Detection, Estimation, and Modulation Theory, Part I:
Detection, Estimation, and Linear Modulation Theory. Harry L. Van Trees
ISBNs: 0-471-09517-6 (Paperback); 0-471-22108-2 (Electronic)

7

Discussion

The next step in our development is to apply the results in Chapter 5 to the nonlinear estimation problem. In Chapter 4 we studied the problem of estimating a single parameter which was contained in the observed signal in a nonlinear manner. The discrete frequency modulation example we discussed illustrated the type of difficulties met. We anticipate that even more problems will appear when we try to estimate a sample function from a random process. This conjecture turns out to be true and it forces us to make several approximations in order to arrive at a satisfactory answer. To make useful approximations it is necessary to consider specific nonlinear estimation problems rather than the general case. In view of this specialization it seems appropriate to pause and summarize what we have already accomplished. In Chapter 2 of Part II we shall discuss nonlinear estimation. The remainder of Part II discusses random channels, radar/sonar signal processing, and array processing. In this brief chapter we discuss three issues.

1. The problem areas that we have studied. These correspond to the first two levels in the hierarchy outlined in Chapter 1.
2. The problem areas that remain to be covered when our development is resumed in Part II.
3. Some areas that we have encountered in our discussion that we do not pursue in Part II.

7.1 SUMMARY

Our initial efforts in this book were divided into developing background in the area of classical detection and estimation theory and random process representation. Chapter 2 developed the ideas of binary and *M*-ary hypothesis testing for Bayes and Neyman–Pearson criteria. The fundamental role of the likelihood ratio was developed. Turning to estimation theory, we studied Bayes estimation for random parameters and maximum-likelihood estimation for nonrandom parameters. The idea of likelihood functions, efficient estimates, and the Cramér-Rao inequality were of central importance in this discussion. The close relation between detection and estimation theory was emphasized in the composite hypothesis testing problem. A particularly important section from the standpoint of the remainder of the text was the general Gaussian problem. The model was simply the finite-dimensional version of the signals in noise problems which were the subject of most of our subsequent work. A section on performance bounds which we shall need in Chapter II.3 concluded the discussion of the classical problem.

In Chapter 3 we reviewed some of the techniques for representing random processes which we needed to extend the classical results to the waveform problem. Emphasis was placed on the Karhunen-Loéve expansion as a method for obtaining a set of uncorrelated random variables as a representation of the process over a finite interval. Other representations, such as the Fourier series expansion for periodic processes, the sampling theorem for bandlimited processes, and the integrated transform for stationary processes on an infinite interval, were also developed. As preparation for some of the problems that were encountered later in the text, we discussed various techniques for solving the integral equation that specified the eigenfunctions and eigenvalues. Simple characterizations for vector processes were also obtained. Later, in Chapter 6, we returned to the representation problem and found that the state-variable approach provided another method of representing processes in terms of a set of random variables. The relation between these two approaches is further developed in [1] (see Problem 6.6.2 of Chapter 6).

The next step was to apply these techniques to the solution of some basic problems in the communications and radar areas. In Chapter 4 we studied a large group of basic problems. The simplest problem was that of detecting one of two known signals in the presence of additive white Gaussian noise. The matched filter receiver was the first important result. The ideas of linear estimation and nonlinear estimation of parameters were derived. As pointed out at that time, these problems corresponded to engineering systems commonly referred to as uncoded PCM, PAM, and PFM, respectively. The extension to nonwhite Gaussian noise followed easily. In this case it was necessary to solve an integral equation to find the functions used in the optimum receiver, so we developed techniques to obtain these solutions. The method for dealing with unwanted parameters such as a random amplitude or random phase angle was derived and some typical cases were examined in detail. Finally, the extensions to multiple-parameter estimation and multiple-channel systems were accomplished. At this point in our development we could design and analyze digital communication systems and time-sampled analog communication systems operating in a reasonably general environment.

The third major area was the estimation of continuous waveforms. In Chapter 5 we approached the problem from the viewpoint of *maximum a posteriori probability* estimation. The primary result was a pair of integral equations that the optimum estimate had to satisfy. To obtain solutions we divided the problem into linear estimation which we studied in Chapter 6 and nonlinear estimation which we shall study in Chapter II.2.

