Nonlinear Analysis of Electroencephalogram with Different Brain Functions and Its Application

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Abstract: *To investigate the nonlinear interaction of electroencephalograph (EEG signal), bispectral analysis is used for the study of clinical EEG in different brain functions. The parametric bispectral estimation is proposed in the paper for the purpose of extracting the nonlinear information beyond second order statistics or power spectra involved in the EEG. The EEG signal with normal subjects ubder different brain functional states and the abnormal EEG are analyzed in terms of the bispectral structure. The experimental results show that all spontanuous EEG exhibit obvious quadratic nonlinear interactions presence among the rhythms, but the bispectral pattern of the EEG changes with different functional states of brain.*

Keywords: Bispectral estimation, EEG signal processing, nonlinear analysis.

Topic Catogory: 5.12

1. Introduction

This paper investigates the relation of quadratic nonlinear interactions among the electroencephalograph (EEG) components by employing bispectral techniques. Biological information in the brain is carried by patterns of neural activity that are manifested in electrical fields of potential known as electroencephalograph. Understanding the generation mechanism of these brain electrical activities is very important for the further investigation of brain functions. As a noninvasive testing method, EEG analysis has been playing an key role in the diagnosis of brain diseases and the functional determination of brain. EEG signal processing is an important clinical procedure for diagnosing, monitoring and managing neurological function related to the brain electrical activities in different brain functions, although many modern techniques continuously coming into use. Over the years, the EEG signals has been extensively analyzed by using a great number of digital signal processing techniques because many results have demonstrated that the measurement and processing of EEG signals are clinically significant due to much of useful information related to different patterns of the EEG signals. A variety of methods, such as FFT technique, power spectral analysis and various parametric models, are widely used to detect the information of different EEG signals for many purposes [1,2].

Various methods, however, based on power spectrum or correlation for EEG signal processing start with the assumption of linearity, Gaussianility and minimum phase systems. Spectral analysis is a very useful tool to determine the energy distribution of an EEG signal in frequency domain, but it cannot distinguish nonlinearly coupled frequencies from spontaneously excited signals with the same resonance condition. In other words, all these methods depend on the second-order spectral or second-order statistics. Additional information stored in higher-order spectra (HOS) is therefore ignored in linear analysis of the EEG signal. To overcome the drawback of spectral methods, the bispectral techniques is proposed to exhibit the nonlinearity and extract more information of EEG signal beyond power spectra. There is a body of evidence showing that the EEG signals exhibit typical non-Gaussian and nonlinear behavior in practical application. To understand the nonlinear feature and extract more higher-order information involved in the EEG, this paper proposes the higher-order spectra, particularly bispectrum, for analyzing the EEG signals in different functional states. The parametric bispectral estimate is presented to detect the non-Gaussianility and extract the quadratic nonlinearities. Several EEG signals recorded under different experimental conditions are modeled and examined in terms of the third-order statistics. Both bispectral and cross-bispectral structure is applied to investigate the quadratic nonlinear coupling phenomena of the multichannel EEG signals.

2. Non-Gaussian EEG Model

Recent attention has been given to improve the bispectral estimation to obtain more accurate bispectral structures and extract as many features contained in the signals as possible. Bispectral estimation methods can be grouped into two main categories: the conventional FFT-based techniques and the parametric modelling methods. The FFT-type based bispectral estimation methods requires longer data length, while the parametric bispectral estimation such as the non-Gaussian parametric modeling requires shorter data length to achieve the same accuracy. Since in clinical practice, EEG data is of finite length, parametric modeling is a better approach for bispectral estimation [3-7].

1. *Identification of the Model Parameters*

Consider a non-Gaussian third-order stationary random process $\{x(t)\}$, its kth dimensional cumulant is defined as [25]

$$\begin{aligned} C_{ky}(\tau_1, \tau_2, \cdots, \tau_k) &= E[x(t)x(t+\tau_1)\cdots x(t+\tau_k)] \\ &- E[g(t)g(t+\tau_1)\cdots g(t+\tau_k)] \end{aligned} \quad (1)$$

where the noise $\{g(t)\}$ is a white Gaussian process with zero-mean, and both $\{x(t)\}$ and $\{g(t)\}$ have the same autocovariance. When k equals to two, we can define the third-order cumulant of the signal in discrete form as [22,23]

$$C_{3x}(m,n) = E[x(k)x(k+m)x(k+n)] \quad (2)$$

The corresponding bispectrum is defined as

$$B_x(w_1, w_2) = \sum_m \sum_n C_{3x}(m,n) e^{-j(mw_1 + nw_2)} \quad (3)$$

To obtain the bispectral estimation with high resolution for finite data length, an AR model driven by a non-Gaussian white noise is proposed for the parametric bispectral estimation of the EEG signals. Let EEG data $\{ y(n) \}$ be modeled by a non-Gaussian AR model with order p:

