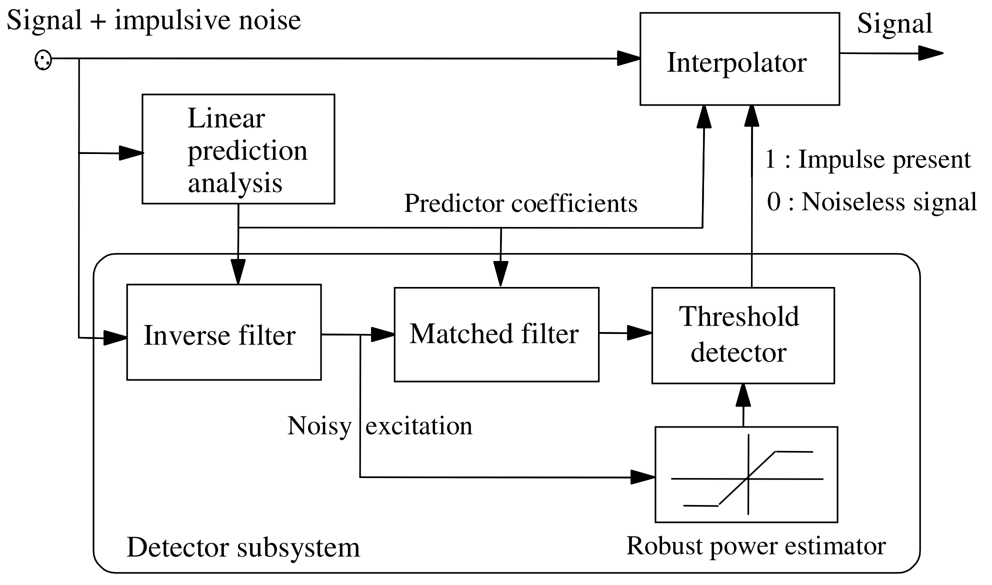

Figure 12.8 Configuration of an impulsive noise removal system incorporating a detector and interpolator subsystems.

using the samples on both sides of the impulsive noise. The detector is composed of a linear prediction analysis system, a matched filter and a threshold detector. The output of the detector is a binary switch and controls the interpolator. A detector output of "0" signals the absence of impulsive noise and the interpolator is bypassed. A detector output of "1" signals the presence of impulsive noise, and the interpolator is activated to replace the samples obliterated by noise.

12.4.1 Impulsive Noise Detection

A simple method for detection of impulsive noise is to employ an amplitude threshold, and classify those samples with an amplitude above the threshold as noise. This method works fairly well for relatively large-amplitude impulses, but fails when the noise amplitude falls below the signal. Detection can be improved by utilising the characteristic differences between the impulsive noise and the signal. An impulsive noise, or a short-duration pulse, introduces uncharacteristic discontinuity in a correlated signal. The discontinuity becomes more detectable when the signal is differentiated. The differentiation (or, for digital signals, the differencing) operation is equivalent to decorrelation or spectral whitening. In this section, we describe a model-based decorrelation method for improving impulsive noise detectability. The correlation structure of the signal is modelled by a linear predictor, and the process of decorrelation is achieved by inverse filtering. Linear prediction and inverse filtering are covered in Chapter 8. Figure 12.9 shows a model for a noisy signal. The noise-free signal $x(m)$ is described by a linear prediction model as

$$x(m)=\sum_{k=1}^{P} a_k\, x(m-k)+e(m) \tag{12.25}$$

where $\boldsymbol{a}=[a_1,\ a_2,\ \ldots,a_P]^{\mathrm{T}}$ is the coefficient vector of a linear predictor of order P, and the excitation $e(m)$ is either a noise-like signal or a mixture of a random noise and a quasi-periodic train of pulses as illustrated in Figure 12.9. The impulsive noise detector is based on the observation that linear predictors are a good model of the correlated signals but not the uncorrelated binary-state impulsive-type noise. Transforming the noisy signal $y(m)$ to the excitation signal of the predictor has the following effects:

(a) The scale of the signal amplitude is reduced to almost that of the original excitation signal, whereas the scale of the noise amplitude remains unchanged or increases.
(b) The signal is decorrelated, whereas the impulsive noise is smeared and transformed to a scaled version of the impulse response of the inverse filter.

