Detection, Estimation, and Modulation Theory, Part III: Radar–Sonar Signal Processing and Gaussian Signals in Noise. Harry L. Van Trees

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Introduction

This book is the third in a set of four volumes. The purpose of these four volumes is to present a unified approach to the solution of detection, estimation, and modulation theory problems. In this volume we study two major problem areas. The first area is the detection of random signals in noise and the estimation of random process parameters. The second area is signal processing in radar and sonar systems. As we pointed out in the Preface, Part III does not use the material in Part II and can be read directly after Part I.

In this chapter we discuss three topics briefly. In Section 1.1, we review Parts I and II so that we can see where the material in Part III fits into the over-all development. In Section 1.2, we introduce the first problem area and outline the organization of Chapters 2 through 7. In Section 1.3, we introduce the radar–sonar problem and outline the organization of Chapters 8 through 14.

1.1 REVIEW OF PARTS I AND II

In the introduction to Part I [1], we outlined a hierarchy of problems in the areas of detection, estimation, and modulation theory and discussed a number of physical situations in which these problems are encountered.

We began our technical discussion in Part I with a detailed study of classical detection and estimation theory. In the classical problem the observation space is finite-dimensional, whereas in most problems of interest to us the observation is a waveform and must be represented in an infinite-dimensional space. All of the basic ideas of detection and parameter estimation were developed in the classical context.

In Chapter I-3, we discussed the representation of waveforms in terms of series expansions. This representation enabled us to bridge the gap between the classical problem and the waveform problem in a straightforward manner. With these two chapters as background, we began our study of the hierarchy of problems that we had outlined in Chapter I-1.

In the first part of Chapter I-4, we studied the detection of known signals in Gaussian noise. A typical problem was the binary detection problem in which the received waveforms on the two hypotheses were

$$r(t) = s_1(t) + n(t), \qquad T_i \leq t \leq T_f : H_1, \tag{1}$$

$$r(t) = s_0(t) + n(t), \qquad T_i \leq t \leq T_f : H_0, \tag{2}$$

where $s_1(t)$ and $s_0(t)$ were known functions. The noise $n(t)$ was a sample function of a Gaussian random process.

We then studied the parameter-estimation problem. Here, the received waveform was

$$r(t) = s(t, \mathbf{A}) + n(t), \qquad T_i \leq t \leq T_f. \tag{3}$$

The signal $s(t, \mathbf{A})$ was a known function of t and $\mathbf{A}$. The parameter $\mathbf{A}$ was a vector, either random or nonrandom, that we wanted to estimate.

We referred to all of these problems as known signal-in-noise problems, and they were in the first level in the hierarchy of problems that we outlined in Chapter I-1. The common characteristic of first-level problems is the presence of a *deterministic signal* at the receiver. In the binary detection problem, the receiver decides which of the two deterministic waveforms is present in the received waveform. In the estimation problem, the receiver estimates the value of a parameter contained in the signal. In all cases it is the additive noise that limits the performance of the receiver.

We then generalized the model by allowing the signal component to depend on a finite set of unknown parameters (either random or nonrandom). In this case, the received waveforms in the binary detection problem were

$$\begin{aligned} r(t) &= s_1(t, \boldsymbol{\theta}) + n(t), \qquad T_i \leq t \leq T_f : H_1, \\ r(t) &= s_0(t, \boldsymbol{\theta}) + n(t), \qquad T_i \leq t \leq T_f : H_0. \end{aligned} \tag{4}$$

In the estimation problem the received waveform was

$$r(t) = s(t, \mathbf{A}, \boldsymbol{\theta}) + n(t), \qquad T_i \leq t \leq T_f. \tag{5}$$

The vector $\boldsymbol{\theta}$ denoted a set of unknown and unwanted parameters whose presence introduced a new uncertainty into the problem. These problems were in the second level of the hierarchy. The additional degree of freedom in the second-level model allowed us to study several important physical channels such as the random-phase channel, the Rayleigh channel, and the Rician channel.

In Chapter I-5, we began our discussion of modulation theory and continuous waveform estimation. After formulating a model for the problem, we derived a set of integral equations that specify the optimum demodulator.

In Chapter I-6, we studied the linear estimation problem in detail. Our analysis led to an integral equation,

$$K_{dr}(t, u) = \int_{T_i}^{T_f} h_o(t, \tau) K_r(\tau, u) \, d\tau, \qquad T_i < t, u < T_f, \tag{6}$$

that specified the optimum receiver. We first studied the case in which the observation interval was infinite and the processes were stationary. Here, the spectrum-factorization techniques of Wiener enabled us to solve the problem completely. For finite observation intervals and nonstationary processes, the state-variable formulation of Kalman and Bucy led to a complete solution. We shall find that the integral equation (6) arises frequently in our development in this book. Thus, many of the results in Chapter I-6 will play an important role in our current discussion.

