Detection, Estimation, and Modulation Theory, Part I: Detection, Estimation, and Linear Modulation Theory. Harry L. Van Trees

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Introduction

In these two books, we shall study three areas of statistical theory, which we have labeled detection theory, estimation theory, and modulation theory. The goal is to develop these theories in a common mathematical framework and to demonstrate how they can be used to solve a wealth of practical problems in many diverse physical situations.

In this chapter we present three outlines of the material. The first is a topical outline in which we develop a qualitative understanding of the three areas by examining some typical problems of interest. The second is a logical outline in which we explore the various methods of attacking the problems. The third is a chronological outline in which we explain the structure of the books.

1.1 TOPICAL OUTLINE

An easy way to explain what is meant by detection theory is to examine several physical situations that lead to detection theory problems.

A simple digital communication system is shown in Fig. 1.1. The source puts out a binary digit every T seconds. Our object is to transmit this sequence of digits to some other location. The channel available for transmitting the sequence depends on the particular situation. Typically, it could be a telephone line, a radio link, or an acoustical channel. For

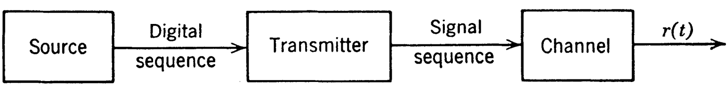

Fig. 1.1 Digital communication system.

purposes of illustration, we shall consider a radio link. In order to transmit the information, we must put it into a form suitable for propagating over the channel. A straightforward method would be to build a device that generates a sine wave,

$$s_1(t) = \sin \omega_1 t, \tag{1}$$

for T seconds if the source generated a "one" in the preceding interval, and a sine wave of a different frequency,

$$s_0(t) = \sin \omega_0 t, \tag{2}$$

for T seconds if the source generated a "zero" in the preceding interval. The frequencies are chosen so that the signals $s_0(t)$ and $s_1(t)$ will propagate over the particular radio link of concern. The output of the device is fed into an antenna and transmitted over the channel. Typical source and transmitted signal sequences are shown in Fig. 1.2. In the simplest kind of channel the signal sequence arrives at the receiving antenna attenuated but essentially undistorted. To process the received signal we pass it through the antenna and some stages of rf-amplification, in the course of which a thermal noise $n(t)$ is added to the message sequence. Thus in any T-second interval we have available a waveform $r(t)$ in which

$$r(t) = s_1(t) + n(t), \qquad 0 \le t \le T, \tag{3}$$

if $s_1(t)$ was transmitted, and

$$r(t) = s_0(t) + n(t), \qquad 0 \le t \le T, \tag{4}$$

if $s_0(t)$ was transmitted. We are now faced with the problem of deciding which of the two possible signals was transmitted. We label the device that does this a decision device. It is simply a processor that observes $r(t)$ and guesses whether $s_1(t)$ or $s_0(t)$ was sent according to some set of rules. This is equivalent to guessing what the source output was in the preceding interval. We refer to designing and evaluating the processor as a detection

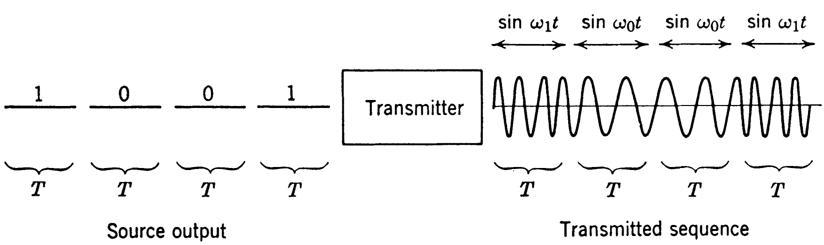

Fig. 1.2 Typical sequences.

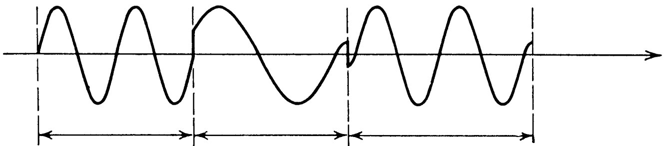

Fig. 1.3 Sequence with phase shifts.

theory problem. In this particular case the only possible source of error in making a decision is the additive noise. If it were not present, the input would be completely known and we could make decisions without errors. We denote this type of problem as the *known signal in noise problem.* It corresponds to the lowest level (i.e., simplest) of the detection problems of interest.