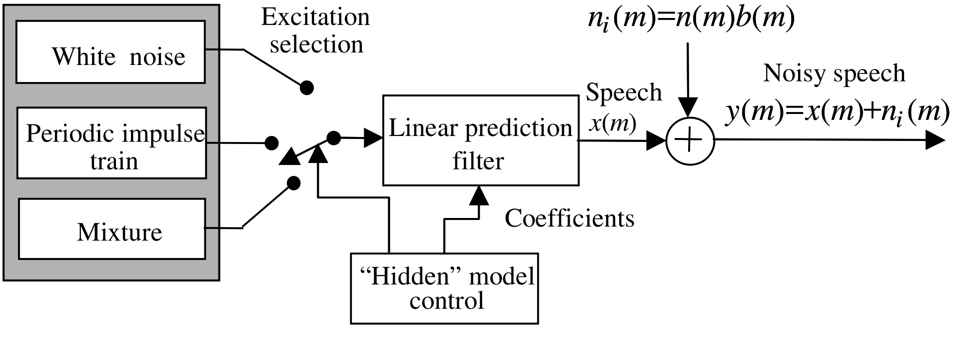

Figure 12.9 Noisy speech model. The signal is modelled by a linear predictor. Impulsive noise is modelled as an amplitude-modulated binary-state process.

Both effects improve noise delectability. Speech or music is composed of random excitations spectrally shaped and amplified by the resonances of vocal tract or the musical instruments. The excitation is more random than the speech, and often has a much smaller amplitude range. The improvement in noise pulse detectability obtained by inverse filtering can be substantial and depends on the time-varying correlation structure of the signal. Note that this method effectively reduces the impulsive noise detection to the problem of separation of outliers from a random noise excitation signal using some optimal thresholding device.

12.4.2 Analysis of Improvement in Noise Detectability

In the following, the improvement in noise detectability that results from inverse filtering is analysed. Using Equation (12.25), we can rewrite a noisy signal model as

$$\begin{aligned} y(m) &= x(m)+n_i(m) \\ &= \sum_{k=1}^{P} a_k\, x(m-k)+e(m)+n_i(m) \end{aligned} \tag{12.26}$$

where $y(m)$, $x(m)$ and $n_i(m)$ are the noisy signal, the signal and the noise respectively. Using an estimate $\hat{\boldsymbol{a}}$ of the predictor coefficient vector $\boldsymbol{a}$, the noisy signal $y(m)$ can be inverse-filtered and transformed to the noisy excitation signal $v(m)$ as

$$\begin{aligned} v(m) &= y(m)-\sum_{k=1}^{P} \hat{a}_k\, y(m-k) \\ &= x(m)+n_i(m)-\sum_{k=1}^{P} (a_k-\tilde{a}_k)[x(m-k)+n_i(m-k)] \end{aligned} \tag{12.27}$$

where $\tilde{a}_k$ is the error in the estimate of the predictor coefficient. Using Equation (12.25) Equation (12.27) can be rewritten in the following form:

$$v(m)=e(m)+n_i(m)+\sum_{k=1}^{P}\tilde{a}_k x(m-k)-\sum_{k=1}^{P}\hat{a}_k n_i(m-k) \tag{12.28}$$

From Equation (12.28) there are essentially three terms that contribute to the noise in the excitation sequence:

(a) the impulsive disturbance $n_i(m)$ which is usually the dominant term;
(b) the effect of the past P noise samples, smeared to the present time by the action of the inverse filtering, $\sum \hat{a}_k n_i(m-k)$;
(c) the increase in the variance of the excitation signal, caused by the error in the parameter vector estimate, and expressed by the term $\sum \tilde{a}_k x(m-k)$.

