

Signal Analysis: Wavelets, Filter Banks, Time-Frequency Transforms and Applications. Alfred Mertins

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Print ISBN 0-471-98626-7 Electronic ISBN 0-470-84183-4

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Wavelets, Filter Banks, Time-Frequency Transforms and Applications

Alfred Mertins

University of Wollongong, Australia

JOHN WILEY & SONS

Chichester · New York · Weinheim · Brisbane · Singapore · Toronto

© B.G. Teubner Stuttgart 1996, Mertins, Signaltheorie
Translation arranged with the approval of the publisher B.G. Teubner Stuttgart, from the original German edition into English.

English (revised edition) Copyright © 1999 by John Wiley & Sons Ltd,
Baffins Lane, Chichester,
West Sussex PO19 1UD, England
National 01243 779777
International (+44) 1243 779777

e-mail (for orders and customer service enquiries): cs-books@wiley.co.uk
Visit our Home Page on <http://www.wiley.co.uk> or <http://www.wiley.com>

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John Wiley & Sons, Inc., 605 Third Avenue,
New York, NY 10158-0012, USA

Wiley-VCH Verlag GmbH, Pappelallee 3,
D-69469 Weinheim, Germany

Jacaranda Wiley Ltd, 33 Park Road, Milton,
Queensland 4064, Australia

John Wiley & Sons (Asia) Pte Ltd, 2 Clementi Loop #02-01,
Jin Xing Distripark, Singapore 129809

John Wiley & Sons (Canada) Ltd, 22 Worcester Road,
Rexdale, Ontario M9W 1L1, Canada

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library.

ISBN 0-471-98626-7

Produced from PostScript files supplied by the author.

Printed and bound in Great Britain by Bookcraft (Bath) Ltd.

This book is printed on acid-free paper responsibly manufactured from sustainable forestry, in which at least two trees are planted for each one used in paper production.

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Preface

A central goal in signal analysis is to extract information from signals that are related to real-world phenomena. Examples are the analysis of speech, images, and signals in medical or geophysical applications. One reason for analyzing such signals is to achieve better understanding of the underlying physical phenomena. Another is to find compact representations of signals which allow compact storage or efficient transmission of signals through real-world environments. The methods of analyzing signals are wide spread and range from classical Fourier analysis to various types of linear time-frequency transforms and model-based and non-linear approaches. This book concentrates on transforms, but also gives a brief introduction to linear estimation theory and related signal analysis methods. The text is self-contained for readers with some background in system theory and digital signal processing, as typically gained in undergraduate courses in electrical and computer engineering.

The first five chapters of this book cover the classical concepts of signal representation, including integral and discrete transforms. Chapter 1 contains an introduction to signals and signal spaces. It explains the basic tools for classifying signals and describing their properties. Chapter 2 gives an introduction to integral signal representation. Examples are the Fourier, Hartley and Hilbert transforms. Chapter 3 discusses the concepts and tools for discrete signal representation. Examples of discrete transforms are given in Chapter 4. Some of the latter are studied comprehensively, while others are only briefly introduced, to a level required in the later chapters. Chapter 5 is dedicated to the processing of stochastic processes using discrete transforms and model-based approaches. It explains the Karhunen–Loève transform and the whitening transform, gives an introduction to linear estimation theory and optimal filtering, and discusses methods of estimating autocorrelation sequences and power spectra.

The final four chapters of this book are dedicated to transforms that provide time-frequency signal representations. In Chapter 6, multirate filter banks are considered. They form the discrete-time variant of time-frequency transforms. The chapter gives an introduction to the field and provides an overview of filter design methods. The classical method of time-frequency analysis is the short-time Fourier transform, which is discussed in Chapter 7. This transform was introduced by Gabor in 1946 and is used in many applications, especially in the form of spectrograms. The most prominent example of linear transforms with time-frequency localization is the wavelet transform. This transform attracts researchers from almost any field of science, because

it has many useful features: a time–frequency resolution that is matched to many real-world phenomena, a multiscale representation, and a very efficient implementation based on multirate filter banks. Chapter 8 discusses the continuous wavelet transform, the discrete wavelet transform, and the wavelet transform of discrete-time signals. Finally, Chapter 9 is dedicated to quadratic time–frequency analysis tools like the Wigner distribution, the distributions of Cohen’s class, and the Wigner–Ville spectrum.

The history of this book is relatively long. It started in 1992 when I produced the first lecture notes for courses on signal theory and linear time–frequency analysis at the Hamburg University of Technology, Germany. Parts of the material were included in a thesis (“Habilitationsschrift”) that I submitted in 1994. In 1996, the text was published as a textbook on Signal Theory in German. This book appeared in a series on Information Technology, edited by Prof. Norbert J. Fliege and published by B.G. Teubner, Stuttgart, Germany. It was Professor Fliege who encouraged me to write the book, and I would like to thank him for that and for his support throughout many years. The present book is mainly a translation of the original German. However, I have rearranged some parts, expanded some of the chapters, and shortened others in order to obtain a more homogeneous and self-contained text. During the various stages, from the first lecture notes, over the German manuscript to the present book, many people helped me by proofreading and commenting on the text. Marcus Benthin, Georg Dickmann, Frank Filbir, Sabine Hohmann, Martin Schönle, Frank Seide, Ursula Seifert, and Jens Wohlers read portions of the German manuscript. Their feedback significantly enhanced the quality of the manuscript. My sister, Inge Mertins–Obbelode, translated the text from German into English and also proofread the new material that was not included in the German book. Tanja Karp and Jörg Kliewer went through the chapters on filter banks and wavelets, respectively, in the English manuscript and made many helpful suggestions. Ian Burnett went through a complete draft of the present text and made many suggestions that helped to improve the presentation. I would like to thank them all. Without their effort and enthusiasm this project would not have been realizable.

Alfred Mertins

Wollongong, December 1998

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