Detection, Estimation, and Modulation Theory, Part III: Radar–Sonar Signal Processing and Gaussian Signals in Noise. Harry L. Van Trees Copyright © 2001 John Wiley & Sons, Inc. ISBNs: 0-471-10793-X (Paperback); 0-471-22109-0 (Electronic)

## Glossary

In this section we discuss the conventions, abbreviations, and symbols used in the book.

## CONVENTIONS

The following conventions have been used:

1. Boldface roman denotes a vector or matrix.

2. The symbol | | means the magnitude of the vector or scalar contained within.

3. The determinant of a square matrix  $\mathbf{A}$  is denoted by  $|\mathbf{A}|$  or det  $\mathbf{A}$ .

4. The script letters  $\mathscr{F}(\cdot)$  and  $\mathscr{L}(\cdot)$  denote the Fourier transform and Laplace transform respectively.

5. Multiple integrals are frequently written as,

$$\int d\tau f(\tau) \int dt g(t, \tau) \triangleq \int f(\tau) \left\{ \int dt g(t) \right\} d\tau,$$

that is, an integral is inside all integrals to its left unless a multiplication is specifically indicated by parentheses.

6.  $E[\cdot]$  denotes the statistical expectation of the quantity in the bracket. The overbar  $\overline{x}$  is also used infrequently to denote expectation.

7. The symbol  $\otimes$  denotes convolution.

$$x(t) \otimes y(t) \triangleq \int_{-\infty}^{\infty} x(t-\tau)y(\tau) d\tau$$

8. Random variables are lower case (e.g., x and x). Values of random variables and nonrandom parameters are capital (e.g., X and X). In some estimation theory problems much of the discussion is valid for both random and nonrandom parameters. Here we depart from the above conventions to avoid repeating each equation.

605

606 Glossary

9. The probability density of x is denoted by  $p_x(\cdot)$  and the probability distribution by  $P_x(\cdot)$ . The probability of an event A is denoted by Pr [A]. The probability density of x, given that the random variable a has a value A, is denoted by  $P_{x|a}(X \mid A)$ . When a probability density depends on non-random parameter A we also use the notation  $p_{x|a}(X \mid A)$ . (This is non-standard but convenient for the same reasons as 8.)

10. A vertical line in an expression means "such that" or "given that"; that is  $\Pr[A \mid x \leq X]$  is the probability that event A occurs given that the random variable x is less than or equal to the value of X.

11. Fourier transforms are denoted by both  $F(j\omega)$  and  $F(\omega)$ . The latter is used when we want to emphasize that the transform is a real-valued function of  $\omega$ . The form used should always be clear from the context.

12. Some common mathematical symbols used include,

(i) $\propto$	proportional to
(ii) $t \rightarrow T^-$	t approaches T from below
(iii) $A + B \stackrel{\Delta}{=} A \cup B$	A or B or both
(iv) l.i.m.	limit in the mean
(v) $\int_{-\infty}^{\infty} d\mathbf{R}$ (vi) $\mathbf{A}^{T}$ (vii) $\mathbf{A}^{-1}$ (viii) $0$	an integral over the same dimension as the vector transpose of <b>A</b> inverse of <b>A</b> matrix with all zero elements
(ix) $\binom{N}{k}$	binomial coefficient $\left(=\frac{N!}{k! (N-k)!}\right)$
$(\mathbf{x}) \stackrel{\Delta}{\rightarrow}$	defined as
(xi) $\int_{\Omega} d\mathbf{R}$	integral over the set $\Omega$

## ABBREVIATIONS

Some abbreviations used in the text are:

ML	maximum likelihood
MAP	maximum a posteriori probability
PFM	pulse frequency modulation
PAM	pulse amplitude modulation
FM	frequency modulation
DSB-SC-AM	double-sideband-suppressed carrier-amplitude modula- tion

DSB-AM	double sideband-amplitude modulation
PM	phase modulation
NLNM	nonlinear no-memory
FM/FM	two-level frequency modulation
MMSE	minimum mean-square error
ERB	equivalent rectangular bandwidth
UMP	uniformly most powerful
ROC	receiver operating characteristic
LRT	likelihood ratio test
LEC	low energy coherence
SPLOT	stationary process-long observation time
SK	separable kernel

## SYMBOLS

The principal symbols used are defined below. In many cases the vector symbol is an obvious modification of the scalar symbol and is not included. Similarly, if the complex symbol is an obvious modification of the real symbol, it may be omitted.

