Detection, Estimation, and Modulation Theory, Part I:
Detection, Estimation, and Linear Modulation Theory. Harry L. Van Trees

ISBNs: 0-471-09517-6 (Paperback); 0-471-22108-2 (Electronic)

Glossary

In this section we discuss the conventions, abbreviations, and symbols used in the book.

CONVENTIONS

The following conventions have been used:

1. Boldface roman denotes a vector or matrix.
2. The symbol $|\ \ |$ means the magnitude of the vector or scalar contained within.
3. The determinant of a square matrix **A** is denoted by $|\mathbf{A}|$ or det **A**.
4. The script letters $\mathcal{F}(\cdot)$ and $\mathcal{L}(\cdot)$ denote the Fourier transform and Laplace transform respectively.
5. Multiple integrals are frequently written as,

$$\int d\tau\, f(\tau) \int dt\, g(t, \tau) \triangleq \int f(\tau) \left\{\int dt\, g(t, \tau)\right\} d\tau,$$

that is, an integral is inside all integrals to its left unless a multiplication is specifically indicated by parentheses.

6. $E[\cdot]$ denotes the statistical expectation of the quantity in the bracket. The overbar $\bar{x}$ is also used infrequently to denote expectation.
7. The symbol $\otimes$ denotes convolution.

$$x(t) \otimes y(t) \triangleq \int_{-\infty}^{\infty} x(t - \tau) y(\tau)\, d\tau$$

8. Random variables are lower case (e.g., x and $\mathbf{x}$). Values of random variables and nonrandom parameters are capital (e.g., X and $\mathbf{X}$). In some estimation theory problems much of the discussion is valid for both random and nonrandom parameters. Here we depart from the above conventions to avoid repeating each equation.

9. The probability density of x is denoted by $p_x(\cdot)$ and the probability distribution by $P_x(\cdot)$. The probability of an event A is denoted by $\Pr[A]$. The probability density of x, given that the random variable a has a value A, is denoted by $P_{x|a}(X|A)$. When a probability density depends on non-random parameter A we also use the notation $p_{x|a}(X|A)$. (This is non-standard but convenient for the same reasons as 8.)

10. A vertical line in an expression means "such that" or "given that"; that is $\Pr[A|x \leq X]$ is the probability that event A occurs given that the random variable x is less than or equal to the value of X.

11. Fourier transforms are denoted by both $F(j\omega)$ and $F(\omega)$. The latter is used when we want to emphasize that the transform is a real-valued function of ω. The form used should always be clear from the context.

12. Some common mathematical symbols used include,

(i) $\propto$	proportional to
(ii) $t \to T^-$	t approaches T from below
(iii) $A + B \triangleq A \cup B$	A or B or both
(iv) l.i.m.	limit in the mean
(v) $\int_{-\infty}^{\infty} d\mathbf{R}$	an integral over the same dimension as the vector
(vi) $\mathbf{A}^T$	transpose of $\mathbf{A}$
(vii) $\mathbf{A}^{-1}$	inverse of $\mathbf{A}$
(viii) $\mathbf{0}$	matrix with all zero elements
(ix) $\binom{N}{k}$	binomial coefficient $\left(= \frac{N!}{k!\,(N-k)!}\right)$
(x) $\triangleq$	defined as
(xi) $\int_{\Omega} d\mathbf{R}$	integral over the set Ω

ABBREVIATIONS

Some abbreviations used in the text are:

ML	maximum likelihood
MAP	maximum a posteriori probability
PFM	pulse frequency modulation
PAM	pulse amplitude modulation
FM	frequency modulation
DSB-SC-AM	double-sideband–suppressed carrier–amplitude modulation
DSB-AM	double sideband-amplitude modulation

PM	phase modulation
NLNM	nonlinear no-memory
FM/FM	two-level frequency modulation
MMSE	minimum mean-square error
ERB	equivalent rectangular bandwidth
UMP	uniformly most powerful
ROC	receiver operating characteristic
LRT	likelihood ratio test

SYMBOLS

The principal symbols used are defined below. In many cases the vector symbol is an obvious modification of the scalar symbol and is not included.

