Detection, Estimation, and Modulation Theory, Part I:
Detection, Estimation, and Linear Modulation Theory. Harry L. Van Trees

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Appendix: *A Typical Course Outline*

It would be presumptious of us to tell a professor how to teach a course at this level. On the other hand, we have spent a great deal of time experimenting with different presentations in search of an efficient and pedagogically sound approach. The concepts are simple, but the great amount of detail can cause confusion unless the important issues are emphasized. The following course outline is the result of these experiments; it should be useful to an instructor who is using the book or teaching this type of material for the first time.

The course outline is based on a 15-week term of three hours of lectures a week. The homework assignments, including reading and working the assigned problems, will take 6 to 15 hours a week. The prerequisite assumed is a course in random processes. Typically, it should include Chapters 1 to 6, 8, and 9 of Davenport and Root or Chapters 1 to 10 of Papoulis. Very little specific material in either of these references is used, but the student needs a certain level of sophistication in applied probability theory to appreciate the subject material.

Each lecture unit corresponds to a one-and-one-half-hour lecture and contains a topical outline, the corresponding text material, and additional comments when necessary. Each even-numbered lecture contains a problem assignment. A set of solutions for this collection of problems is available.

In a normal term we get through the first 28 lectures, but this requires a brisk pace and leaves no room for making up background deficiencies. An ideal format would be to teach the material in two 10-week quarters. The expansion from 32 to 40 lectures is easily accomplished (probably without additional planning). A third alternative is two 15-week terms, which would allow time to cover more material in class and reduce the homework load. We have not tried either of the last two alternatives, but student comments indicate they should work well.

One final word is worthwhile. There is a great deal of difference between reading the text and being able to apply the material to solve actual problems of interest. This book (and therefore presumably any course using it) is designed to train engineers and scientists to solve new problems. The only way for most of us to acquire this ability is by practice. Therefore any effective course must include a fair amount of problem solving and a critique (or grading) of the students' efforts.

	Lecture 1 **pp. 1–18**
Chapter 1	Discussion of the physical situations that give rise to detection, estimation, and modulation theory problems
1.1	Detection theory, 1–5 Digital communication systems (known signal in noise), 1–2 Radar/sonar systems (signal with unknown parameters), 3 Scatter communication, passive sonar (random signals), 4 Show hierarchy in Fig. 1.4, 5 Estimation theory, 6–8 PAM and PFM systems (known signal in noise), 6 Range and Doppler estimation in radar (signal with unknown parameters), 7 Power spectrum parameter estimation (random signal), 8 Show hierarchy in Fig. 1.7, 8 Modulation theory, 9–12 Continuous modulation systems (AM and FM), 9 Show hierarchy in Fig. 1.10, 11
1.2	Various approaches, 12–15 Structured versus nonstructured, 12–15 Classical versus waveform, 15
1.3	Outline of course, 15–18

	Lecture 2 **pp. 19–33**
Chapter 2	Formulation of the hypothesis testing problem, 19–23
2.2.1	Decision criteria (Bayes), 23–30 Necessary inputs to implement test; a priori probabilities and costs, 23–24 Set up risk expression and find LRT, 25–27 Do three examples in text, introduce idea of sufficient statistic, 27–30 Minimax test, 31–33 Minimum Pr(ϵ) test, idea of maximum a posteriori rule, 30 **Problem Assignment 1** **1.** 2.2.1 **2.** 2.2.2

	Lecture 3 **pp. 33–52**
	Neyman–Pearson tests, 33–34 Fundamental role of LRT, relative unimportance of the criteria, 34 Sufficient statistics, 34 Definition, geometric interpretation

2.2.2	Performance, idea of ROC, 36–46
	    Example 1 on pp. 36–38; Bound on $\text{erfc}_*(X)$, (72)
	Properties of ROC, 44–48
	    Concave down, slope, minimax

2.3	M-Hypotheses, 46–52
	    Set up risk expression, do $M = 3$ case and demonstrate that the decision space has at most two dimensions; emphasize that regardless of the observation space dimension the decision space has at most $M - 1$ dimensions, 52
	    Develop idea of maximum a posteriori probability test (109)

Comments

The randomized tests discussed on p. 43 are not important in the sequel and may be omitted in the first reading.