Although MAP interval estimation served as a motivation and introduction to Chapter 6, the actual emphasis in the chapter was on minimum mean-square point estimators. After a brief discussion of general properties we concentrated on two classes of problems. The first was the optimum realizable point estimator for the case in which the processes were stationary and the infinite past was available. Using the Wiener-Hopf spectrum factorization techniques, we derived an algorithm for obtaining the form expressions for the errors. The second class was the optimum realizable point estimation problem for finite observation times and possibly nonstationary processes. After characterizing the processes by using state-variable methods, a differential equation that implicitly specified the optimum estimator was derived. This result, due to Kalman and Bucy, provided a computationally feasible way of finding optimum processors for complex systems. Finally, the results were applied to the types of linear modulation scheme encountered in practice, such as DSB-AM and SSB.

It is important to re-emphasize two points. The linear processors resulted from our initial Gaussian assumption and were the best processors of any kind. The use of the minimum mean-square error criterion was not a restriction because we showed in Chapter 2 that when the a posteriori density is Gaussian the conditional mean is the optimum estimate for any convex (upward) error criterion. Thus the results of Chapter 6 are of far greater generality than might appear at first glance.

7.2 PREVIEW OF PART II

In Part II we deal with four major areas. The first is the solution of the nonlinear estimation problem that was formulated in Chapter 5. In Chapter II.2 we study angle modulation in detail. The first step is to show how the integral equation that specifies the optimum MAP estimate suggests a demodulator configuration, commonly referred to as a phase-lock loop (PLL). We then demonstrate that under certain signal and noise conditions the output of the phase-lock loop is an asymptotically efficient estimate of the message. For the simple case which arises when the PLL is used for carrier synchronization an exact analysis of the performance in the nonlinear region is derived. Turning to frequency modulation, we investigate the design of optimum demodulators in the presence of bandwidth and power constraints. We then compare the performance of these optimum demodulators with conventional limiter-discriminators. After designing the optimum receiver the next step is to return to the transmitter and modify it to improve the overall system performance. This modification, analogous to the pre-emphasis problem in conventional frequency modulation, leads to an optimum angle modulation system. Our results in this chapter and Chapter I.4. give us the background to answer the following question: If we have an analog message to transmit, should we (a) sample and quantize it and use a digital transmission system, (b) sample it and use a continuous amplitude-time discrete system, or (c) use a continuous analog system? In order to answer this question we first use the rate-distortion results of Shannon to derive bounds on how well *any* system could do. We then compare the various techniques discussed above with these bounds. As a final topic in this chapter we show how state-variable techniques can be used to derive optimum nonlinear demodulators.

In Chapter II.3 we return to a more general problem. We first consider the problem of observing a received waveform $r(t)$ and deciding to which of two random processes it belongs. This type of problem occurs naturally in radio astronomy, scatter communication, and passive sonar. The likelihood ratio test leads to a quadratic receiver which can be realized in several mathematically equivalent ways. One receiver implementation, the estimator-correlator realization, contains the optimum linear filter as a component and lends further importance to the results in Chapter I.6. In a parallel manner we consider the problem of estimating parameters contained in the covariance function of a random process. Specific problems encountered in practice, such as the estimation of the center frequency of a bandpass process or the spectral width of a process, are discussed in detail. In both the detection and estimation cases particularly simple solutions are obtained when the processes are stationary and the observation times are long. To complete the hierarchy of problems outlined in Chapter 1 we study the problem of transmitting an analog message over a randomly time-varying channel. As a specific example, we study an angle modulation system operating over a Rayleigh channel.

In Chapter II.4 we show how our earlier results can be applied to solve detection and parameter estimation problems in the radar-sonar area. Because we are interested in narrow-band signals, we develop a representation for them by using complex envelopes. Three classes of target models are considered. The simplest is a slowly fluctuating point target whose range and velocity are to be estimated. We find that the issues of accuracy, ambiguity, and resolution must all be considered when designing signals and processors. The radar ambiguity function originated by Woodward plays a central role in our discussion. The targets in the next class are referred to as singly spread. This class includes fast-fluctuating point targets which spread the transmitted signal in frequency and slowly fluctuating dispersive targets which spread the signal in time (range). The third class consists of doubly spread targets which spread the signal in both time and frequency. Many diverse physical situations, such as reverberation in active sonar systems, communication over scatter channels, and resolution in mapping radars, are included as examples. The overall development provides a unified picture of modern radar-sonar theory.

The fourth major area in Part II is the study of multiple-process and multivariable process problems. The primary topic in this area is a detailed study of array processing in passive sonar (or seismic) systems. Optimum processors for typical noise fields are analyzed from the standpoint of signal waveform estimation accuracy, detection performance, and beam patterns. Two other topics, multiplex transmission systems and multivariable systems (e.g., continuous receiving apertures or optical systems), are discussed briefly.