An example of the next level of detection problem is shown in Fig. 1.3. The oscillators used to generate $s_1(t)$ and $s_0(t)$ in the preceding example have a phase drift. Therefore in a particular T-second interval the received signal corresponding to a "one" is

$$r(t) = \sin(\omega_1 t + \theta_1) + n(t), \qquad 0 \le t \le T, \tag{5}$$

and the received signal corresponding to a "zero" is

$$r(t) = \sin(\omega_0 t + \theta_0) + n(t), \qquad 0 \le t \le T, \tag{6}$$

where θ_0 and θ_1 are unknown constant phase angles. Thus even in the absence of noise the input waveform is not completely known. In a practical system the receiver may include auxiliary equipment to measure the oscillator phase. If the phase varies slowly enough, we shall see that essentially perfect measurement is possible. If this is true, the problem is the same as above. However, if the measurement is not perfect, we must incorporate the signal uncertainty in our model.

A corresponding problem arises in the radar and sonar areas. A conventional radar transmits a pulse at some frequency ω_c with a rectangular envelope:

$$s_t(t) = \sin \omega_c t, \qquad 0 \le t \le T. \tag{7}$$

If a target is present, the pulse is reflected. Even the simplest target will introduce an attenuation and phase shift in the transmitted signal. Thus the signal available for processing in the interval of interest is

$$\begin{aligned} r(t) &= V_r \sin[\omega_c(t - \tau) + \theta_r] + n(t), && \tau \le t \le \tau + T, \\ &= n(t), && 0 \le t < \tau, \tau + T < t < \infty, \end{aligned} \tag{8}$$

if a target is present and

$$r(t) = n(t), \qquad 0 \le t < \infty, \tag{9}$$

if a target is absent. We see that in the absence of noise the signal still contains three unknown quantities: V_r, the amplitude, θ_r, the phase, and τ, the round-trip travel time to the target.

These two examples represent the second level of detection problems. We classify them as *signal with unknown parameters in noise problems.*

Detection problems of a third level appear in several areas. In a passive sonar detection system the receiver listens for noise generated by enemy vessels. The engines, propellers, and other elements in the vessel generate acoustical signals that travel through the ocean to the hydrophones in the detection system. This composite signal can best be characterized as a sample function from a random process. In addition, the hydrophone generates self-noise and picks up sea noise. Thus a suitable model for the detection problem might be

$$r(t) = s_\Omega(t) + n(t) \tag{10}$$

if the target is present and

$$r(t) = n(t) \tag{11}$$

if it is not. In the absence of noise the signal is a sample function from a random process (indicated by the subscript Ω).

In the communications field a large number of systems employ channels in which randomness is inherent. Typical systems are tropospheric scatter links, orbiting dipole links, and chaff systems. A common technique is to transmit one of two signals separated in frequency. (We denote these frequencies as ω_1 and ω_0.) The resulting received signal is

$$r(t) = s_{\Omega_1}(t) + n(t) \tag{12}$$

if $s_1(t)$ was transmitted and

$$r(t) = s_{\Omega_0}(t) + n(t) \tag{13}$$

if $s_0(t)$ was transmitted. Here $s_{\Omega_1}(t)$ is a sample function from a random process centered at ω_1, and $s_{\Omega_0}(t)$ is a sample function from a random process centered at ω_0. These examples are characterized by the lack of any deterministic signal component. Any decision procedure that we design will have to be based on the difference in the statistical properties of the two random processes from which $s_{\Omega_0}(t)$ and $s_{\Omega_1}(t)$ are obtained. This is the third level of detection problem and is referred to as a *random signal in noise problem.*

In our examination of representative examples we have seen that detection theory problems are characterized by the fact that we must decide which of several alternatives is true. There were only two alternatives in the examples cited; therefore we refer to them as binary detection problems. Later we will encounter problems in which there are M alternatives available (the M-ary detection problem). Our hierarchy of detection problems is presented graphically in Fig. 1.4.

There is a parallel set of problems in the estimation theory area. A simple example is given in Fig. 1.5, in which the source puts out an analog message $a(t)$ (Fig. 1.5*a*). To transmit the message we first sample it every T seconds. Then, every T seconds we transmit a signal that contains

	Detection theory
Level 1. Known signals in noise	1. Synchronous digital communication 2. Pattern recognition problems
Level 2. Signals with unknown parameters in noise	1. Conventional pulsed radar or sonar, target detection 2. Target classification (orientation of target unknown) 3. Digital communication systems without phase reference 4. Digital communication over slowly-fading channels
Level 3. Random signals in noise	1. Digital communication over scatter link, orbiting dipole channel, or chaff link 2. Passive sonar 3. Seismic detection system 4. Radio astronomy (detection of noise sources)

Fig. 1.4 Detection theory hierarchy.