The improvement resulting from the inverse filter can be formulated as follows. The impulsive noise to signal ratio for the noisy signal is given by

$$\frac{\text{impulsive noise power}}{\text{signal power}} = \frac{\mathcal{E}[n_i^2(m)]}{\mathcal{E}[x^2(m)]} \tag{12.29}$$

where $\mathcal{E}[\cdot]$ is the expectation operator. Note that in impulsive noise detection, the signal of interest is the impulsive noise to be detected from the accompanying signal. Assuming that the dominant noise term in the noisy excitation signal $v(m)$ is the impulse $n_i(m)$, the impulsive noise to excitation signal ratio is given by

$$\frac{\text{impulsive noise power}}{\text{excitation power}} = \frac{\mathcal{E}[n_i^2(m)]}{\mathcal{E}[e^2(m)]} \tag{12.30}$$

The overall gain in impulsive noise to signal ratio is obtained, by dividing Equations (12.29) and (12.30), as

$$\frac{\mathcal{E}[x^2(m)]}{\mathcal{E}[e^2(m)]} = \text{gain} \tag{12.31}$$

This simple analysis demonstrates that the improvement in impulsive noise detectability depends on the power amplification characteristics, due to resonances, of the linear predictor model. For speech signals, the scale of the amplitude of the noiseless speech excitation is on the order of 10^{-1} to 10^{-4} of that of the speech itself; therefore substantial improvement in impulsive noise detectability can be expected through inverse filtering of the noisy speech signals.

Figure 12.10 illustrates the effect of inverse filtering in improving the detectability of impulsive noise. The inverse filtering has the effect that the signal $x(m)$ is transformed to an uncorrelated excitation signal $e(m)$, whereas the impulsive noise is smeared to a scaled version of the inverse filter impulse response [1, $-a_1$, ...,$-a_P$], as indicated by the term $\sum \hat{a}_k n_i(m-k)$ in Equation (12.28). Assuming that the excitation is a white noise Gaussian signal, a filter matched to the inverse filter coefficients may enhance the delectability of the smeared impulsive noise from the excitation signal.

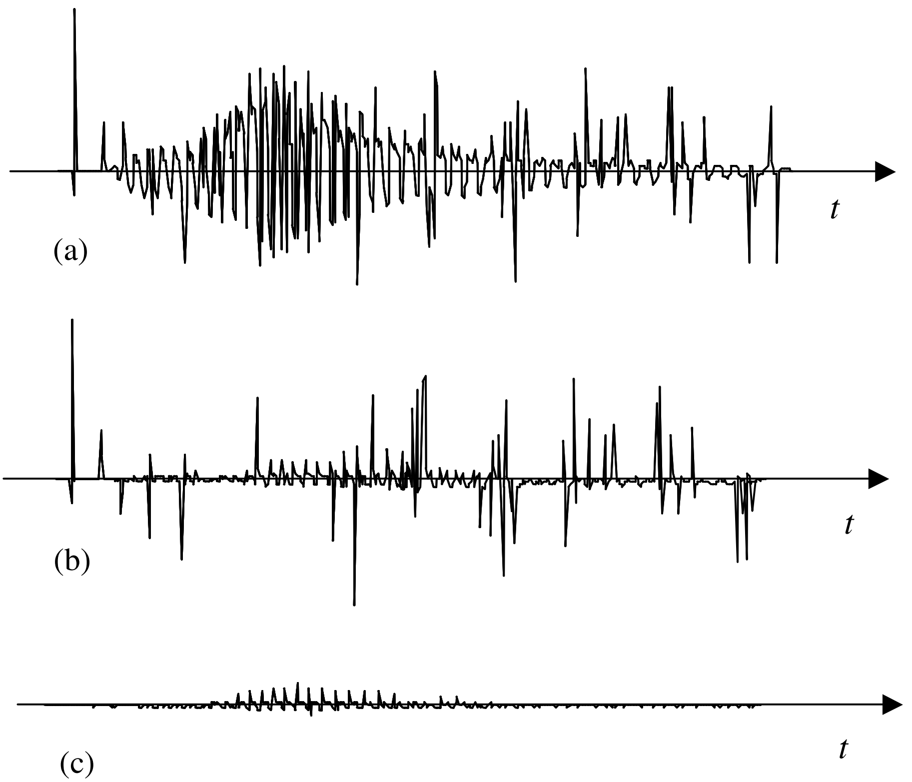

Figure 12.10 Illustration of the effects of inverse filtering on detectability of Impulsive noise: (a) Impulsive noise contaminated speech with 5% impulse contamination at an average SINR of 10dB, (b) Speech excitation of impulse-contaminated speech, and (c) Speech excitation of impulse-free speech.