A_a	actual value of parameter
A_i	sample at t_i
$\tilde{a}(t)$	Hilbert transform of $a(t)$
$\hat{a}_{\text{abs}}$	minimum absolute error estimate of a
$\hat{a}_{\text{map}}$	maximum a posteriori probability estimate of a
$\hat{a}_{\text{ml}}$	maximum likelihood estimate of A
$\hat{a}_{\text{ml}}(t)$	maximum likelihood estimate of $a(t)$
$\hat{a}_{\text{ms}}$	minimum mean-square estimate of a
α	amplitude weighting of specular component in Rician channel
α	constraint on P_F (in Neyman-Pearson test)
α	delay or prediction time (in context of waveform estimation)
B	constant bias
$B(A)$	bias that is a function of A
$\mathbf{B}_d(t)$	matrix in state equation for desired signal
β	parameter in PFM and angle modulation
C	channel capacity
$C(a_\epsilon)$	cost of an estimation error, a_ϵ
$C(\hat{a}, a)$	cost of estimating a when a is the actual parameter
$C(d_\epsilon(t))$	cost function for point estimation
C_F	cost of a false alarm (say H_1 when H_0 is true)
C_{ij}	cost of saying H_i is true when H_j is true
C_M	cost of a miss (say H_0 when H_1 is true)
C_∞	channel capacity, infinite bandwidth
$\mathbf{C}(t)$	modulation (or observation) matrix
$\mathbf{C}_d(t)$	observation matrix, desired signal

$\mathbf{C}_M(t)$	message modulation matrix
$\mathbf{C}_N(t)$	noise modulation matrix
χ	parameter space
χ_a	parameter space for a
χ_θ	parameter space for θ
χ^2	chi-square (description of a probability density)
$D(\omega^2)$	denominator of spectrum
d	desired function of parameter
d	performance index parameter on ROC for Gaussian problems
$\hat{d}$	estimate of desired function
$d(t)$	desired signal
$\hat{d}(t)$	estimate of desired signal
d_a	actual performance index
$\hat{d}_B(t)$	Bayes point estimate
d_f	parameter in FM system (frequency deviation)
$\hat{d}_o(t)$	optimum MMSE estimate
$d_s(t, a(t))$	derivative of $s(t, a(t))$ with respect to $a(t)$
$d_\epsilon(t)$	error in desired point estimate
$d_*(t)$	output of arbitrary nonlinear operation
δ	phase of specular component (Rician channel)
Δ	interval in PFM detector
Δd	change in performance index
Δd_x	desired change in d
ΔN	change in white noise level
Δ_n	constraint on covariance function error
$\mathbf{\Delta m}$	mean difference vector (i.e., vector denoting the difference between two mean vectors)
$\mathbf{\Delta Q}$	matrix denoting difference between two inverse covariance matrices
E	energy (no subscript when there is only one energy in the problem)
E_a	expectation over the random variable a only
$E_e(N)$	energy in error waveform (as a function of the number of terms in approximating series)
E_I	energy in interfering signal
E_i	energy on ith hypothesis
$\bar{E}_r$	expected value of received energy
E_t	transmitted energy
E_y	energy in $y(t)$