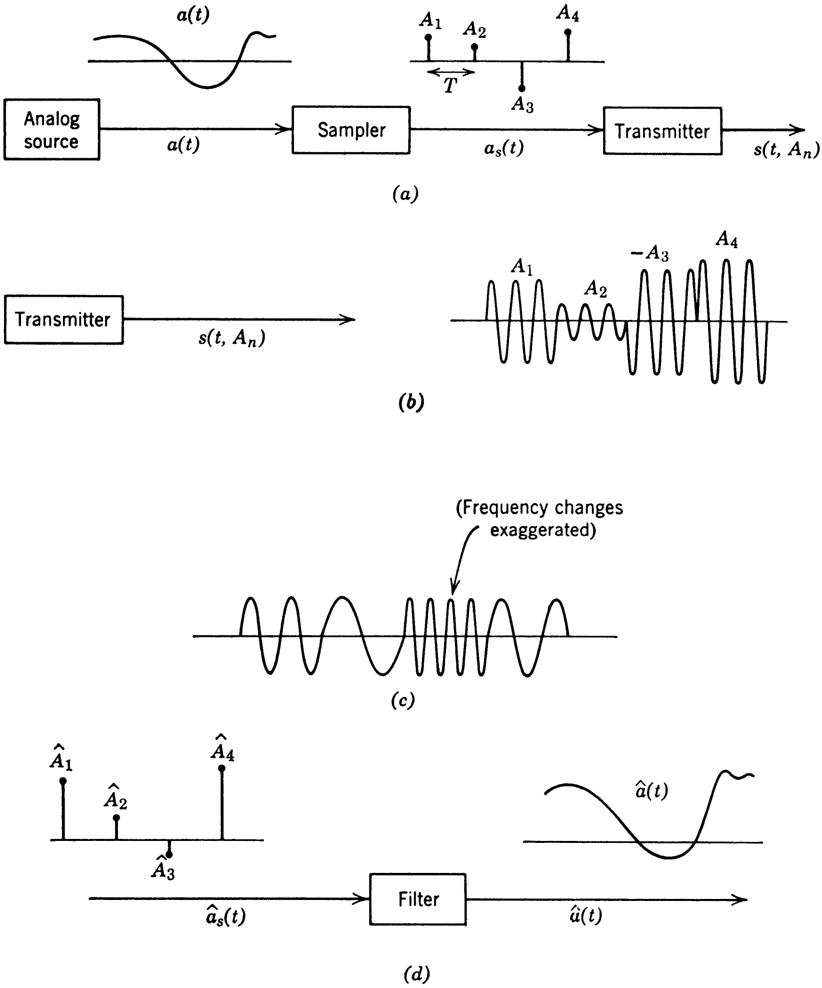

Fig. 1.5 (*a*) Sampling an analog source; (*b*) pulse-amplitude modulation; (*c*) pulse-frequency modulation; (*d*) waveform reconstruction.

a parameter which is uniquely related to the last sample value. In Fig. 1.5*b* the signal is a sinusoid whose amplitude depends on the last sample. Thus, if the sample at time nT is A_n, the signal in the interval $[nT, (n + 1)T]$ is

$$s(t, A_n) = A_n \sin \omega_c t, \qquad nT \leq t \leq (n + 1)T. \tag{14}$$

A system of this type is called a pulse amplitude modulation (PAM) system. In Fig. 1.5*c* the signal is a sinusoid whose frequency in the interval differs from the reference frequency ω_c by an amount proportional to the preceding sample value,

$$s(t, A_n) = \sin(\omega_c t + A_n t), \qquad nT \leq t \leq (n+1)T. \tag{15}$$

A system of this type is called a pulse frequency modulation (PFM) system. Once again there is additive noise. The received waveform, given that A_n was the sample value, is

$$r(t) = s(t, A_n) + n(t), \qquad nT \leq t \leq (n+1)T. \tag{16}$$

During each interval the receiver tries to estimate A_n. We denote these estimates as $\hat{A}_n$. Over a period of time we obtain a sequence of estimates, as shown in Fig. 1.5*d*, which is passed into a device whose output is an estimate of the original message $a(t)$. If $a(t)$ is a band-limited signal, the device is just an ideal low-pass filter. For other cases it is more involved.

If, however, the parameters in this example were known and the noise were absent, the received signal would be completely known. We refer to problems in this category as *known signal in noise problems*. If we assume that the mapping from A_n to $s(t, A_n)$ in the transmitter has an inverse, we see that if the noise were not present we could determine A_n unambiguously. (Clearly, if we were allowed to design the transmitter, we should always choose a mapping with an inverse.) The *known signal in noise problem* is the first level of the estimation problem hierarchy.