	Lecture 4	**pp. 52–62**

2.4	Estimation theory, 52
	Model, 52–54
	    Parameter space, question of randomness, 53
	    Mapping into observation space, 53
	    Estimation rule, 53
	Bayes estimation, 54–63
	    Cost functions, 54
	    Typical single-argument expressions, 55
	      Mean-square, absolute magnitude, uniform, 55
	    Risk expression, 55
	    Solve for $\hat{a}_{ms}(\mathbf{R})$, $\hat{a}_{map}(\mathbf{R})$, and $\hat{a}_{mae}(\mathbf{R})$, 56–58
	    Linear example, 58–59
	    Nonlinear example, 62
	
	Problem Assignment 2
	1. 2.2.10 **5.** 2.3.3
	2. 2.2.15 **6.** 2.3.5
	3. 2.2.17 **7.** 2.4.2
	4. 2.3.2 **8.** 2.4.3	

	Lecture 5 **pp. 60–69**

	Bayes estimation (*continued*)
	  Convex cost criteria, 60–61
	  Optimality of $\hat{a}_{\mathrm{ms}}(\mathbf{R})$, 61

2.4.2	Nonrandom parameter estimation, 63–73
	  Difficulty with direct approach, 64
	  Bias, variance, 64
	Maximum likelihood estimation, 65
	Bounds
	  Cramér-Rao inequality, 66–67
	  Efficiency, 66
	  Optimality of $\hat{a}_{\mathrm{m}l}(\mathbf{R})$ when efficient estimate exists, 68
	Linear example, 68–69

	Lecture 6 **pp. 69–98**

	Nonrandom parameter estimation (*continued*)
	Nonlinear example, 69
	    Asymptotic results, 70–71
	    Intuitive explanation of when C–R bound is accurate, 70–71
	Bounds for random variables, 72–73

2.4.3, 2.4.4 **2.5**	Assign the sections on multiple parameter estimation and composite hypothesis testing for reading, 74–96

2.6	General Gaussian problem, 96–116
	    Definition of a Gaussian random vector, 96
	    Expressions for $M_{\mathbf{r}}(j\mathbf{v})$, $p_{\mathbf{r}}(\mathbf{R})$, 97
	    Derive LRT, define quadratic forms, 97–98

	Problem Assignment 3
	1. 2.4.9 **5.** 2.6.1 **2.** 2.4.12 *Optional* **3.** 2.4.27 **6.** 2.5.1 **4.** 2.4.28

	Lecture 7 **pp. 98–133**
2.6	General Gaussian problem (*continued*) Equal covariance matrices, unequal mean vectors, 98–107 Expression for d^2, interpretation as distance, 99–100 Diagonalization of **Q**, eigenvalues, eigenvectors, 101–107 Geometric interpretation, 102 Unequal covariance matrices, equal mean vectors, 107–116 Structure of LRT, interpretation as estimator, 107 Diagonal matrices with identical components, χ^2 density, 107–111 Computational problem, motivation of performance bounds, 116
2.7	Assign section on performance bounds as reading, 116–133

Comments

1. (p. 99) Note that $\text{Var}[l \mid H_1] = \text{Var}[l \mid H_0]$ and that d^2 completely characterizes the test because l is Gaussian on both hypotheses.
2. (p. 119) The idea of a "tilted" density seems to cause trouble. Emphasize the motivation for tilting.

	Lecture 8	**pp. 166–186**
Chapter 3		

3.1, 3.2	Extension of results to waveform observations
	Deterministic waveforms
	    Time-domain and frequency-domain characterizations, 166–169
	    Orthogonal function representations, 169–174
	    Complete orthonormal sets, 171
	    Geometric interpretation, 172–174	

3.3, 3.3.1	Second-moment characterizations, 174–178 Positive definiteness, nonnegative definiteness, and symmetry of covariance functions, 176–177	

3.3.3	Gaussian random processes, 182
	    Difficulty with usual definitions, 185
	    Definition in terms of a linear functional, 183
	    Jointly Gaussian processes, 185
	    Consequences of definition; joint Gaussian density at any set of times, 184–185	

	Problem Assignment 4 **1.** 2.6.2 *Optional* **2.** 2.6.4. **5.** 2.7.1 **3.** 2.6.8 **6.** 2.7.2 **4.** 2.6.10	

	Lecture 9	**pp. 178–194**
3.3.2	Orthogonal representation for random processes, 178–182 Choice of coefficients to minimize mean-square representation error, 178 Choice of coordinate system, 179 Karhunen-Loeve expansion, 179 Properties of integral equations, 180–181 Analogy with finite-dimensional case in Section 2.6 The following comparison should be made:	

