

14

Discussion

In this chapter we discuss three topics briefly. In Section 14.1, we summarize some of the major results of our radar-sonar discussion. In Section 14.2, we outline the contents of *Array Processing*, the final volume of this series. In Section 14.3, we make some concluding comments on the over-all sequence.

14.1 SUMMARY: SIGNAL PROCESSING IN RADAR AND SONAR SYSTEMS

In Chapter 8 we introduced the radar-sonar problem and discussed the hierarchy of target and channel models of interest. We then detoured to the Appendix and developed a complex representation for narrow-band signals, systems, and processes. For signals,

$$f(t) = \sqrt{2} \operatorname{Re} [\tilde{f}(t)e^{j\omega_c t}], \quad (1)$$

where $\tilde{f}(t)$ is the complex envelope. For systems,

$$h(t, u) = 2 \operatorname{Re} [\tilde{h}(t, u)e^{j\omega_c t}], \quad (2)$$

where $\tilde{h}(t, u)$ is the complex impulse response. For random processes,

$$n(t) = \sqrt{2} \operatorname{Re} [\tilde{n}(t)e^{j\omega_c t}], \quad (3)$$

where $\tilde{n}(t)$ is the complex envelope process. By restricting our attention to processes where

$$E[\tilde{n}(t)\tilde{n}(u)] = 0, \quad (4)$$

we have a one-to-one relationship between the covariance function of the complex envelope process $\tilde{K}_{\tilde{n}}(t, u)$ and the covariance function of the actual process,

$$K_n(t, u) = \sqrt{2} \operatorname{Re} [\tilde{K}_{\tilde{n}}(t, u)e^{j\omega_c(t-u)}]. \quad (5)$$

This class of processes includes all stationary processes and the non-stationary processes that we encounter in practice. We also introduced complex state variables and developed their properties. The complex notation enabled us to see the important features in the problems more clearly. In addition, it simplified all of the analyses, because we could work with one complex quantity instead of two real quantities.

In Chapter 9, we studied the problem of detecting the return from a slowly fluctuating point target in the presence of noise. The likelihood ratio test was

$$|\tilde{I}|^2 \triangleq \left| \int_{T_i}^{T_f} \tilde{r}(t) \tilde{g}^*(t) dt \right|_{H_0}^2 \underset{H_1}{\geq} \gamma, \tag{6}$$

where $\tilde{g}(t)$ satisfies the integral equation

$$\tilde{f}(t) = \int_{T_i}^{T_f} \tilde{K}_n(t, u) \tilde{g}(u) du, \quad T_i \leq t \leq T_f. \tag{7}$$

The performance was completely characterized by the quantity

$$\Delta \triangleq \frac{E\{|\tilde{I}|^2 | H_1\} - E\{|\tilde{I}|^2 | H_0\}}{E\{|\tilde{I}|^2 | H_0\}}. \tag{8}$$

This quantity could be used in (9.50) to determine the error probabilities. In addition, we specified the receiver and its performance in terms of a set of differential equations that could be readily solved using numerical techniques. Although we formulated the optimal signal design problem, we did not study it in detail.

In Chapter 10 we discussed the problem of estimating the range and velocity of a slowly fluctuating point target in the presence of additive white noise. We found that the time-frequency correlation function,

$$\phi\{\tau, f\} = \int_{-\infty}^{\infty} \tilde{f}\left(t - \frac{\tau}{2}\right) \tilde{f}^*\left(t + \frac{\tau}{2}\right) e^{j2\pi f t} dt, \tag{9}$$

and the ambiguity function,

$$\theta\{\tau, f\} = |\phi\{\tau, f\}|^2, \tag{10}$$

played a key part in most of our discussion. When the estimation errors were small, the accuracy was directly related to the shape of the ambiguity function at the origin. However, if the ambiguity function had subsidiary peaks whose heights were close to unity, the probability of making a large error was increased. These two issues were related by the radar uncertainty principle, which said that the total volume under the ambiguity function

was unity for *any* transmitted signal,

$$\iint_{-\infty}^{\infty} \theta\{\tau, f\} d\tau df = 1. \quad (11)$$

It is important to re-emphasize that the ambiguity function is important because the receiver has been designed to be optimum in the presence of additive white Gaussian noise. We found that, in some environments, we want to use a different filter [e.g., $\tilde{v}^*(t)$]. This function, $\tilde{v}^*(t)$, could correspond to the $\tilde{g}^*(t)$ specified by (7), or it could be a function chosen for ease in receiver implementation. Now the cross-ambiguity function

$$\theta_{fv}\{\tau, f\} = \left| \int_{-\infty}^{\infty} \tilde{f}\left(t - \frac{\tau}{2}\right) \tilde{v}^*\left(t + \frac{\tau}{2}\right) e^{j2\pi ft} dt \right|^2 \quad (12)$$

played the central role in our analyses.