A	class of detection problem
$A_a$	actual value of parameter
$A_i$	sample at $t_i$
$A_w$	class of detection problem, white noise present
$\hat{a}_0$	solution to likelihood equation
$\hat{a}_{ m abs}$	minimum absolute error estimate of a
$\hat{a}_{ ext{map}}$	maximum a posteriori probability estimate of a
$\hat{a}_{\mathrm{m}l}$	maximum likelihood estimate of A
$\hat{a}_{ m ms}$	minimum mean-square estimate of a
α	amplitude weighting of specular component in Rician
	channel
В	constant bias
В	Bhattacharyya distance (equals $-\mu(1/2)$ )
В	class of detection problem
В	signal bandwidth
B(A)	bias that is a function of A
$egin{array}{c} B_w \  ilde b \end{array}$	class of detection problem, white noise present
	random variable describing target or channel reflection
$\tilde{b}_D(t)$	complex Gaussian process describing reflection from
	Doppler-spread target
${ ilde b}_R(\lambda)$	complex Gaussian process describing reflection from range-spread target

608 Glossary	
$\mathbf{B}_{d}(t)$	matrix in state equation for desired signal
$\beta$	parameter in PFM and angle modulation
Ċ	channel capacity
$C(a_{\epsilon})$	cost of an estimation error, $a_{\epsilon}$
$C_F$	cost of a false alarm (say $H_1$ when $H_0$ is true)
$C_{ij}^{r}$	cost of saying $H_i$ is true when $H_j$ is true
$C_M$	cost of a miss (say $H_0$ when $H_1$ is true)
$C_{\infty}^{m}$	channel capacity, infinite bandwidth
$\widetilde{C}(t: \widetilde{\mathbf{x}}(t, \lambda))$	modulation functional
c	velocity of propagation
$\mathbf{C}(t)$	modulation (or observation) matrix
$\mathbf{C}_{d}(t)$	observation matrix, desired signal
$\mathbf{C}_{M}(t)$	message modulation matrix
$\mathbf{C}_{N}(t)$	noise modulation matrix
χ	parameter space
Xa	parameter space for a
Хө	parameter space for $\theta$
$\chi^2$	chi-square (description of a probability density)
$D(\omega^2)$	denominator of spectrum
$D_{\mathscr{F}}(\cdot)$	Fredholm determinant
$D_{\min}$	minimum diversity
$D_{ m opt}$	optimum diversity
$D_o$	optimum diversity
d	desired function of parameter
d	performance index parameter on ROC for Gaussian
	problems
d(t)	desired signal
$\hat{d}(t)$	estimate of desired signal
$\hat{d}_o(t)$	optimum MMSE estimate
$d_{\epsilon}(t)$	error in desired point estimate
δ	phase of specular component (Rician channel)
$\Delta$	performance measure (9.49)
$\Delta_{dg}$	performance degradation due to colored noise
$\Delta_o$	performance measure in optimum receiver
$\Delta_{\omega}$	width of Doppler cell
$\Delta_{\tau}$	length of range cell
$\Delta_v$	performance measure in suboptimum test
$\Delta_{wo}$	performance measure in "white-optimum" receiver
Δm	mean difference vector (i.e., vector denoting the dif- ference between two mean vectors)
ΔQ	matrix denoting difference between two inverse co-
ц	variance matrices
	variance matrices