Gaussian definition $z = \mathbf{g}^T\mathbf{x}$	$z = \int_0^T g(u)\, x(u)\, du$
Symmetry $K_{ij} = K_{ji}$	$K_x(t, u) = K_x(u, t)$
Nonnegative definiteness $\mathbf{x}^T\mathbf{K}\mathbf{x} \geq 0$	$\int_0^T dt \int_0^T du\, x(t)\, K_x(t, u)\, x(u) \geq 0$
Coordinate system $\mathbf{K}\boldsymbol{\phi} = \lambda\boldsymbol{\phi}$	$\int_0^T K_x(t, u)\, \phi(u)\, du = \lambda\phi(t) \quad 0 \leq t \leq T$
Orthogonality $\boldsymbol{\phi}_i^T\boldsymbol{\phi}_j = \delta_{ij}$	$\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \delta_{ij}$

Mercer's theorem, 181
Convergence in mean-square sense, 182

3.4, 3.4.1, 3.4.2	Assign Section 3.4 on integral equation solutions for reading, 186–194

	Lecture 10	**pp. 186–226**
3.4.1	Solution of integral equations, 186–196	
3.4.2	Basic technique, obtain differential equation, solve, and satisfy boundary conditions, 186–191	
3.4.3	Example: Wiener process, 194–196	
3.4.4	White noise and its properties, 196–198 Impulsive covariance function, flat spectrum, orthogonal representation, 197–198	
3.4.5	Optimum linear filter, 198–204 This derivation illustrates variational procedures; the specific result is needed in Chapter 4; the integral equation (144) should be emphasized because it appears in many later discussions; a series solution in terms of eigenvalues and eigenfunctions is adequate for the present	
3.4.6–3.7	Assign the remainder of Chapter 3 for reading; Sections 3.5 and 3.6 are not used in the text (some problems use the results); Section 3.7 is not needed until Section 4.5 (the discussion of vector processes can be avoided until Section 6.3, when it becomes essential), 204–226	
	Problem Assignment 5 **1.** 3.3.1 **5.** 3.4.4 **2.** 3.3.6 **6.** 3.4.6 **3.** 3.3.19 **7.** 3.4.8 **4.** 3.3.22	

Comment

At the beginning of the derivation on p. 200 we assumed $h(t, u)$ was continuous. Whenever there is a white noise component in $r(t)$ (143), this restriction does not affect the performance. When $K_r(t, u)$ does not contain an impulse, a discontinuous filter may perform better. The optimum filter satisfies (138) for $0 \leq u \leq T$ if discontinuities are allowed.

	Lecture 11 **pp. 239–257**
Chapter 4	
4.1	Physical situations in which detection problem arises Communication, radar/sonar, 239–246
4.2	Detection of known signals in additive white Gaussian noise, 246–271 Simple binary detection, 247 Sufficient statistic, reduction to scalar problem, 248 Applicability of ROC in Fig. 2.9 with $d^2 = 2E/N_0$, 250–251 Lack of dependence on signal shape, 253 General binary detection, 254–257 Coordinate system using Gram-Schmidt, 254 LRT, reduction to single sufficient statistic, 256 Minimum-distance rule; "largest-of" rule, 257 Expression for d^2, 256 Optimum signal choice, 257

	Lecture 12 **pp. 257–271**
4.2.1	M-ary detection in white Gaussian noise, 257 Set of sufficient statistics; Gram-Schmidt procedure leads to at most M (less if signals have some linear dependence); Illustrate with PSK and FSK set, 259 Emphasize equal a priori probabilities and minimum $\Pr(\epsilon)$ criterion. This leads to minimum-distance rules. Go through three examples in text and calculate $\Pr(\epsilon)$ Example 1 illustrates symmetry and rotation, 261 Example 3 illustrates complexity of exact calculation for a simple signal set. Derive bound and discuss accuracy, 261–264 Example 4 illustrates the idea of transmitting sequences of digits and possible performance improvements, 264–267 Sensitivity, 267–271 Functional variation and parameter variation; this is a mundane topic that is usually omitted; it is a crucial issue when we try to implement optimum systems and must measure the quantities needed in the mathematical model
	Problem Assignment 6 **1.** 4.2.4 **5.** 4.2.16 **2.** 4.2.6 **6.** 4.2.23 **3.** 4.2.8 **4.** 4.2.9