E_1, E_0	energy of signals on H_1 and H_0 respectively
E_ϵ	energy in error signal (sensitivity context)
$e_N(t)$	error waveform
ϵ_I	interval error
ϵ_T	total error
$\mathrm{erf}(\cdot)$	error function (conventional)
$\mathrm{erf}_*(\cdot)$	error function (as defined in text)
$\mathrm{erfc}(\cdot)$	complement of error function (conventional)
$\mathrm{erfc}_*(\cdot)$	complement of error function (as defined in text)
η	(eta) threshold in likelihood ratio test
$E(\cdot)$	expectation operation (also denoted by $\overline{(\cdot)}$ infrequently)
F	function to minimize or maximize that includes Lagrange multiplier
$f(t)$	envelope of transmitted signal
$f(t)$	function used in various contexts
$f(t: r(u), T_i \le u \le T_f)$	nonlinear operation on $r(u)$ (includes linear operation as special case)
f_c	oscillator frequency ($\omega_c = 2\pi f_c$)
$f_\Delta(t)$	normalized difference signal
$\mathbf{F}$	matrix in differential equation
$\mathbf{F}(t)$	time-varying matrix in differential equation
$\mathbf{F}_d(t)$	matrix in equation describing desired signal
$G^+(j\omega)$	factor of $S_r(\omega)$ that has all of the poles and zeros in LHP (and $\frac{1}{2}$ of the zeros on $j\omega$-axis). Its transform is zero for negative time.
$g(t)$	function in colored noise correlator
$g(t, A)$, $g(t, \mathbf{A})$	function in problem of estimating A (or $\mathbf{A}$) in colored noise
$g(\lambda_i)$	a function of an eigenvalue
$g_h(t)$	homogeneous solution
$g_l(\tau)$	filter in loop
$g_{lo}(\tau)$, $G_{lo}(j\omega)$	impulse response and transfer function optimum loop filter
$g_{pu}(\tau)$	unrealizable post-loop filter
$g_{puo}(\tau)$, $G_{puo}(j\omega)$	optimum unrealizable post-loop filter
$g_\delta(t)$	impulse solution
$g_\Delta(t)$	difference function in colored noise correlator
g_λ	a weighted sum of $g(\lambda_i)$
$g_\infty(t)$, $G_\infty(j\omega)$	infinite interval solution

$\mathbf{G}$	matrix in differential equation
$\mathbf{G}(t)$	time-varying matrix in differential equation
$\mathbf{G}_d$	linear transformation describing desired vector $\mathbf{d}$
$\mathbf{G}_d(t)$	matrix in differential equation for desired signal
$\mathbf{g}(t)$	function for vector correlator
$\mathbf{g}_{\mathbf{d}}(\mathbf{A})$	nonlinear transformation describing desired vector $\mathbf{d}$
$\Gamma(x)$	Gamma function
γ	parameter ($\gamma = k\sqrt{1 + \Lambda}$)
γ	threshold for arbitrary test (frequently various constants absorbed in γ)
γ_a	factor in nonlinear modulation problem which controls the error variance
$H_0, H_1, \ldots, H_i$	hypotheses in decision problem
$h(t, u)$	impulse response of time-varying filter (output at t due to impulse input at u)
$h_{\text{ch}}(t, u)$	channel impulse response
$h_L(t)$	low pass function (envelope of bandpass filter)
$h_o(t, u)$	optimum linear filter
$h_o'(\tau), H_o'(j\omega)$	optimum processor on whitened signal: impulse response and transfer function, respectively
$h_{ou}(\tau), H_{ou}(j\omega)$	optimum unrealizable filter (impulse response and transfer function)
$h_w(t, u)$	whitening filter
$h_\epsilon(t, u)$	arbitrary linear filter
$h_*(t, u)$	linear filter in uniqueness discussion
$\mathbf{H}$	linear matrix transformation
$\mathbf{h}_o(t, u)$	optimum linear matrix filter
$I_0(\cdot)$	modified Bessel function of 1st kind and order zero
I_1, I_2	integrals
I_Γ	incomplete Gamma function
$\mathbf{I}$	identity matrix
$J(t, u)$	information kernel
J^{ij}	elements in $\mathbf{J}^{-1}$
$J^{-1}(t, u)$	inverse information kernel
J_{ij}	elements in information matrix
$J_K(t, u)$	kth term approximation to information kernel
$\mathbf{J}$	information matrix (Fisher's)
$\mathbf{J}_D$	data component of information matrix
$\mathbf{J}_P$	a priori component of information matrix