12.4.3 Two-Sided Predictor for Impulsive Noise Detection

In the previous section, it was shown that impulsive noise detectability can be improved by decorrelating the speech signal. The process of decorrelation can be taken further by the use of a two-sided linear prediction model. The two-sided linear prediction of a sample $x(m)$ is based on the P past samples and the P future samples, and is defined by the equation

$$x(m)=\sum_{k=1}^{P} a_k x(m-k)+\sum_{k=1}^{P} a_{k+P} x(m+k)+e(m) \tag{12.32}$$

where a_k are the two-sided predictor coefficients and $e(m)$ is the excitation signal. All the analysis used for the case of one-sided linear predictor can be extended to the two-sided model. However, the variance of the excitation input of a two-sided model is less than that of the one-sided predictor because in Equation (12.32) the correlations of each sample with the future, as well as the past, samples are modeled. Although Equation (12.32) is a non-causal filter, its inverse, required in the detection subsystem, is causal. The use of a two-sided predictor can result in further improvement in noise detectability.

12.4.4 Interpolation of Discarded Samples

Samples irrevocably distorted by an impulsive noise are discarded and the gap thus left is interpolated. For interpolation imperfections to remain inaudible a high-fidelity interpolator is required. A number of interpolators for replacement of a sequence of missing samples are introduced in Chapter 10. The least square autoregressive (LSAR) interpolation algorithm of Section 10.3.2 produces high-quality results for a relatively small number of missing samples left by an impulsive noise. The LSAR interpolation method is a two-stage process. In the first stage, the available samples on both sides of the noise pulse are used to estimate the parameters of a linear prediction model of the signal. In the second stage, the estimated model parameters, and the samples on both sides of the gap are used to interpolate the missing samples. The use of this interpolator in replacement of audio signals distorted by impulsive noise has produced high-quality results.

12.5 Robust Parameter Estimation

In Figure 12.8, the threshold used for detection of impulsive noise from the excitation signal is derived from a nonlinear robust estimate of the excitation power. In this section, we consider robust estimation of a parameter, such as the signal power, in the presence of impulsive noise.

A *robust* estimator is one that is not over-sensitive to deviations of the input signal from the assumed distribution. In a robust estimator, an input sample with unusually large amplitude has only a limited effect on the estimation results. Most signal processing algorithms developed for adaptive filtering, speech recognition, speech coding, etc. are based on the assumption that the signal and the noise are Gaussian-distributed, and employ a mean square distance measure as the optimality criterion. The mean square error criterion is sensitive to non-Gaussian events such as impulsive noise. A large impulsive noise in a signal can substantially