	Lecture 13 **pp. 271–278**

4.2.2	Estimation of signal parameters
	  Derivation of likelihood function, 274
	  Necessary conditions on $\hat{a}_{\text{map}}(r(t))$ and $\hat{a}_{\text{ml}}(r(t))$, 274
	  Generalization of Cramér-Rao inequality, 275
	  Conditions for efficient estimates, 276
	Linear estimation, 271–273
	  Simplicity of solution, 272
	  Relation to detection problem, 273

4.2.3	Nonlinear estimation, 273–286
	  Optimum receiver for estimating arrival time (equivalently, the PPM problem), 276
	  Intuitive discussion of system performance, 277

	Lecture 14 **pp. 278–289**
4.2.3	Nonlinear estimation (*continued*), 278–286 Pulse frequency modulation (PFM), 278 An approximation to the optimum receiver (interval selector followed by selection of local maximum), 279 Performance in weak noise, effect of βT product, 280 Threshold analysis, using orthogonal signal approximation, 280–281 Bandwidth constraints, 282 Design of system under threshold and bandwidth constraints, 282 Total mean-square error, 283–285
4.2.4	Summary: known signals in white noise, 286–287
4.3	Introduction to colored noise problem, 287–289 Model, observation interval, motivation for including white noise component
	Problem Assignment 7 **1.** 4.2.25 **3.** 4.2.28 **2.** 4.2.26

Comments on Lecture 15 (lecture is on p. 651)

1. As in Section 3.4.5, we must be careful about the endpoints of the interval. If there is a white noise component, we may choose a continuous $g(t)$ without affecting the performance. This leads to an integral equation on an open interval. If there is no white component, we must use a closed interval and the solution will usually contain singularities.

2. On p. 296 it is useful to emphasize the analogy between an inverse kernel and an inverse matrix.

	Lecture 15 **pp. 289–333**
4.3	Colored noise problem
4.3.1, 4.3.2, 4.3.3	Possible approaches: Karhunen-Loéve expansion, prewhitening filter, generation of sufficient statistic, 289
	Reversibility proof (this idea is used several times in the text), 289–290
	Whitening derivation, 290–297
	Define whitening filter, $h_w(t, u)$, 291
	Define inverse kernel $Q_n(t, u)$ and function for correlator $g(t)$, 292
	Derive integral equations for above functions, 293–297
	Draw the three realizations for optimum receiver (Fig. 4.38), 293
	Construction of $Q_n(t, u)$
	Interpretation as impulse minus optimum linear filter, 294–295
	Series solutions, 296–297
4.3.4	Performance, 301–307
	Expression for d^2 as quadratic form and in terms of eigenvalues, 302
	Optimum signal design, 302–303
	Singularity, 303–305
	Importance of white noise assumption and effect of removing it
4.3.8	Duality with known channel problem, 331–333
4.3.5–4.3.8	Assign as reading,
	1. Estimation (4.3.5)
	2. Solution to integral equations (4.3.6)
	3. Sensitivity (4.3.7)
	4. Known linear channels (4.3.8)
	Problem Set 7 (*continued*)
	4. 4.3.4 **7.** 4.3.12
	5. 4.3.7 **8.** 4.3.21
	6. 4.3.8

	Lecture 16 **pp. 333–377**
4.4	Signals with unwanted parameters, 333–366 Example of random phase problem to motivate the model, 336 Construction of LRT by integrating out unwanted parameters, 334 Models for unwanted parameters, 334
4.4.1	Random phase, 335–348 Formulate bandpass model, 335–336 Go to $\Lambda(r(t))$ by inspection, define quadrature sufficient statistics L_c and L_s, 337 Introduce phase density (364), motivate by brief phaselock loop discussion, 337 Obtain LRT, discuss properties of $\ln I_0(x)$, point out it can be eliminated here but will be needed in diversity problems, 338–341 Compute ROC for uniform phase case, 344 Introduce Marcum's Q function Discuss extension to binary and M-ary case, signal selection in partly coherent channels
4.4.2	Random amplitude and phase, 349–366 Motivate Rayleigh channel model, piecewise constant approximation, possibility of continuous measurement, 349–352 Formulate in terms of quadrature components, 352 Solve general Gaussian problem, 352–353 Interpret as filter-squarer receiver and estimator-correlator receiver, 354 Apply Gaussian result to Rayleigh channel, 355 Point out that performance was already computed in Chapter 2 Discuss Rician channel, modifications necessary to obtain receiver, relation to partially-coherent channel in Section 4.4.1, 360–364
4.5–4.7	Assign Section 4.6 to be read before next lecture; Sections 4.5 and 4.7 may be read later, 366–377