In spite of the length of the two volumes, a number of interesting topics have been omitted. Some of them are outlined in the next section.

7.3 UNEXPLORED ISSUES

Several times in our development we have encountered interesting ideas whose complete discussion would have taken us too far afield. In this section we provide suitable references for further study.

Coded Digital Communication Systems. The most important topic we have not discussed is the use of coding techniques to reduce errors in systems that transmit sequences of digits. Shannon's classical information theory results indicate how well we can do, and a great deal of research has been devoted to finding ways to approach these bounds. Suitable texts in this area are Wozencraft and Jacobs [2], Fano [3], Gallager [4], Peterson [5], and Golomb [6]. Bibliographies of current papers appear in the "Progress in Information Theory" series [7] and [8].

Sequential Detection and Estimation Schemes. All our work has dealt with a fixed observation interval. Improved performance can frequently be obtained if a variable length test is allowed. The fundamental work in this area is due to Wald [10]. It was applied to the waveform problem by

Peterson, Birdsall, and Fox [11] and Bussgang and Middleton [12]. Since that time a great many papers have considered various aspects of the subject (e.g. [13] to [22]).

Nonparametric Techniques. Throughout our discussion we have assumed that the random variables and processes have known density functions. The nonparametric statistics approach tries to develop tests that do not depend on the density function. As we might expect, the problem is more difficult, and even in the classical case there are a number of unsolved problems. Two books in the classical area are Fraser [23] and Kendall [24]. Other references are given on pp. 261–264 of [25] and in [26]. The progress in the waveform case is less satisfactory. (A number of models start by sampling the input and then use a known classical result.) Some recent interesting papers include [27] to [32].

Adaptive and Learning Systems. Ever-popular adjectives are the words "adaptive" and "learning." By a suitable definition of adaptivity many familiar systems can be described as adaptive or learning systems. The basic notion is straightforward. We want to build a system to operate efficiently in an unknown or changing environment. By allowing the system to change its parameters or structure as a function of its input, the performance can be improved over that of a fixed system. The complexity of the system depends on the assumed model of the environment and the degrees of freedom that are allowed in the system. There are a large number of references in this general area. A representative group is [33] to [50] and [9].

Pattern Recognition. The problem of interest in this area is to recognize (or classify) patterns based on some type of imperfect observation. Typical areas of interest include printed character recognition, target classification in sonar systems, speech recognition, and various medical areas such as cardiology. The problem can be formulated in either a statistical or non-statistical manner. Our work would be appropriate to a statistical formulation.

In the statistical formulation the observation is usually reduced to an N-dimensional vector. If there are M possible patterns, the problem reduces to the finite dimensional M-hypothesis testing problem of Chapter 2. If we further assume a Gaussian distribution for the measurements, the general Gaussian problem of Section 2.6 is directly applicable. Some typical pattern recognition applications were developed in the problems of Chapter 2. A more challenging problem arises when the Gaussian assumption is not valid; [8] discusses some of the results that have been obtained and also provides further references.

Discrete Time Processes. Most of our discussion has dealt with observations that were sample functions from continuous time processes. Many of the results can be reformulated easily in terms of discrete time processes. Some of these transitions were developed in the problems.

Non-Gaussian Noise. Starting with Chapter 4, we confined our work to Gaussian processes. As we pointed out at the end of Chapter 4, various types of result can be obtained for other forms of interference. Some typical results are contained [51] to [57].

Receiver-to-Transmitter Feedback Systems. In many physical situations it is reasonable to have a feedback link from the receiver to the transmitter. The addition of the link provides a new dimension in system design and frequently enables us to achieve efficient performance with far less complexity than would be possible in the absence of feedback. Green [58] describes the work in the area up to 1961. Recent work in the area includes [59] to [63] and [76].

Physical Realizations. The end product of most of our developments has been a block diagram of the optimum receiver. Various references discuss practical realizations of these block diagrams. Representative systems that use the techniques of modern communication theory are described in [64] to [68].

Markov Process-Differential Equation Approach. With the exception of Section 6.3, our approach to the detection and estimation problem could be described as a "covariance function-impulse response" type whose success was based on the fact that the processes of concern were Gaussian. An alternate approach could be based on the Markovian nature of the processes of concern. This technique might be labeled the "state-variable-differential equation" approach and appears to offer advantages in many problems: [69] to [75] discuss this approach for some specific problems.

Undoubtedly there are other related topics that we have not mentioned, but the foregoing items illustrate the major ones.

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