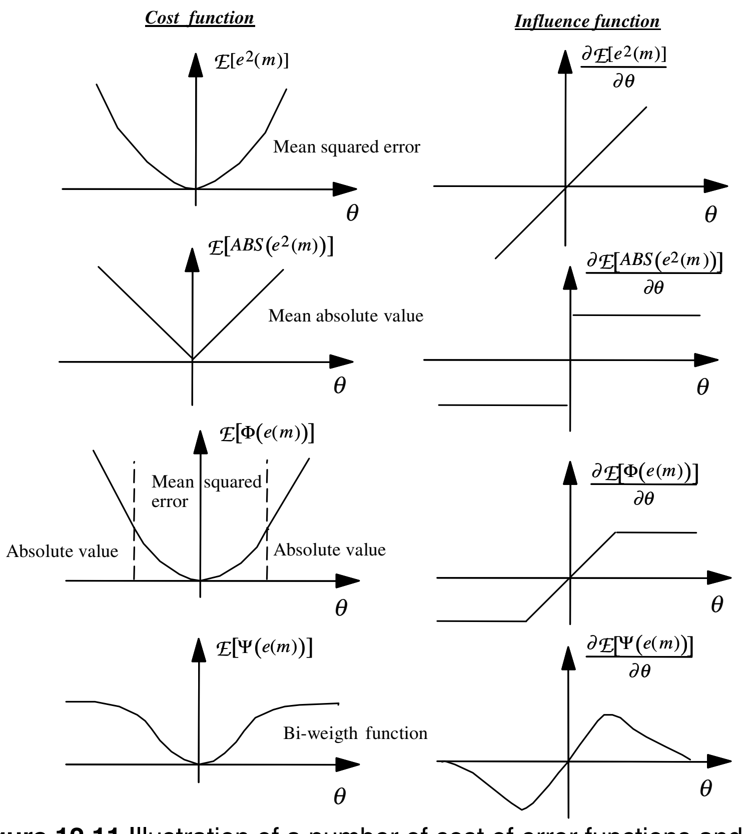

Figure 12.11 Illustration of a number of cost of error functions and the corresponding influence functions.

overshadow the influence of noise-free samples.

Figure 12.11 illustrates the variations of several cost of error functions with a parameter θ. Figure 12.11(a) shows a least square error cost function and its influence function. The influence function is the derivative of the cost function, and, as the name implies, it has a direct influence on the estimation results. It can be seen from the influence function of Figure 12.11(a) that an unbounded sample has an unbounded influence on the estimation results.

A method for introducing robustness is to use a non-linear function and limit the influence of any one sample on the overall estimation results. The absolute value of error is a robust cost function, as shown by the influence function in Figure 12.11(b). One disadvantage of this function is that it is not continuous at the origin. A further drawback is that it does not allow for the fact that, in practice, a large proportion of the samples are not contaminated with impulsive noise, and may well be modelled with Gaussian densities.

Many processes may be regarded as Gaussian for the sample values that cluster about the mean. For such processes, it is desirable to have an influence function that limits the influence of outliers and at the same time is linear and optimal for the large number of relatively small-amplitude samples that may be regarded as Gaussian-distributed. One such function is Huber's function, defined as

$$\psi[e(m)] = \begin{cases} e^2(m) & \text{if } |e(m)| \le k \\ k|e(m)| & \text{otherwise} \end{cases} \tag{12.33}$$

Huber's function, shown in Figure 12.11(c), is a hybrid of the least mean square and the absolute value of error functions. Tukeys bi-weight function, which is a redescending robust objective function, is defined as

$$\psi[e(m)] = \begin{cases} \{1 - [1 - e^2(m)]^3\}/6 & \text{if } |e(m)| \le 1 \\ 1/6 & \text{otherwise} \end{cases} \tag{12.34}$$

As shown in Figure 12.11(d), the influence function is linear for small signal values but introduces attenuation as the signal value exceeds some threshold. The threshold may be obtained from a robust median estimate of the signal power.