Returning to the area of radar, we consider a somewhat different problem. We assume that we know a target is present but do not know its range or velocity. Then the received signal is

$$\begin{aligned} r(t) &= V_r \sin[(\omega_c + \omega_d)(t - \tau) + \theta_r] + n(t), && \tau \leq t \leq \tau + T, \\ &= n(t), && 0 \leq t < \tau, \tau + T < t < \infty, \end{aligned} \tag{17}$$

where ω_d denotes a Doppler shift caused by the target's motion. We want to estimate τ and ω_d. Now, even if the noise were absent and τ and ω_d were known, the signal would still contain the unknown parameters V_r and θ_r. This is a typical second-level estimation problem. As in detection theory, we refer to problems in this category as *signal with unknown parameters in noise problems*.

At the third level the signal component is a random process whose statistical characteristics contain parameters we want to estimate. The received signal is of the form

$$r(t) = s_\Omega(t, A) + n(t), \tag{18}$$

where $s_\Omega(t, A)$ is a sample function from a random process. In a simple case it might be a stationary process with the narrow-band spectrum shown in Fig. 1.6. The shape of the spectrum is known but the center frequency

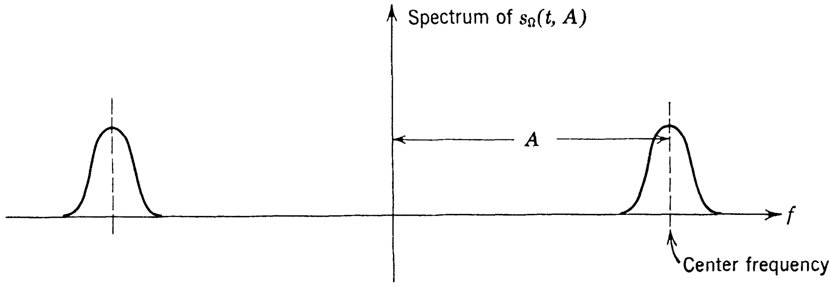

Fig. 1.6 Spectrum of random signal.

	Estimation Theory
Level 1. Known signals in noise	1. PAM, PFM, and PPM communication systems with phase synchronization 2. Inaccuracies in inertial systems (e.g., drift angle measurement)
Level 2. Signals with unknown parameters in noise	1. Range, velocity, or angle measurement in radar/sonar problems 2. Discrete time, continuous amplitude communication system (with unknown amplitude or phase in channel)
Level 3. Random signals in noise	1. Power spectrum parameter estimation 2. Range or Doppler spread target parameters in radar/sonar problem 3. Velocity measurement in radio astronomy 4. Target parameter estimation: passive sonar 5. Ground mapping radars

Fig. 1.7 Estimation theory hierarchy.

is not. The receiver must observe $r(t)$ and, using the statistical properties of $s_\Omega(t, A)$ and $n(t)$, estimate the value of A. This particular example could arise in either radio astronomy or passive sonar. The general class of problem in which the signal containing the parameters is a sample function from a random process is referred to as the *random signal in noise problem.* The hierarchy of estimation theory problems is shown in Fig. 1.7.

We note that there appears to be considerable parallelism in the detection and estimation theory problems. We shall frequently exploit these parallels to reduce the work, but there is a basic difference that should be emphasized. In binary detection the receiver is either "right" or "wrong." In the estimation of a continuous parameter the receiver will seldom be exactly right, but it can try to be close most of the time. This difference will be reflected in the manner in which we judge system performance.

The third area of interest is frequently referred to as modulation theory. We shall see shortly that this term is too narrow for the actual problems. Once again a simple example is useful. In Fig. 1.8 we show an analog message source whose output might typically be music or speech. To convey the message over the channel, we transform it by using a modulation scheme to get it into a form suitable for propagation. The transmitted signal is a continuous waveform that depends on $a(t)$ in some deterministic manner. In Fig. 1.8 it is an amplitude modulated waveform:

$$s[t, a(t)] = [1 + ma(t)] \sin(\omega_c t). \tag{19}$$

(This is conventional double-sideband AM with modulation index m.) In Fig. 1.8c the transmitted signal is a frequency modulated (FM) waveform:

$$s[t, a(t)] = \sin\left[\omega_c t + \int_{-\infty}^{t} a(u)\, du\right]. \tag{20}$$

When noise is added the received signal is

$$r(t) = s[t, a(t)] + n(t). \tag{21}$$

Now the receiver must observe $r(t)$ and put out a continuous estimate of the message $a(t)$, as shown in Fig. 1.8. This particular example is a first-level modulation problem, for if $n(t)$ were absent and $a(t)$ were known the received signal would be completely known. Once again we describe it as a *known signal in noise problem.*