In Part II, we studied nonlinear modulation theory [2]. Because the subject matter in Part II is essentially disjoint from that in Part III, we shall not review the contents in detail. The material in Chapters I-4 through Part II is a detailed study of the first and second levels of our hierarchy of detection, estimation, and modulation theory problems.

There are a large number of physical situations in which the models in the first and second level do not adequately describe the problem. In the next section we discuss several of these physical situations and indicate a more appropriate model.

1.2 RANDOM SIGNALS IN NOISE

We begin our discussion by considering several physical situations in which our previous models are not adequate. Consider the problem of detecting the presence of a submarine using a *passive* sonar system. The engines, propellers, and other elements in the submarine generate acoustic signals that travel through the ocean to the hydrophones in the detection system. This signal can best be characterized as a sample function from a random process. In addition, a hydrophone generates self-noise and picks up sea noise. Thus a suitable model for the detection problem might be

$$r(t) = s(t) + n(t), \qquad T_i \le t \le T_f : H_1, \tag{7}$$

$$r(t) = n(t), \qquad T_i \le t \le T_f : H_0. \tag{8}$$

Now $s(t)$ is a sample function from a random process. The new feature in this problem is that the mapping from the hypothesis (or source output) to the signal $s(t)$ is no longer deterministic. The detection problem is to decide whether $r(t)$ is a sample function from a signal plus noise process or from the noise process alone.

A second area in which we decide which of two processes is present is the digital communications area. A large number of digital systems operate over channels in which randomness is inherent in the transmission characteristics. For example, tropospheric scatter links, orbiting dipole links, chaff systems, atmospheric channels for optical systems, and underwater acoustic channels all exhibit random behavior. We discuss channel models in detail in Chapters 9–13. We shall find that a typical method of communicating digital data over channels of this type is to transmit one of two signals that are separated in frequency. (We denote these two frequencies as ω_1 and ω_0). The resulting received signal is

$$\begin{aligned} r(t) &= s_1(t) + n(t), \qquad T_i \le t \le T_f : H_1, \\ r(t) &= s_0(t) + n(t), \qquad T_i \le t \le T_f : H_0. \end{aligned} \tag{9}$$

Now $s_1(t)$ is a sample function from a random process whose spectrum is centered at ω_1, and $s_0(t)$ is a sample function from a random process whose spectrum is centered at ω_0. We want to build a receiver that will decide between H_1 and H_0.

Problems in which we want to estimate the parameters of random processes are plentiful. Usually when we model a physical phenomenon using a stationary random process we assume that the power spectrum is known. In practice, we frequently have a sample function available and must determine the spectrum by observing it. One procedure is to parameterize the spectrum and estimate the parameters. For example, we assume

$$S(\omega, A) = \frac{A_1}{\omega^2 + A_2{}^2}, \qquad -\infty < \omega < \infty, \tag{10}$$

and try to estimate A_1 and A_2 by observing a sample function of $s(t)$ corrupted by measurement noise. A second procedure is to consider a small frequency interval and try to estimate the average height of spectrum over that interval.

A second example of estimation of process parameters arises in such diverse areas as radio astronomy, spectroscopy, and passive sonar. The source generates a narrow-band random process whose center frequency identifies the source. Here we want to estimate the center frequency of the spectrum.

A closely related problem arises in the radio astronomy area. Various sources in our galaxy generate a narrow-band process that would be centered at some known frequency if the source were not moving. By estimating the center frequency of the received process, the velocity of the source can be determined. The received waveform may be written as

$$r(t) = s(t, v) + n(t), \qquad T_i \leq t \leq T_f, \tag{11}$$

where $s(t, v)$ is a sample function of a random process whose statistical properties depend on the velocity v.

These examples of detection and estimation theory problems correspond to the third level in the hierarchy that we outlined in Chapter I-1. They have the common characteristic that the information of interest is imbedded in a random process. Any detection or estimation procedure must be based on how the statistics of $r(t)$ vary as a function of the hypothesis or the parameter value.

In Chapter 2, we formulate a quantitative model of the simple binary detection problem in which the received waveform consists of a white Gaussian noise process on one hypothesis and the sum of a Gaussian signal process and the white Gaussian noise process on the other hypothesis. In Chapter 3, we study the general problem in which the received signal is a sample function from one of two Gaussian random processes. In both sections we derive optimum receiver structures and investigate the resulting performance.