$$y(n) = \sum_{k=1}^{p} a_k y(n-k) + e(n) \quad (4)$$

where $e(n)$ is a white non-Gaussian process with zero mean and third-order stationarity. $E[e^2(n)] = Q$ and $E[e^3(n)] = b \neq 0$. The system given in equation (4) is assumed to be stable and $y(n)$ is statistically independent of $e(n)$ for $k \leq n$. Since the driving non-Gaussian white noise process satisfies third-order stationarity, $y(k)$ is also a third-order stationary process. The third-order cumulant of equation (4) is given by:

$$\begin{aligned} &C_{3y}(-m,-n) = b\delta(m,n) - \\ &\sum_{k=1}^{p} a_k C_{3y}(k-m, k-n); \qquad m,n \geq 0 \end{aligned} \quad (5)$$

The model parameters $\{a_k\}$ can be estimated by solving equation (5) along the lines

$m = n = 0,1,\cdots,p$, the diagonal slices of the third-order cumulant sequences, and is given by the following matrix equation [5-6]:

$$\mathbf{C}_{3\mathbf{y}} \cdot \mathbf{a} = \mathbf{b} \qquad (6)$$

where

$$\mathbf{C}_{3\mathbf{y}} = \begin{bmatrix} C_{3y}(0,0) & C_{3y}(1,1) & \cdots & C_{3y}(p,p) \\ C_{3y}(-1,-1) & C_{3y}(0,0) & \cdots & C_{3y}(p-1,p-1) \\ \vdots & \vdots & & \vdots \\ C_{3y}(-p,-p) & C_{3y}(-p+1,-p+1) & \cdots & C_{3y}(0,0) \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} 1, & a_1, & \cdots, & a_p \end{bmatrix}^T$$

$$\mathbf{b} = \begin{bmatrix} b, & 0, & \cdots, & 0 \end{bmatrix}^T$$

The parametric bispectrum can be estimated via the following relation:

$$B_y(f_1, f_2) = bH(f_1)H(f_2)H^*(f_1, f_2) \qquad (7)$$

where $H(f)$, the transfer function of the AR filter. Note that the third-order statistics is contained in the parameters $a_i \ (i = 1,2,\cdots,p)$.

3 Model Order Determination

To estimate the model parameters, the order of the non-Gaussian AR model must first be determined. Several techniques for order determination of non-Gaussian parametric models were reported, such as the singular value decomposition (SVD) approach, and the information theory criteria [8-10]. Third-order statistics are found suitable for order selection of the non-Gaussian ARMA model of a non-Gaussian process in additive Gaussian noise with unknown covariance. In this paper, the SVD method based on the non-Gaussian ARMA model is employed to estimate the optimal order of the non-Gaussian AR model. Let the process $y(k)$ satisfies third-order stationarity, and can be modeled by the following ARMA model:

$$\sum_{i=0}^{p} a(i)y(k-i) = \sum_{j=0}^{q} b(j)e(k-j) \qquad (8)$$

where the $e(k)$ is the non-Gaussian white noise with zero-mean. A block matrix based on the third-order cumulant slice is defined [11]. The block matrix has a full rank p, which is also the model order. The order parameters p^* and q^* are selected to be greater than the order p and q of the ARMA model. The order determination problem is thus to estimate the rank of the block matrix. The SVD method can provide robust estimate of the rank. To determine the model order of the AR(p) model of the EEG data, the rank of the block matrix can be estimated by setting $q = 0$ and $p^* > p$. Let V_i be the ith non-zero SVD of the block matrix, then the optimal order of the non-Gaussian AR model can be determined by finding the largest difference of the adjoined SVD among all the non-zero SVD $V_i \ (i = 1,2,\cdots,p^*)$ of the block matrix. So the optimal order $p_{opt} = i$ is given in terms of $\Delta = V_i - V_{i+1} = \Delta_{\max}$. The optimal order is chosen for the maximum value of Δ.

4 Real EEG Analysis and Results

A personal computer-based system for EEG data acquisition and processing is implemented. The sampled rate is set in 100 Hz. Several real EEG data are obtained to examine the quadratic phase coupling phenomena among the rhythms components. Both normal and pathological EEG signals are analyzed in terms of bispectral estimation. Fig. 1 shows four kinds of EEG records in different brain functions. Fig. 2 shows the average bispectral estimation of the normal EEG data when subjects were concentrating on the task of computing. Fig. 3 shows the bispectral structure of a typical record of seizure EEG data. Fig. 4 shows the average bispectral estimation of the EEG signals of five subjects in rest with eyes closed. Fig. 5 gives the average bispectral structure of a record of EEG data from a schizophrenic patient.