Lecture 16 *(continued)*

Problem Assignment 8

1.	4.4.3	**5.**	4.4.29
2.	4.4.5	**6.**	4.4.42
3.	4.4.13	**7.**	4.6.6
4.	4.4.27		

	Lecture 17 **pp. 370–460**
4.6	Multiple-parameter estimation, 370–374 Set up a model and derive MAP equations for the colored noise case, 374 The examples can be left as a reading assignment but the MAP equations are needed for the next topic
Chapter 5	Continuous waveform estimation, 423–460
5.1, 5.2	Model of problem, typical continuous systems such as AM, PM, and FM; other problems such as channel estimation; linear and nonlinear modulation, 423–426 Restriction to no-memory modulation, 427 Definition of $\hat{a}_{\text{map}}(r(t))$ in terms of an orthogonal expansion, complete equivalence to multiple parameter problem, 429–430 Derivation of MAP equations (31–33), 427–431 Block diagram interpretation, 432–433 Conditions for an efficient estimate to exist, 439
5.3–5.6	Assign remainder of Chapter 5 for reading, 433–460

Lecture 18	pp. 467–481

Chapter 6 Linear modulation

6.1 Model for linear problem, equations for MAP interval estimation, 467–468

Property 1: MAP estimate can be obtained by using linear processor; derive integral equation, 468

Property 2: MAP and MMSE estimates coincide for linear modulation because efficient estimate exists, 470

Formulation of linear *point* estimation problem, 470

Gaussian assumption, 471

Structured approach, linear processors

Property 3: Derivation of optimum linear processor, 472

Property 4: Derivation of error expression [emphasize (27)], 473

Property 5: Summary of information needed, 474

Property 6: Optimum error and received waveform are uncorrelated, 474

Property 6A: In addition, $e_0(t)$ and $r(u)$ are statistically independent under Gaussian assumption, 475

Optimality of linear filter under Gaussian assumption, 475–477

Property 7: Prove no other processor could be better, 475

Property 7A: Conditions for uniqueness, 476

Generalization of criteria (Gaussian assumption), 477

Property 8: Extend to convex criteria; uniqueness for strictly convex criteria; extend to monotone increasing criteria, 477–478

Relationship to interval and MAP estimators

Property 9: Interval estimate is a collection of point estimates, 478

Property 10: MAP point estimates and MMSE point estimates coincide, 479

Lecture 18 ***(continued)***

Summary, 481

Emphasize interplay between structure, criteria, and Gaussian assumption

Point out that the optimum linear filter plays a central role in nonlinear modulation (Chapter II.2) and detection of random signals in random noise (Chapter II.3)

Problem Assignment 9

1. 5.2.1 **3.** 6.1.1

2. 5.3.5 **4.** 6.1.4

	Lecture 19 **pp. 481–515**
6.2	Realizable linear filters, stationary processes, infinite time (Wiener-Hopf problem) Modification of general equation to get Wiener-Hopf equation, 482 Solution of Wiener-Hopf equation, rational spectra, 482 Whitening property; illustrate with one-pole example and then indicate general case, 483–486 Demonstrate a unique spectrum factorization procedure, 485 Express $H_0'(j\omega)$ in terms of original quantities; define "realizable part" operator, 487 Combine to get final solution, 488 Example of one-pole spectrum plus white noise, 488–493 Desired signal is message shifted in time Find optimum linear filter and resulting error, 494 Prove that the mean-square error is a monotone increasing function of α, the prediction time; emphasize importance of filtering with delay to reduce the mean-square error, 493–495 Unrealizable filters, 496–497 Solve equation by using Fourier transforms Emphasize that error performance is easy to compute and bounds the performance of a realizable system; unrealizable error represents ultimate performance; filters can be approximated arbitrarily closely by allowing delay Closed-form error expressions in the presence of white noise, 498–508 Indicate form of result and its importance in system studies Assign derivation as reading Discuss error behavior for Butterworth family, 502 Assign the remainder of Section 6.2 as reading, 508–515 **Problem Assignment 9** (*continued*) **5.** 6.2.1 **7.** 6.2.7 **6.** 6.2.3 **8.** 6.2.43