Another type of physical situation in which we want to estimate a continuous function is shown in Fig. 1.9. The channel is a time-invariant linear system whose impulse response $h(\tau)$ is unknown. To estimate the impulse response we transmit a known signal $x(t)$. The received signal is

$$r(t) = \int_{0}^{\infty} h(\tau)\, x(t - \tau)\, d\tau + n(t). \tag{22}$$

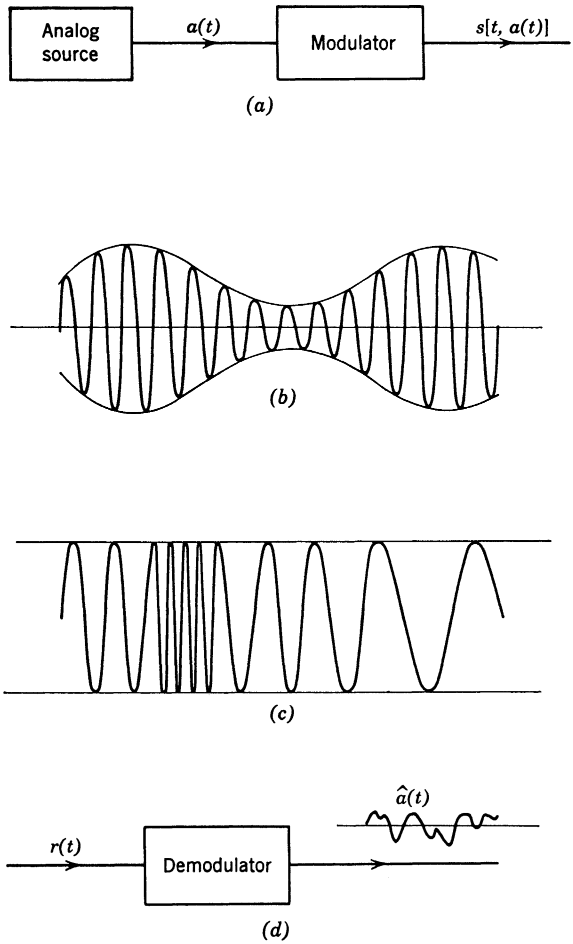

Fig. 1.8 **A modulation theory example: (*a*) analog transmission system; (*b*) amplitude modulated signal; (*c*) frequency modulated signal; (*d*) demodulator.**

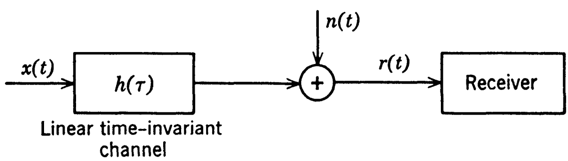

Fig. 1.9 **Channel measurement.**

The receiver observes $r(t)$ and tries to estimate $h(\tau)$. This particular example could best be described as a continuous estimation problem. Many other problems of interest in which we shall try to estimate a continuous waveform will be encountered. For convenience, we shall use the term *modulation theory* for this category, even though the term continuous waveform estimation might be more descriptive.

The other levels of the modulation theory problem follow by direct analogy. In the amplitude modulation system shown in Fig. 1.8*b* the receiver frequently does not know the phase angle of the carrier. In this case a suitable model is

$$r(t) = (1 + ma(t)) \sin (\omega_c t + \theta) + n(t), \tag{23}$$

	Modulation Theory (Continuous waveform estimation)
1. Known signals in noise	1. Conventional communication systems such as AM (DSB-AM, SSB), FM, and PM with phase synchronization 2. Optimum filter theory 3. Optimum feedback systems 4. Channel measurement 5. Orbital estimation for satellites 6. Signal estimation in seismic and sonar classification systems 7. Synchronization in digital systems
2. Signals with unknown parameters in noise	1. Conventional communication systems without phase synchronization 2. Estimation of channel characteristics when phase of input signal is unknown
3. Random signals in noise	1. Analog communication over randomly varying channels 2. Estimation of statistics of time-varying processes 3. Estimation of plant characteristics

Fig. 1.10 Modulation theory hierarchy.

where θ is an unknown parameter. This is an example of a *signal with unknown parameter problem* in the modulation theory area.

A simple example of a third-level problem (*random signal in noise*) is one in which we transmit a frequency-modulated signal over a radio link whose gain and phase characteristics are time-varying. We shall find that if we transmit the signal in (20) over this channel the received waveform will be

$$r(t) = V(t) \sin\left[\omega_c t + \int_{-\infty}^{t} a(u)\, du + \theta(t)\right] + n(t), \tag{24}$$

where $V(t)$ and $\theta(t)$ are sample functions from random processes. Thus, even if $a(u)$ were known and the noise $n(t)$ were absent, the received signal would still be a random process. An over-all outline of the problems of interest to us appears in Fig. 1.10. Additional examples included in the table to indicate the breadth of the problems that fit into the outline are discussed in more detail in the text.