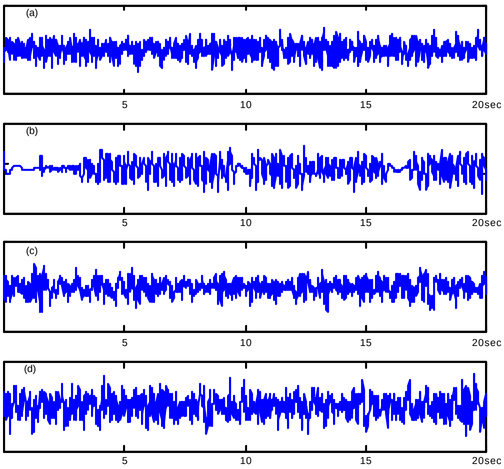

Fig. 1 Four kinds of EEG signals with different brain functions. From top to bottom: (a)when subject was concentrating on computing task. (b)during seizure period. (c)eyes closed state. (d)a record from a schizophrenic patient.

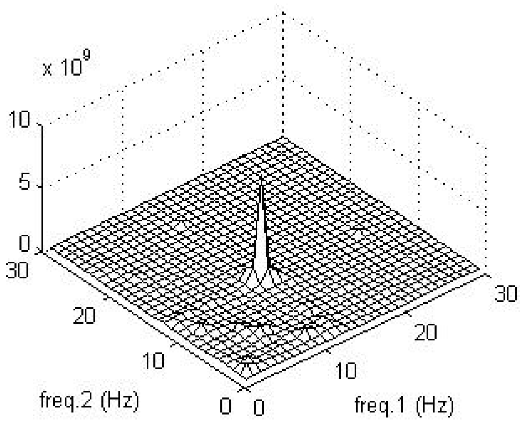

Fig. 2 The average bispectral estimation of the normal EEG series when subjects were concentrating on the task of computing.

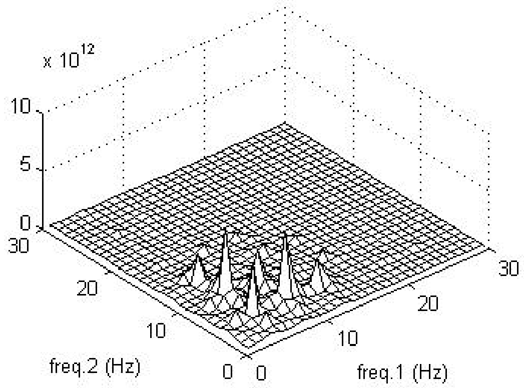

Fig. 3 The bispectral structure of a typical record of seizure EEG signal.

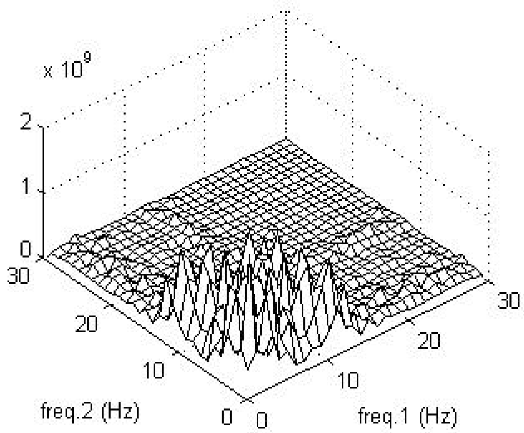

Fig. 4 shows the average bispectral estimation of the EEG signals of five subjects in rest with eyes closed.

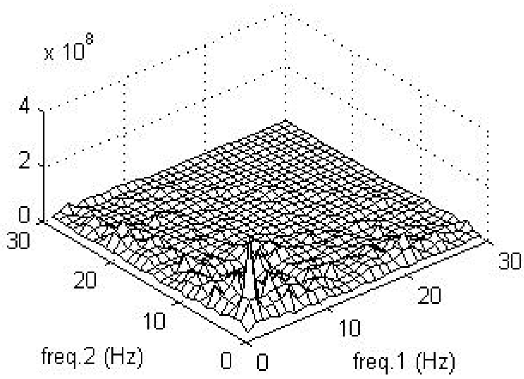

Fig. 5 The average bispectral structure of a record of EEG signal from a schizophrenic patient.

Based on the experimental results of the bispectral estimations of the spontaneous electroencephalogram in different brain functions, it can be seen that there exists one bispectral peak at 10 Hz in bifrequency domain during the subject was concentrating on the specific computing task. In Fig. 3, five bispectral peaks can be found in the bifrequency plane for a record of seizure EEG. The main bispectral peaks in bifrequency domain are concentrated among alpha rhythms when subjects' eyes were closed. Finally, the bispectral pattern was estimated for a record of EEG of schizophrenic patient, with one bispectral peak in bifrequency domain of delta rhythm.