	Lecture 20*	**pp. 515–538**
6.3	State-variable approach to optimum filters (Kalman-Bucy problem), 515–575	

6.3.1	Motivation for differential equation approach, 515
	State variable representation of system, 515–526
	    Differential equation description
	    Initial conditions and state variables
	    Analog computer realization
	    Process generation
	Example 1: First-order system, 517
	Example 2: System with poles only, introduction of vector differential equation, vector block diagrams, 518
	Example 3: General linear differential equation, 521
	Vector-inputs, time-varying coefficients, 527
	System observation model, 529
	State transition matrix, $\boldsymbol{\phi}(t, \tau)$, 529
	    Properties, solution for time-invariant case, 529–531
	    Relation to impulse response, 532
	Statistical properties of system driven by white noise
	    Properties 13 and 14, 532–534
	Linear modulation model in presence of white noise, 534
	Generalizations of model, 535–538	

	Problem Assignment 10 **1.** 6.3.1 **4.** 6.3.9 **2.** 6.3.4 **5.** 6.3.12 **3.** 6.3.7(a, b) **6.** 6.3.16	

*Lectures 20–22 may be omitted if time is a limitation and the material in Lectures 28–30 on the radar/sonar problem is of particular interest to the audience. For graduate students the material in 20–22 should be used because of its fundamental nature and importance in current research.

	Lecture 22 **pp. 546–586**
6.3.3	Applications to typical estimation problems, 546–566 Example 1: One-pole spectrum, transient behavior, 546 Example 3: Wiener process, relation to pole-splitting, 555 Example 4: Canonic receiver for stationary messages in single channel, 556 Example 5: FM problem; emphasize that optimum realizable estimation commutes with linear transformations, *not* linear filtering (this point seems to cause confusion unless discussed explicitly), 557–561 Example 7: Diversity system, maximal ratio combining, 564–565
6.3.4	Generalizations, 566–575 List the eight topics in Section 6.3.4 and explain why they are of interest; assign derivations as reading Compare state-variable approach to conventional Wiener approach, 575
6.4	Amplitude modulation, 575–584 Derive synchronous demodulator; assign the remainder of Section 6.4 as reading
6.5–6.6	Assign remainder of Chapter 6 as reading. Emphasize the importance of optimum linear filters in other areas
	Problem Assignment 11 **1.** 6.3.23 **4.** 6.3.37 **2.** 6.3.27 **5.** 6.3.43 **3.** 6.3.32 **6.** 6.3.44

	Lecture 23*

Chapter II–2	Nonlinear modulation
	Model of angle modulation system
	    Applications; synchronization, analog communication
	    Intuitive discussion of what optimum demodulator should be

II–2.2	MAP estimation equations

II–2.3	Derived in Chapter I–5, specialize to phase modulation
	    Interpretation as unrealizable block diagram
	    Approximation by realizable loop followed by unrealizable postloop filter
	    Derivation of linear model, loop error variance constraint
	Synchronization example
	    Design of filters
	Nonlinear behavior
	    Cycle-skipping
	    Indicate method of performing exact nonlinear analysis

*The problem assignments for Lectures 23–32 will be included in Appendix 1 of Part II.

	Lecture 24
II–2.5	Frequency modulation Design of optimum demodulator Optimum loop filters and postloop filters Signal-to-noise constraints Bandwidth constraints Indicate comparison of optimum demodulator and conventional limiter-discriminator Discuss other design techniques
II–2.6	Optimum angle modulation Threshold and bandwidth constraints Derive optimum pre-emphasis filter Compare with optimum FM systems

	Lecture 25

II–2.7	Comparison of various systems for transmitting analog messages
	Sampled and quantized systems
	    Discuss simple schemes such as binary and M-ary signaling
	    Derive expressions for system transmitting at channel capacity
	Sampled, continuous amplitude systems
	    Develop PFM system, use results from Chapter I–4, and compare with continuous FM system
	Bounds on analog transmission
	    Rate-distortion functions
	    Expression for Gaussian sources
	    Channel capacity formulas
	    Comparison for infinite-bandwidth channel of continuous modulation schemes with the bound
	    Bandlimited message and bandlimited channel
	        Comparison of optimum FM with bound
	        Comparison of simple companding schemes with bound
	Summary of analog message transmission and continuous waveform estimation