$\mathbf{J}_T$	total information matrix
$\mathfrak{J}(\omega)$	transform of $J(\tau)$
$\mathfrak{J}^{-1}(\omega)$	transform of $J^{-1}(\tau)$
$K_{na}(t, u)$	actual noise covariance (sensitivity discussion)
$K_{ne}(t, u)$	effective noise covariance
$K_{n\epsilon}(t, u)$	error in noise covariance (sensitivity discussion)
$K_x(t, u)$	covariance function of $x(t)$
k	Boltzmann's constant
$k(t, r(u))$	operation in reversibility proof
$\mathbf{K}$	covariance matrix
$\mathbf{k}_d(t)$	linear transformation of $\mathbf{x}(t)$
$\mathbf{k}_d(t, v)$	matrix filter with p inputs and q outputs relating $\mathbf{a}(v)$ and $\mathbf{d}(t)$
$\mathbf{k}_f(u, v)$	matrix filter with p inputs and n outputs relating $\mathbf{a}(v)$ and $\mathbf{x}(u)$
$l(\mathbf{R}), l$	sufficient statistic
$l(A)$	likelihood function
l_a	actual sufficient statistic (sensitivity problem)
l_c, l_s	sufficient statistics corresponds to cosine and sine components
I	a set of sufficient statistics
Λ	a parameter which frequently corresponds to a signal-to-noise ratio in message ERB
$\Lambda(\mathbf{R})$	likelihood ratio
$\Lambda(r_K(t))$	likelihood ratio
$\Lambda(r_K(t), A)$	likelihood function
Λ_B	signal-to-noise ratio in reference bandwidth for Butterworth spectra
Λ_{ef}	effective signal-to-noise ratio
Λ_g	generalized likelihood ratio
Λ_m	parameter in phase probability density
Λ_{3db}	signal-to-noise ratio in 3-db bandwidth
$\mathbf{\Lambda}_{\mathbf{x}}$	covariance matrix of vector $\mathbf{x}$
$\mathbf{\Lambda}_{\mathbf{x}}(t)$	covariance matrix of state vector ($= \mathbf{K}_{\mathbf{x}}(t, t)$)
λ	Lagrange multiplier
λ_i	eigenvalue of matrix or integral equation
λ_i^{ch}	eigenvalues of channel quadratic form
λ_i^T	total eigenvalue
ln	natural logarithm
$\log_a$	logarithm to the base a

$M_x(jv)$, $M_{\mathbf{x}}(j\mathbf{v})$	characteristic function of random variable x (or $\mathbf{x}$)
$m_x(t)$	mean-value function of process
$\mathbf{M}$	matrix used in colored noise derivation
$\mathbf{m}$	mean vector
$\mu(s)$	exponent of moment-generating function
N	dimension of observation space
N	number of coefficients in series expansion
$N(m, \sigma)$	Gaussian (or Normal) density with mean m and standard deviation σ
$N(\omega^2)$	numerator of spectrum
N_{ef}	effective noise level
N_0	spectral height (joules)
$n(t)$	noise random process
$n_c(t)$	colored noise (does not contain white noise)
$n_{Ei}(t)$	external noise
n_i	ith noise component
$n_{Ri}(t)$	receiver noise
$n_*(t)$	noise component at output of whitening filter
$\hat{n}_{c_r}(t)$	MMSE realizable estimate of colored noise component
$\hat{n}_{c_u}(t)$	MMSE unrealizable estimate of colored noise component
$\mathbf{N}$	noise correlation matrix numbers)
n, $\mathbf{n}$	noise random variable (or vector variable)
$\xi_{a\epsilon}$	cross-correlation between error and actual state vector
ξ_I	expected value of interval estimation error
$\xi_{ij}(t)$	elements in error covariance matrix
ξ_{ml}	variance of ML interval estimate
$\xi_P(t)$	expected value of *realizable* point estimation error
$\xi_{Pi}(t)$	variance of error of point estimate of ith signal
$\xi_{Pn}(t)$	normalized realizable point estimation error
ξ_{Pn}^{α}	normalized error as function of prediction (or lag) time
$\xi_{P\infty}$	expected value of point estimation error, statistical steady state
ξ_u	optimum unrealizable error
ξ_{un}	normalized optimum unrealizable error
$\xi_*(t)$	mean-square error using nonlinear operation
$\boldsymbol{\xi}_{ac}$	actual covariance matrix
$\boldsymbol{\xi}_d(t)$	covariance matrix in estimating $d(t)$
$\boldsymbol{\xi}_{P\infty}$	steady-state error covariance matrix