Now that we have outlined the areas of interest it is appropriate to determine how to go about solving them.

1.2 POSSIBLE APPROACHES

From the examples we have discussed it is obvious that an inherent feature of all the problems is randomness of source, channel, or noise (often all three). Thus our approach must be statistical in nature. Even assuming that we are using a statistical model, there are many different ways to approach the problem. We can divide the possible approaches into two categories, which we denote as "structured" and "nonstructured." Some simple examples will illustrate what we mean by a structured approach.

Example 1. The input to a linear time-invariant system is $r(t)$:

$$\begin{aligned} r(t) &= s(t) + w(t) \qquad && 0 \le t \le T, \\ &= 0, && \text{elsewhere.} \end{aligned} \tag{25}$$

The impulse response of the system is $h(\tau)$. The signal $s(t)$ is a known function with energy E_s,

$$E_s = \int_0^T s^2(t)\, dt, \tag{26}$$

and $w(t)$ is a sample function from a zero-mean random process with a covariance function:

$$K_w(t, u) = \frac{N_0}{2}\, \delta(t-u). \tag{27}$$

We are concerned with the output of the system at time T. The output due to the signal is a deterministic quantity:

$$s_o(T) = \int_0^T h(\tau)\, s(T-\tau)\, d\tau. \tag{28}$$

The output due to the noise is a random variable:

$$n_o(T) = \int_0^T h(\tau)\, n(T - \tau)\, d\tau. \tag{29}$$

We can define the output signal-to-noise ratio at time T as

$$\frac{S}{N} \triangleq \frac{s_o^2(T)}{E[n_o^2(T)]}, \tag{30}$$

where $E(\cdot)$ denotes expectation.

Substituting (28) and (29) into (30), we obtain

$$\frac{S}{N} = \frac{\left[\int_0^T h(\tau)\, s(T - \tau)\, d\tau\right]^2}{E\left[\iint_0^T h(\tau)\, h(u)\, n(T - \tau)\, n(T - u)\, d\tau\, du\right]}. \tag{31}$$

By bringing the expectation inside the integral, using (27), and performing the integration with respect to u, we have

$$\frac{S}{N} = \frac{\left[\int_0^T h(\tau)\, s(T - \tau)\, d\tau\right]^2}{N_0/2 \int_0^T h^2(\tau)\, d\tau}. \tag{32}$$

The problem of interest is to choose $h(\tau)$ to maximize the signal-to-noise ratio. The solution follows easily, but it is not important for our present discussion. (See Problem 3.3.1.)

This example illustrates the three essential features of the structured approach to a statistical optimization problem:

Structure. The processor was required to be a linear time-invariant filter. We wanted to choose the best system in this class. Systems that were not in this class (e.g., nonlinear or time-varying) were not allowed.

Criterion. In this case we wanted to maximize a quantity that we called the signal-to-noise ratio.

Information. To write the expression for S/N we had to know the signal shape and the covariance function of the noise process.

If we knew more about the process (e.g., its first-order probability density), we could not use it, and if we knew less, we could not solve the problem. Clearly, if we changed the criterion, the information required might be different. For example, to maximize x

$$x = \frac{s_o^4(T)}{E[n_o^4(T)]}, \tag{33}$$

the covariance function of the noise process would not be adequate. Alternatively, if we changed the structure, the information required might change. Thus the three ideas of structure, criterion, and information are closely related. It is important to emphasize that the structured approach does not imply a linear system, as illustrated by Example 2.

Example 2. The input to the nonlinear no-memory device shown in Fig. 1.11 is $r(t)$, where

$$r(t) = s(t) + n(t), \qquad -\infty < t < \infty. \tag{34}$$

At any time t, $s(t)$ is the value of a random variable s with known probability density $p_s(S)$. Similarly, $n(t)$ is the value of a statistically independent random variable n with known density $p_n(N)$. The output of the device is $y(t)$, where

$$y(t) = a_0 + a_1[r(t)] + a_2[r(t)]^2 \tag{35}$$

is a quadratic no-memory function of $r(t)$. [The adjective no-memory emphasizes that the value of $y(t_0)$ depends *only* on $r(t_0)$.] We want to choose the coefficients a_0, a_1, and a_2 so that $y(t)$ is the minimum mean-square error estimate of $s(t)$. The mean-square error is

$$\begin{aligned} \xi(t) &\triangleq E\{[y(t) - s(t)^2]\} \\ &= E(\{a_0 + a_1[r(t)] + a_2[r^2(t)] - s(t)\}^2) \end{aligned} \tag{36}$$

and a_0, a_1, and a_2 are chosen to minimize $\xi(t)$. The solution to this particular problem is given in Chapter 3.