	Lecture 26
Chapter II-3	Gaussian signals in Gaussian noise
3.2.1	Simple binary problem, white Gaussian noise on H_0 and H_1, additional colored Gaussian noise on H_1 Derivation of LRT using Karhunen-Loéve expansion Various receiver realizations Estimator-correlator Filter-squarer Structure with optimum realizable filter as component (this discussion is most effective when Lectures 20–22 are included; it should be mentioned, however, even if they were not studied) Computation of bias terms Performance bounds using $\mu(s)$ and tilted probability densities (at this point we must digress and develop the material in Section 2.7 of Chapter I–2). Interpretation of $\mu(s)$ in terms of realizable filtering errors Example: Structure and performance bounds for the case in which additive colored noise has a one-pole spectrum

	Lecture 27

3.2.2	General binary problem
	    Derive LRT using whitening approach
	    Eliminate explicit dependence on white noise
	    Singularity
	    Derive $\mu(s)$ expression
	Symmetric binary problems
	    $\Pr(\epsilon)$ expressions, relation to Bhattacharyya distance
	    Inadequacy of signal-to-noise criterion

	Lecture 28
Chapter II–3	Special cases of particular importance
3.2.3	Separable kernels Time diversity Frequency diversity Eigenfunction diversity Optimum diversity
3.2.4	Coherently undetectable case Receiver structure Show how $\mu(s)$ degenerates into an expression involving d^2
3.2.5	Stationary processes, long observation times Simplifications that occur in receiver structure Use the results in Lecture 26 to show when these approximations are valid Asymptotic formulas for $\mu(s)$ Example: Do same example as in Lecture 26 Plot P_D versus kT for various E/N_0 ratios and P_F's Find the optimum kT product (this is continuous version of the optimum diversity problem)
	Assign remainder of Chapter II–3 (Sections 3.3–3.6) as reading

	Lecture 29
Chapter II–4	Radar-sonar problem
4.1	Representation of narrow-band signals and processes Typical signals, quadrature representation, complex signal representation. Derive properties: energy, correlation, moments; narrow-band random processes; quadrature and complex waveform representation. Complex state variables Possible target models; develop target hierarchy in Fig. 4.6
4.2	Slowly-fluctuating point targets System model Optimum receiver for estimating range and Doppler Develop time-frequency autocorrelation function and radar ambiguity function Examples: Rectangular pulse Ideal ambiguity function Sequence of pulses Simple Gaussian pulse Effect of frequency modulation on the signal ambiguity function Accuracy relations

	Lecture 30
4.2.4	Properties of autocorrelation functions and ambiguity functions. Emphasize: Property 3: Volume invariance Property 4: Symmetry Property 6: Scaling Property 11: Multiplication Property 13: Selftransform Property 14: Partial volume invariances Assign the remaining properties as reading
4.2.5	Pseudo-random signals Properties of interest Generation using shift-registers
4.2.6	Resolution Model of problem, discrete and continuous resolution environments, possible solutions: optimum or "conventional" receiver Performance of conventional receiver, intuitive discussion of optimal signal design
	Assign the remainder of Section 4.2.6 and Section 4.2.7 as reading.

	Lecture 31

4.3	Singly spread targets (or channels)
	    Frequency spreading—delay spreading
	Model for Doppler-spread channel
	    Derivation of statistics (output covariance)
	    Intuitive discussion of time-selective fading
	Optimum receiver for Doppler-spread target (simple example)
	
	Assign the remainder of Section 4.3 as reading
	
	Doubly spread targets
	    Physical problems of interest: reverberation, scatter communication
	    Model for doubly spread return, idea of target or channel scattering function
	Reverberation (resolution in a dense environment)
	    Conventional or optimum receiver
	    Interaction between targets, scattering function, and signal ambiguity function
	    Typical target-reverberation configurations
	      Optimum signal design
	
	Assign the remainder of Chapter II–4 as reading

	Lecture 32
Chapter II–5	
5.1	Physical situations in which multiple waveform and multiple variable problems arise Review vector Karhunen-Loéve expansion briefly (Chapter I–3)
5.3	Formulate array processing problem for sonar
5.3.1	Active sonar Consider single-signal source, develop array steering Homogeneous noise case, array gain Comparison of optimum space-time system with conventional space-optimum time system Beam patterns Distributed noise fields Point noise sources
5.3.2	Passive sonar Formulate problem, indicate result. Assign derivation as reading Assign remainder of Chapter II–5 and Chapter II–6 as reading