ω_c	carrier frequency (radians/second)
ω_D	Doppler shift
P	power
$Pr(\epsilon)$	probability of error
P_D	probability of detection (a conditional probability)
P_{ef}	effective power
P_F	probability of false alarm (a conditional probability)
P_i	a priori probability of *i*th hypothesis
P_M	probability of a miss (a conditional probability)
$P_D(\theta)$	probability of detection for a particular value of θ
p	operator to denote d/dt (used infrequently)
p_0	fixed probability of interval error in PFM problems
$p_{\mathbf{r}\mid H_i}(\mathbf{R}\mid H_i)$	probability density of $\mathbf{r}$, given that H_i is true
$p_{x_t}(X_t)$ or $p_{x_t}(X: t)$	probability density of a random process at time t
$\phi(t)$	eigenfunction
$\phi_i(t)$	*i*th coordinate function, *i*th eigenfunction
$\phi_x(s)$	moment generating function of random variable x
$\phi(t)$	phase of signal
$\psi_L(t)$	low pass phase function
$\mathbf{P}(t)$	cross-correlation matrix between input to message generator and additive channel noise
$\boldsymbol{\phi}(t, \tau)$	state transition matrix, time-varying system
$\boldsymbol{\phi}(t - t_0) \triangleq \boldsymbol{\phi}(\tau)$	state transition matrix, time-invariant system
$\Pr[\cdot], \Pr(\cdot)$	probability of event in brackets or parentheses
$Q(\alpha, \beta)$	Marcum's Q function
$Q_n(t, u)$	inverse kernel
q	height of scalar white noise drive
$\mathbf{Q}$	covariance matrix of vector white noise drive (Section 6.3)
$\mathbf{Q}$	inverse of covariance matrix $\mathbf{K}$
$\mathbf{Q}_1, \mathbf{Q}_0$	inverse of covariance matrix $\mathbf{K}_1, \mathbf{K}_0$
$\mathbf{Q}_n(u, z)$	inverse matrix kernel
R	rate (digits/second)
$R_x(t, u)$	correlation function
$\mathcal{R}$	risk
$\mathcal{R}(d(t), t)$	risk in point estimation
$\mathcal{R}_{abs}$	risk using absolute value cost function
$\mathcal{R}_B$	Bayes risk

$\mathcal{R}_F$	risk using fixed test
$\mathcal{R}_{ms}$	risk using mean-square cost function
$\mathcal{R}_{unf}$	risk using uniform cost function
$r(t)$	received waveform (denotes both the random process and a sample function of the process)
$r_c(t)$	combined received signal
$r_g(t)$	output when inverse kernel filter operates on $r(t)$
$r_K(t)$	K term approximation
$r_*(t)$	output of whitening filter
$r_{*a}(t)$	actual output of whitening filter (sensitivity context)
$r_{**}(t)$	output of $S_Q(\omega)$ filter (equivalent to cascading two whitening filters)
ρ_{ij}	normalized correlation $s_i(t)$ and $s_j(t)$ (normalized signals)
ρ_{12}	normalized covariance between two random variables
$\mathbf{R}(t)$	covariance matrix of vector white noise $\mathbf{w}(t)$
$\mathbf{R}$	correlation matrix of errors
$\mathbf{R}_{\epsilon_I}$	error correlation matrix, interval estimate
$\mathbf{r}$, $\mathbf{R}$	observation vector
$R_{on}{}^1[\cdot, \cdot]$	radial prolate spheroidal function
$S(j\omega)$	Fourier transform of $s(t)$
$S_c(\omega)$	spectrum of colored noise
$S_{on}[\cdot, \cdot]$	angular prolate spheroidal function
$S_Q(\omega)$	Fourier transform of $Q(\tau)$
$S_r(\omega)$	power density spectrum of received signal
$S_x(\omega)$	power density spectrum
$S_{\epsilon_o}(j\omega)$	transform of optimum error signal
$s(t)$	signal component in $r(t)$, no subscript when only one signal
$s(t, A)$	signal depending on A
$s(t, a(t))$	modulated signal
$s_a(t)$	actual $s(t)$ (sensitivity context)
$s_I(t)$	interfering signal
$s_{Ia}(t)$	actual interfering signal (sensitivity context)
s_i	coefficient in expansion of $s(t)$
s_i	ith signal component
$s_r(t, \theta)$	received signal component
$s_t(t)$	signal transmitted
$s_0(t)$	signal on H_0
$s_1(t)$	signal on H_1
$s_1(t, \boldsymbol{\theta})$, $s_0(t, \boldsymbol{\theta})$	signal with unwanted parameters
$s_\epsilon(t)$	error signal (sensitivity context)