In Chapter 4, we study four special categories of detection problems for which complete solutions can be obtained. In Chapter 5, we consider the M-ary problem, the performance of suboptimum receivers for the binary problem, and summarize our detection theory results.

In Chapters 6 and 7, we treat the parameter estimation problem. In Chapter 6, we develop the model for the single-parameter estimation problem, derive the optimum estimator, and discuss performance analysis techniques. In Chapter 7, we study four categories of estimation problems in which reasonably complete solutions can be obtained. We also extend our results to include multiple-parameter estimation and summarize our estimation theory discussion.

The first half of the book is long, and several of the discussions include a fair amount of detail. This detailed discussion is necessary in order to develop an ability actually to solve practical problems. Strictly speaking, there are no new concepts. We are simply applying decision theory and estimation theory to a more general class of problems. It turns out that the transition from the concept to actual receiver design requires a significant amount of effort.

The development in Chapters 2 through 7 completes our study of the hierarchy of problems that were outlined in Chapter I-1. The remainder of the book applies these ideas to signal processing in radar and sonar systems.

1.3 SIGNAL PROCESSING IN RADAR–SONAR SYSTEMS

In a conventional active radar system we transmit a pulsed sinusoid. If a target is present, the signal is reflected. The received waveform consists of the reflected signal plus interfering noises. In the simplest case, the only source of interference is an additive Gaussian receiver noise. In the more general case, there is interference due to external noise sources or reflections from other targets. In the detection problem, the receiver processes the signal to decide whether or not a target is present at a particular location. In the parameter estimation problem, the receiver processes the signal to measure some characteristics of the target such as range, velocity, or acceleration. We are interested in the signal-processing aspects of this problem.

There are a number of issues that arise in the signal-processing problem.

1. We must describe the reflective characteristics of the target. In other words, if the transmitted signal is $s_t(t)$, what is the reflected signal?
2. We must describe the effect of the transmission channels on the signals.
3. We must characterize the interference. In addition to the receiver noise, there may be other targets, external noise generators, or clutter.
4. After we develop a quantitative model for the environment, we must design an optimum (or suboptimum) receiver and evaluate its performance.

In the second half of the book we study these issues. In Chapter 8, we discuss the radar–sonar problem qualitatively. In Chapter 9, we discuss the problem of detecting a slowly fluctuating point target at a particular range and velocity. First we assume that the only interference is additive white Gaussian noise, and we develop the optimum receiver and evaluate its performance. We then consider nonwhite Gaussian noise and find the optimum receiver and its performance. We use complex state-variable theory to obtain complete solutions for the nonwhite noise case.

In Chapter 10, we consider the problem of estimating the parameters of a slowly fluctuating point target. Initially, we consider the problem of estimating the range and velocity of a single target when the interference is additive white Gaussian noise. Starting with the likelihood function, we develop the structure of the optimum receiver. We then investigate the performance of the receiver and see how the signal characteristics affect the estimation accuracy. Finally, we consider the problem of detecting a target in the presence of other interfering targets.

The work in Chapters 9 and 10 deals with the simplest type of target and models the received signal as a known signal with unknown random parameters. The background for this problem was developed in Section I-4.4, and Chapters 9 and 10 can be read directly after Chapter I-4.

In Chapter 11, we consider a point target that fluctuates during the time during which the transmitted pulse is being reflected. Now we must model the received signal as a sample function of a random process.

In Chapter 12, we consider a slowly fluctuating target that is distributed in range. Once again we model the received signal as a sample function of a random process. In both cases, the necessary background for solving the problem has been developed in Chapters III-2 through III-4.

In Chapter 13, we consider fluctuating, distributed targets. This model is useful in the study of clutter in radar systems and reverberation in sonar systems. It is also appropriate in radar astronomy and scatter communications problems. As in Chapters 11 and 12, the received signal is modeled as a sample function of a random process. In all three of these chapters we are able to find the optimum receivers and analyze their performance.

Throughout our discussion we emphasize the similarity between the radar problem and the digital communications problem. Imbedded in various chapters are detailed discussions of digital communication over fluctuating channels. Thus, the material will be of interest to communications engineers as well as radar/sonar signal processors.

Finally, in Chapter 14, we summarize the major results of the radar–sonar discussion and outline the contents of the subsequent book on *Array Processing* [3]. In addition to the body of the text, there is an Appendix on the complex representation of signals, systems, and processes.

REFERENCES

[1] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part I,* Wiley, New York, 1968.

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[3] H. L. Van Trees, *Array Processing*, Wiley, New York (to be published).