A particularly important problem is the resolution problem. In Section 10.5, we considered resolution in a discrete environment. A typical situation in which this type of problem arises is when we try to detect a target in the presence of decoys. Although we could always find the optimum receiver, the conventional matched-filter receiver was frequently used because of its simplicity. In this case, the degradation due to the interference was

$$\rho_r = \sum_{i=1}^K \frac{\bar{E}_i}{N_0} \theta(\tau_i - \tau_a, \omega_i - \omega_a). \quad (13)$$

Thus, if we could make the ambiguity function zero at those points in the τ, ω plane where the interfering targets were located, there would be no degradation. In general, this was not a practical solution, but it did provide some insight into the selection of good signals. Whenever ρ_r was appreciable, we could improve the performance by using an optimum receiver. If there were no white noise, the optimum receiver would simply tune out the interference (this eliminates some of the signal energy also). In the presence of white noise the optimum receiver cannot eliminate all of the interference without affecting the detectability, and so the resulting filter is a compromise that maximizes Δ in (8).

We continued our discussion of resolution in Section 13.2. The reverberation (or clutter) return was modeled as a dense, doubly-spread target. Once again we considered both conventional and optimum receivers. In the conventional matched-filter receiver the degradation due to the reverberation was given by the expression

$$\rho_r = \frac{E_t}{N_0} \iint_{-\infty}^{\infty} df d\lambda \tilde{S}_{DR}\{f, \lambda\} \theta\{\lambda - \tau_a, f_a - f\}. \quad (14)$$

Now the signal design problem consisted of minimizing the common volume of the signal ambiguity function and target-scattering function. When ρ_r was appreciable, some improvement was possible using an optimum receiver. In the general case we had to approximate the target by some orthogonal series model, such as the tapped-delay line of Fig. 13.18, in order actually to find the optimum receiver. Several suboptimum configurations for operation in a reverberation environment were developed in the problems.

In Chapter 11 we discussed Doppler-spread point targets. The basic assumption in our model was that the reflection process was a stationary, zero-mean Gaussian process. The covariance function of the complex envelope of the received signal process was

$$\tilde{K}_s(t, u) = E_t \tilde{f}(t - \lambda) \tilde{K}_D(t - u) \tilde{f}^*(u - \lambda), \quad (15)$$

where $\tilde{K}_D(\tau)$ was the covariance function of the reflection process. Equivalently, we could characterize the reflection process by the Doppler scattering function,

$$\tilde{S}_D\{f\} = \int_{-\infty}^{\infty} \tilde{K}_D(\tau) e^{-j2\pi f\tau} d\tau. \quad (16)$$

We saw that whenever the pulse length T was greater than the correlation time of the reflection process ($\simeq B^{-1}$), the target or channel caused time-selective fading. The optimum receiver problem was simply the bandpass version of the Gaussian signal in noise problem that we had studied in Chapters 2–4. Several classes of reflection processes allowed us to obtain complete solutions. In particular, whenever $\tilde{S}_D\{f\}$ was rational or could be approximated by a rational function, we could obtain a complete solution for the optimum receiver and a good approximation to its performance. This rational-spectrum approximation includes most cases of interest. We also studied binary communication over Doppler-spread channels. We found that there is a bound on the probability of error,

$$\Pr(\epsilon) \leq \frac{1}{2} \exp\left(-0.1488 \frac{\bar{E}_r}{N_0}\right), \quad (17)$$

that is independent of the shape of the scattering function. In addition, we were able to demonstrate systems using simple signals and receivers whose performance approached this bound. We found that the key to efficient performance was the use of either implicit or explicit diversity. In addition to being important in its own right, the communication problem gave us further insight into the general detection problem.

In Chapter 12 we discussed dispersive (or range-spread) targets and channels. The basic assumptions in our model were that the return from

disjoint intervals in range were statistically independent and that the received signal was a sample function of a zero-mean Gaussian random process. The covariance function was

$$\tilde{K}_s(t, u) = E_t \int_{-\infty}^{\infty} \tilde{f}(t - \lambda) \tilde{S}_R(\lambda) \tilde{f}^*(u - \lambda) d\lambda, \quad (18)$$

where $\tilde{S}_R(\lambda)$ was the range-scattering function. Equivalently, we could characterize the target in terms of a two-frequency correlation function,

$$\tilde{K}_R\{v\} = \int_{-\infty}^{\infty} \tilde{S}_R(\lambda) e^{j2\pi v\lambda} d\lambda. \quad (19)$$

Whenever the bandwidth of the transmitted signal was greater than the reciprocal of the target length (L^{-1}), we saw that the target caused frequency-selective fading. We next introduced the concept of time-frequency duality. Because the performance of a system is completely determined by the eigenvalues of the received process, we could analyze either a system or its dual. The duality theory enabled us to deal with a large class of range-spread targets that would be difficult to analyze directly. In addition, it offered new insights into the problem. The availability of efficient Fourier transform algorithms makes the synthesis of dual receivers practical.