$s_\Delta(t)$	difference signal $(\sqrt{E_1}\, s_1(t) - \sqrt{E_0}\, s_0(t))$
$s_\Omega(t)$	random signal
$s_{\epsilon *}(t)$	whitened difference signal
$s_*(t)$	signal component at output of whitening filter
$s_{*\epsilon}(t)$	output of whitening filter due to signal error
σ^2	variance
$\sigma_1{}^2$, $\sigma_0{}^2$	variance on H_1, H_0
$\sigma_{\epsilon_t}{}^2$	error variance
$\mathbf{s}(t)$	vector signal
T_e	effective noise temperature
θ, $\boldsymbol{\theta}$	unwanted parameter
$\hat{\theta}$	phase estimate
θ_a	actual phase in binary system
$\theta_{ch}(t)$	phase of channel response
$\hat{\boldsymbol{\theta}}_1$	estimate of $\boldsymbol{\theta}_1$
$\mathbf{T}(t, \tau)$	transition matrix
$[\ ]^T$	transpose of matrix
$u_{-1}(t)$	unit step function
$u(t)$, $\mathbf{u}(t)$	input to system
V	variable in piecewise approximation to $V_{ch}(t)$
$V_{ch}(t)$	envelope of channel response
$\mathbf{v}(t)$	combined drive for correlated noise case
$\mathbf{v}_1(t)$, $\mathbf{v}_2(t)$	vector functions in Property 16 of Chapter 6
W	bandwidth parameter (cps)
$W(j\omega)$	transfer function of whitening filter
W_{ch}	channel bandwidth (cps) single-sided
$W^{-1}(j\omega)$	transform of inverse of whitening filter
$w(t)$	white noise process
$w(\tau)$	impulse response of whitening filter
$\mathbf{W}$	a matrix operation whose output vector has a diagonal covariance matrix
$x(t)$	input to modulator
$x(t)$	random process
$\hat{x}(t)$	estimate of random process
$\mathbf{x}$	random vector
$\mathbf{x}(t)$	state vector
$\mathbf{x}_a(t)$	augmented state vector

$\mathbf{x}_{ac}$	actual state vector
$\mathbf{x}_d(t)$	state vector for desired operation
$\mathbf{x}_f(t)$	prefiltered state vector
$\mathbf{x}_M(t)$	state vector, message
$\mathbf{x}_{mo}$	state vector in model
$\mathbf{x}_N(t)$	state vector, noise
$y(t)$	output of differential equation
$y(t)$	portion of $r(t)$ not needed for decision
$y(t)$	transmitted signal
$\mathbf{y}$	vector component of observation that is not needed for decision
$y = s(A)$	nonlinear function of parameter A
Z	observation space
$Z_c(\omega)$	integrated cosine transform
$Z_s(\omega)$	integrated sine transform
Z_1, Z_2	subspace of observation space
$z(t)$	output of whitening filter
$\mathbf{z}(t)$	gain matrix in state-variable filter ($\triangleq \mathbf{h}_o(t, t)$)