The results have shown that different bispectral structures of the EEG signals are observed with different brain functions. The experimental results have shown that parametric bispectral techniques acan be a useful tools in analyzing the quadratic nonlinear interactions of finite EEG data with relative high resolution of bispectral estimation. Nonlinear analysis may be an effective and useful tool for understanding the basic mechanism of EEG generation and helping the assistant diagnosis of some kinds of EEG signals. It is suggested that the bispectral methods provide more useful information in the study of EEG signal, especially for the feature extraction by employing the parameters of the non-Gaussian AR model.

5 Conclusions

In this contribution we investigate the third-order recursion method for the estimation of the AR model parameters. One dimensional diagonal slice of third-order cumulant are estimated and used for determination of the parameters of the non-Gaussian AR model. In addition, the model order determination is another important problem when using the non-Gaussian AR model. Though various methods have been proposed for the determination of the order of a general ARMA model based on both second and third-order statistics, the problem of selecting a proper order of the non-Gaussian AR model is still a difficult one that has never been solved satisfactorily. For this purpose, bispectral cross-correlation is employed in this paper to estimate the optimal order of the non-Gaussian AR model. The cross correlation between conventional bispectral estimate and parametric bispectral estimate is defined and tested for the clinical EEG data. Moreover, the performance of cancellation of additive Gaussian noise is also tested to demonstrate the high detection probability of EEG signals in the noise by using higher-order spectral analysis.

Based on the higher-order spectral analysis and parametric bispectral estimation of EEG signals, some conclusion can be summarized as follows: 1. A spontaneous EEG is a typical non-Gaussian process which should be investigated by using HOS techniques. 2. Most of the spontaneous EEG signals can be modeled by a non-Gaussian AR model. The parameters of the model give a satisfied bispectral estimation with high resolution in EEG analysis. 3. The location and the amplitude of occurring bispectral peaks in bifrequency domain is quite different depending on the EEG signals in different kinds of functional states of the brain. 4. The cross-bispectrum of symmetrical channel EEG gives the similar structure with the bispectral structure when doing calculation by heart with eyes closed. 5. Higher-order spectral techniques may be an effective tool in the analysis and diagnosis of the EEG signals.

The method presented in this paper shows the capacities of higher-order spectral analysis for the description of non-Gaussian and nonlinear phenomena in brain activity signals. Although the new method in clinical EEG signal processing is addressed, open problems still remain. One is the pattern recognition via higher-order spectra, such as bispectral estimation, which is obviously related to the higher-order spectral characteristics of the EEG signals investigated. It will be an interesting and challenging research project to built an artificial neural network-based classification scheme.

References

[1] S. L. Marple. A New Autoregressive Spectrum Analysis Algorithm. *IEEE Trans. on ASSP.* Vol. 28. 1980.

[2] M. Yokoi. Clinical Evaluation on 5 Years Experience of Automated Phonocardiographic

Analysis. *Japan Heart Journal*, Vol. 18. 1977.
[2] M. L. Mendel Tutorial on Higher-Order Stistics (Spectra) in Signal Processing and System Theory: Theoretical Results and Some Applications. *Proceedings of The IEEE,* Vol. 79, No. 3, March. 1991.
[3] M. M. Jerry and et al. Editorial Applications of Higher Order Statistics, *IEE Proceedings-F,* Vol. 140, No. 6. December 1993.
[4] M. R. Raghuveer and C. L. Nikias. Bispectrum Estimation: a Parametric approach. *IEEE Trans. on ASSP,* Vol. ASSP-33, October. 1985.
[5] R. Subba. *An Introduction to Bispectral Analysis and Bilinear Time Series Models*, New York: Spring-Verg, 1984.
[6] C. L. Nikias and M.R. Raghuveer. Bispectrum estimation: a digital signal processing framwork, *Proceedings of the IEEE,* Vol. 75, No. 7, July 1987.
[7] C. L. Nikias. *Higher-order Spectra Analysis, A Nonlinear Signal Processing Framework* PTR Prentice Hall, Eaglewood Cliff, New Jersey 1993
[8] K. T. Jitendra. Linear Model Validation and Order Selection Using Higher Order Statistics, *IEEE Trans. on Signal Processing*, Vol. 42, No. 7, July 1994
[9] G. Giannakis. et al. Cumulant-Based Order Determination of Non-Gaussian ARMA Models, *IEEE Trans. on ASSP,* Vol. 38(8), Aug 1990.
[10] G. Giannakis and S. Shamsunder. Information Theoretic Criteria for Non-Gaussian ARMA Order Determination and Parameter Estimation. *Proc. of ICASSP'93,* Vol. 4, USA. 1993
[11] J. Noonan and ea al. AR Model Order Selection based on Bispectral Cross Correlation, *IEEE Trans. on Signal Processing* , Vol. 39, No. 6. June 1999