The technique for solving structured problems is conceptually straightforward. We allow the structure to vary within the allowed class and choose the particular system that maximizes (or minimizes) the criterion of interest.

An obvious advantage to the structured approach is that it usually requires only a partial characterization of the processes. This is important because, in practice, we must measure or calculate the process properties needed.

An obvious disadvantage is that it is often impossible to tell if the structure chosen is correct. In Example 1 a simple nonlinear system might

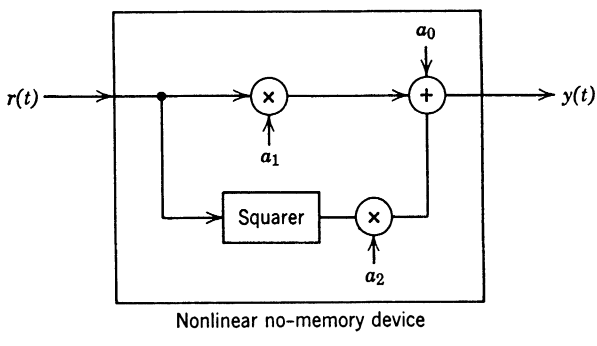

Fig. 1.11 A structured nonlinear device.

be far superior to the best linear system. Similarly, in Example 2 some other nonlinear system might be far superior to the quadratic system. Once a class of structure is chosen we are committed. A number of trivial examples demonstrate the effect of choosing the wrong structure. We shall encounter an important practical example when we study frequency modulation in Chapter II-2.

At first glance it appears that one way to get around the problem of choosing the proper strucutre is to let the structure be an arbitrary nonlinear time-varying system. In other words, the class of structure is chosen to be so large that every possible system will be included in it. The difficulty is that there is no convenient tool, such as the convolution integral, to express the output of a nonlinear system in terms of its input. This means that there is no convenient way to investigate all possible systems by using a structured approach.

The alternative to the structured approach is a nonstructured approach. Here we refuse to make any a priori guesses about what structure the processor should have. We establish a criterion, solve the problem, and implement whatever processing procedure is indicated.

A simple example of the nonstructured approach can be obtained by modifying Example 2. Instead of assigning characteristics to the device, we denote the estimate by $y(t)$. Letting

$$\xi(t) \triangleq E\{[y(t) - s(t)]^2\}, \tag{37}$$

we solve for the $y(t)$ that is obtained from $r(t)$ in *any* manner to minimize ξ. The obvious advantage is that if we can solve the problem we know that our answer, is *with respect to the chosen criterion*, the best processor of all possible processors. The obvious disadvantage is that we must completely characterize all the signals, channels, and noises that enter into the problem. Fortunately, it turns out that there are a large number of problems of practical importance in which this complete characterization is possible. Throughout both books we shall emphasize the nonstructured approach.

Our discussion up to this point has developed the topical and logical basis of these books. We now discuss the actual organization.

1.3 ORGANIZATION

The material covered in this book and Volume II can be divided into five parts. The first can be labeled *Background* and consists of Chapters 2 and 3. In Chapter 2 we develop in detail a topic that we call Classical Detection and Estimation Theory. Here we deal with problems in which the observations are sets of random variables instead of random waveforms. The theory needed to solve problems of this type has been studied by statisticians for many years. We therefore use the adjective classical to describe it. The purpose of the chapter is twofold: first, to derive all the basic statistical results we need in the remainder of the chapters; second, to provide a general background in detection and estimation theory that can be extended into various areas that we do not discuss in detail. To accomplish the second purpose we keep the discussion as general as possible. We consider in detail the binary and M-ary hypothesis testing problem, the problem of estimating random and nonrandom variables, and the composite hypothesis testing problem. Two more specialized topics, the general Gaussian problem and performance bounds on binary tests, are developed as background for specific problems we shall encounter later.

The next step is to bridge the gap between the classical case and the waveform problems discussed in Section 1.1. Chapter 3 develops the necessary techniques. The key to the transition is a suitable method for characterizing random processes. When the observation interval is finite, the most useful characterization is by a series expansion of the random process which is a generalization of the conventional Fourier series. When the observation interval is infinite, a transform characterization, which is a generalization of the usual Fourier transform, is needed. In the process of developing these characterizations, we encounter integral equations and we digress briefly to develop methods of solution. Just as in Chapter 2, our discussion is general and provides background for other areas of application.