In Chapter 13 we discussed the final class of targets in our hierarchy, doubly-spread targets. Here we assumed that the reflection process from each incremental range element was a sample function from a stationary Gaussian process and that the reflections from disjoint intervals were statistically independent. The covariance function of the received signal was given by

$$\tilde{K}_s(t, u) = E_t \int_{-\infty}^{\infty} \tilde{f}(t - \lambda) \tilde{K}_{DR}(t - u, \lambda) \tilde{f}^*(u - \lambda) d\lambda, \quad (20)$$

where $\tilde{K}_{DR}(t - u, \lambda)$ is the covariance function of the received process as a function of λ . Equivalently, we could characterize the target by a range-Doppler scattering function,

$$\tilde{S}_{DR}\{f, \lambda\} = \int_{-\infty}^{\infty} \tilde{K}_{DR}(\tau, \lambda) e^{-j2\pi f\tau} d\tau. \quad (21)$$

If $BL < 1$, we could obtain flat fading by a suitable signal choice. On the other hand, for $BL > 1$, the target was overspread and the received signal had to exhibit either time-selective or frequency-selective fading (or both).

After discussing the reverberation problem, we considered the detection problem for doubly-spread targets. For the low-energy-coherence case the results were straightforward. For the general case we used an orthogonal series model for the channel. The most common model for this type

is the tapped-delay line model. In this case, if we could approximate the spectrum of the tap gain processes by rational functions, we could find a complex state-variable model for the entire system. This enabled us to specify the optimum receiver completely and obtain a good approximation to its performance. A second method of solving the doubly-spread channel problem relied on a differential-equation characterization of the channel. This method also led to a set of equations that could be solved numerically. Although the optimum receivers were complicated, we could obtain a good approximation to them in most situations.

The final topic was the discussion of parameter estimation for doubly-spread targets. After deriving the likelihood function, we introduced the *generalized spread ambiguity function* in order to study the performance. Several specific estimation problems were studied in detail.

This concludes our discussion of signal processing in radar and sonar systems. In the next section we briefly discuss the contents of *Array Processing*.

14.2 OPTIMUM ARRAY PROCESSING

In the subsequent volume [1] we study the array-processing problem for sonar and seismic systems. The first topic is detection of known signals in noise. The basic derivation is just a special case of the results in Chapter I-4. The important problem is a study of the various issues that arise in a particular physical situation. To explore these issues, we first develop a model for spatially distributed noise fields. We then introduce the ideas of array gain, beam patterns, and distortionless filters, and demonstrate their utility in the signal-processing problem.

The next topic is the detection of unknown signals in noise. This model is appropriate for the passive sonar and seismic problem. By exploiting the central role of the distortionless filter, we are able to develop a receiver whose basic structure does not depend on the detailed assumptions of the model.

The final topic in [1] is the study of multivariable processes as encountered in continuous receiving apertures. Although the basic results are a straightforward extension of the multidimensional results, we shall find that both new insight and simplified computational procedures can be obtained from this general approach.

Just as in Parts II and III, we present a large number of new research results in the book. As in this volume, the result is a mixture of a research monograph and a graduate-level text.

14.3 EPILOGUE

Because of the specialized nature of the material in *Array Processing*, many readers will stop at this point, and so a few comments about the over-all development are worthwhile.

We hope that the reader appreciates the close relationships among the various problems that we have considered. A brief glance at the table of contents of the books indicates the wide range of physical situations that we have studied. By exploiting a few fundamental concepts, we were able to analyze them efficiently. An understanding of the relationships among the various areas is important, because it enables one to use results from other problems to solve the problem of current interest.

A second point that the reader should appreciate is the utility of various techniques for solving problems. A standard course in communications theory is no longer adequate. One should understand the techniques and concepts used in control theory, information theory, and other disciplines in order to be an effective analyst. We may have "oversold" the use of state-variable techniques, because it is an item of current research interest to us. We do feel that it is certain to have an important influence on many sophisticated systems in the future.

The reader should remember that we have been working with mathematical models of physical situations. More specifically, we have emphasized Gaussian process models throughout our discussion. In many cases, they are adequate to describe the actual situation, and our predicted performance results can be confirmed experimentally. In other cases, more complicated models employing non-Gaussian processes must be used. There are still other cases in which experimental (or simulation) procedures provide the only feasible approach. These comments do not negate the value of a thorough study of the Gaussian problem, but serve to remind us of its limitations.

Although it is not conventional, we feel that an appropriate final comment is to thank those readers who have followed us through this lengthy development. We hope that you have obtained an appreciation of *Detection, Estimation, and Modulation Theory*.

REFERENCES

- [1] H. L. Van Trees, *Array Processing*, Wiley, New York, 1971.