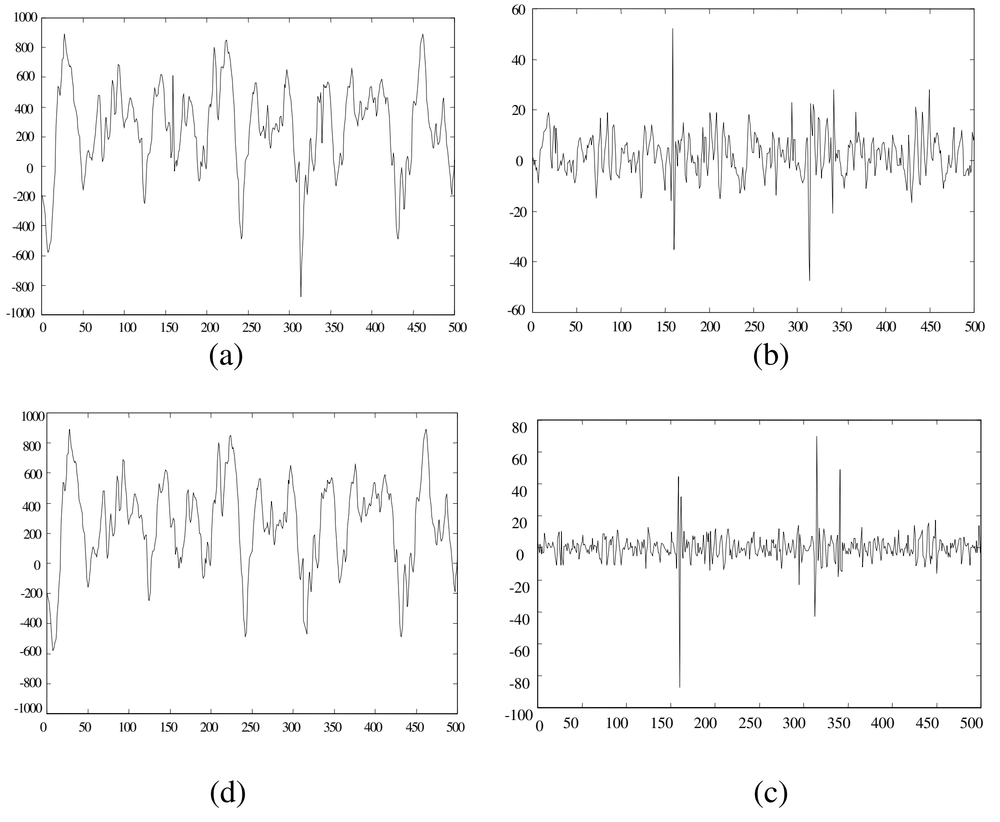

Figure 12.12 (a) A noisy audio signal from a 78 rpm record, (b) Noisy excitation signal, (c) Matched filter output, (d) Restored signal.

12.6 Restoration of Archived Gramophone Records

This Section describes the application of the impulsive noise removal system of Figure 12.8 to the restoration of archived audio records. As the bandwidth of archived recordings is limited to 7–8 kHz, a low-pass, anti-aliasing filter with a cutoff frequency of 8 kHz is used to remove the out of band noise. Playedback signals were sampled at a rate of 20 kHz, and digitised to 16 bits. Figure 12.12(a) shows a 25 ms segment of noisy music and song from an old 78 rpm gramophone record. The impulsive interferences are due to faults in the record stamping process, granularities of the record material or physical damage. This signal is modelled by a predictor of order 20. The excitation signal obtained from the inverse filter and the matched filter output are shown in Figures 12.12(b) and (c) respectively. Close examination of these figures show that some of the ambiguities between the noise pulses and the genuine signal excitation pulses are resolved after matched filtering.

The amplitude threshold for detection of impulsive noise from the excitation signal is adapted on a block basis, and is set to $k\sigma_e^2$, where σ_e^2 is a robust estimate of the excitation power. The robust estimate is obtained by passing the noisy excitation signal through a soft nonlinearity that rejects outliers. The scalar *k* is a tuning parameter; the choice of *k* reflects a trade-off between the hit rate and the false-alarm rate of the detector. As *k* decreases, smaller noise pulses are detected but the false detection rate also increases. When an impulse is detected, a few samples are discarded and replaced by the LSAR interpolation algorithm described in Chapter 10. Figure 12.12(d) shows the signal with the impulses removed. The impulsive noise removal system of Figure 12.8 was successfully applied to restoration of numerous examples of archived gramophone records. The system is also effective in suppressing impulsive noise in examples of noisy telephone conversations.

12.7 Summary

The classic linear time-invariant theory on which many signal processing methods are based is not suitable for dealing with the non-stationary impulsive noise problem. In this chapter, we considered impulsive noise as a random on/off process and studied several stochastic models for impulsive noise, including the Bernoulli–Gaussian model, the Poisson–Gaussian and the hidden Markov model (HMM). The HMM provides a particularly interesting framework, because the theory of HMM studied in Chapter 5 is well developed, and also because the state sequence of an HMM of noise can be used to provide an estimate of the presence or the absence of the noise. By definition, an impulsive noise is a short and sharp event uncharacteristic of the signal that it contaminates. In general, differencing operation enhance the detectibility of impulsive noise. Based on this observation, in Section 12.4, we considered an algorithm based on a linear prediction model of the signal for detection of impulsive noise.

In the next Chapter we expand the materials we considered in this chapter for the modelling, detection, and removal of transient noise pulses.

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