With these two chapters in the first part as background, we are prepared to work our way through the hierarchy of problems outlined in Figs. 1.4, 1.7, and 1.10. The second part of the book (Chapter 4) can be labeled *Elementary Detection and Estimation Theory*. Here we develop the first two levels described in Section 1.1. (This material corresponds to the upper two levels in Figs. 1.4 and 1.7.) We begin by looking at the simple binary digital communication system described in Fig. 1.1 and then proceed to more complicated problems in the communications, radar, and sonar area involving M-ary communication, random phase channels, random amplitude and phase channels, and colored noise interference. By exploiting the parallel nature of the estimation problem, results are obtained easily for the estimation problem outlined in Fig. 1.5 and other more complex systems. The extension of the results to include the multiple channel (e.g., frequency diversity systems or arrays) and multiple parameter (e.g., range and Doppler) problems completes our discussion. The results in this chapter are fundamental to the understanding of modern communication and radar/sonar systems.

The third part, which can be labeled *Modulation Theory or Continuous Estimation Theory*, consists of Chapters 5 and 6 and Chapter 2 of Volume II. In Chapter 5 we formulate a quantitative model for the first two levels of the continuous waveform estimation problem and derive a set of integral equations whose solution is the optimum estimate of the message. We also derive equations that give bounds on the performance of the estimators. In order to study solution techniques, we divide the estimation problem into two categories, linear and nonlinear.

In Chapter 6 we study linear estimation problems in detail. In the first section of the chapter we discuss the relationships between various criteria, process characteristics, and the structure of the processor. In the next section we discuss the special case in which the processes are stationary and the infinite past is available. This case, the Wiener problem, leads to straightforward solution techniques. The original work of Wiener is extended to obtain some important closed-form error expressions. In the next section we discuss the case in which the processes can be characterized by using state-variable techniques. This case, the Kalman-Bucy problem, enables us to deal with nonstationary, finite-interval problems and adds considerable insight to the results of the preceding section.

The material in Chapters 1 through 6 has two characteristics:

1. In almost all cases we can obtain *explicit*, *exact* solutions to the problems that we formulate.

2. Most of the topics discussed are of such fundamental interest that everyone concerned with the statistical design of communication, radar, or sonar systems should be familiar with them.

As soon as we try to solve the nonlinear estimation problem, we see a sharp departure. To obtain useful results we must resort to approximate solution techniques. To decide what approximations are valid, however, we must consider specific nonlinear modulation systems. Thus the precise quantitative results are only applicable to the specific system. In view of this departure, we pause briefly in our logical development and summarize our results in Chapter 7.

After a brief introduction we return to the nonlinear modulation problem in Chapter 2 of Volume II and consider angle modulation systems in great detail. After an approximation to the optimum processor is developed, its performance and possible design modification are analyzed both theoretically and experimentally. More advanced techniques from Markov process theory and information theory are used to obtain significant results.

In the fourth part we revisit the problems of detection, estimation, and modulation theory at the third level of the hierarchy described in Section 1.1. Looking at the bottom boxes in Figs. 1.4, 1.7, and 1.10, we see that this is the *Random Signals in Noise* problem. Chapter II-3 studies it in detail. We find that the linear processors developed in Chapter I-6 play a fundamental role in the random signal problem. This result, coupled with the corresponding result in Chapter II-2, emphasizes the fundamental importance of the results in Chapter I-6. They also illustrate the inherent unity of the various problems. Specific topics such as power-spectrum parameter estimation and analog transmission over time-varying channels are also developed.

The fifth part is labeled *Applications* and includes Chapters II-4 and II-5. Throughout the two books we emphasize applications of the theory to models of practical problems. In most of them the relation of the actual physical situation can be explained in a page or two. The fifth part deals with physical situations in which developing the model from the physical situation is a central issue. Chapter II-4 studies the radar/sonar problem in depth. It builds up a set of target and channel models, starting with slowly fluctuating point targets and culminating in deep targets that fluctuate at arbitrary rates. This set of models enables us to study the signal design problem for radar and sonar, the resolution problem in mapping radars, the effect of reverberation on sonar-system performance, estimation of parameters of spread targets, communication over spread channels, and other important problems.

In Chapter II-5 we study various multidimensional problems such as multiplex communication systems and multivariable processing problems encountered in continuous receiving apertures and optical systems. The primary emphasis in the chapter is on optimum array processing in sonar (or seismic) systems. Both active and passive sonar systems are discussed; specific processor configurations are developed and their performance is analyzed.

Finally, in Chapter II-6 we summarize some of the more important results, mention some related topics that have been omitted, and suggest areas of future research.