E	energy (no subscript when there is only one energy in
	the problem)
$E_a$	expectation over the random variable <i>a</i> only
$E_I$	energy in interfering signal
$E_i$	energy on <i>i</i> th hypothesis
$ar{E}_r$	expected value of received energy
$E_t$	transmitted energy
$E_y$	energy in $y(t)$
$E_{1}, E_{0}$	energy of signals on $H_1$ and $H_0$ respectively
E(R)	exponent in M-ary error bound
$E_{\epsilon}$	energy in error signal (sensitivity context)
$e_N(t)$	error waveform
$\epsilon_I$	interval error
$\epsilon_T$	total error
erf (·)	error function (conventional)
$\operatorname{erf}_{*}(\cdot)$	error function (as defined in text)
$erfc(\cdot)$	complement of error function (conventional)
$erfc_{*}(\cdot)$	complement of error function (as defined in text)
$\eta$	(eta) threshold in likelihood ratio test
$E(\cdot)$	expectation operation (also denoted by $(\cdot)$ infrequently)
F	function to minimize or maximize that includes
	Lagrange multiplier
$ ilde{f}(t)$	complex envelope of signal
$\widetilde{f}_d(t)$	complex envelope of signal returned from desired
	target
$f_c$	oscillator frequency ( $\omega_c = 2\pi f_c$ )
F	matrix in differential equation
$\mathbf{F}(t)$	time-varying matrix in differential equation
$G^+(j\omega)$	factor of $S_r(\omega)$ that has all of the poles and zeros in
	LHP (and $\frac{1}{2}$ of the zeros on <i>j</i> $\omega$ -axis). Its transform is
	zero for negative time.
GB	general binary detection problem
g(t)	function in colored noise correlator
$g(t, A), g(t, \mathbf{A})$	function in problem of estimating $A$ (or $A$ ) in colored
	noise
$g(\lambda_i)$	a function of an eigenvalue
$g_{ILP}(\tau)$	impulse response of ideal low-pass filter
$g_d(\lambda)$	efficiency factor for diversity system
$g_h(t)$	homogeneous solution
$g_l(\tau)$	filter in loop
$g_{\iota o}(\tau),  G_{\iota o}(j\omega)$	impulse response and transfer function optimum loop
	filter

610 Glossary

$g_{pu}(\tau)$	unrealizable post-loop filter
$g_{puo}(\tau), G_{puo}(j\omega)$	optimum unrealizable post-loop filter
$g_{\delta}(t)$	impulse solution
$g_{\Delta}(t)$	difference function in colored noise correlator
8 <sub>2</sub>	a weighted sum of $g(\lambda_i)$
$g_{\infty}(t), G_{\infty}(j\omega)$	infinite interval solution
$\tilde{g}(t)$	complex function for optimum colored noise correlator
G	matrix in differential equation
$\mathbf{G}(t)$	time-varying matrix in differential equation
$\mathbf{G}_{d}$	linear transformation describing desired vector d
$\mathbf{G}_{d}^{u}(t)$	matrix in differential equation for desired signal
$\mathbf{g}(t)$	function for vector correlator
g <sub>d</sub> (A)	nonlinear transformation describing desired vector <b>d</b>
$\Gamma(x)$	Gamma function
γ	parameter ( $\gamma = k\sqrt{1+\Lambda}$ )
$\gamma$	threshold for arbitrary test (frequently various constants
/	absorbed in $\gamma$ )
$\gamma_a$	factor in nonlinear modulation problem which controls
7 a	the error variance
$\mathbb{T}_{L}(\lambda)$	gate function
$H_0, H_1, \ldots, H_i$	hypotheses in decision problem
$h_i$	<i>i</i> th coefficient in orthogonal expansion of $h(t, u)$
$H_n(t)$	<i>n</i> th order Hermite polynomial
h(t, u)	impulse response of time-varying filter (output at t due
	to impulse input at u)
$h_1(\tau, u \mid z)$	optimum unrealizable filter when white noise spectral
	height is z
$h_1(\tau, u:t)$	optimum filter for [0, t] interval
$h_1^{[1/2]}(t,z)$	functional square root of $h_1(t, z)$
$h_{1\infty}^{1}(\tau), H_{1\infty}(j\omega)$	filter using asymptotic approximation
$h_{1d}(t, u)$	filter to give delayed unrealizable MMSE estimate
$h_{\rm ch}(t, u)$	channel impulse response
$h_f(t,z)$	filter in Canonical Realization No. 3
$h_{fr}(t,z)$	realizable filter in Canonical Realization No. 3
$h_{fu}(t,z)$	unrealizable filter in Canonical Realization No. 3
$h_o(t, u)$	optimum linear filter
$h'_o(\tau), H'_o(j\omega)$	optimum processor on whitened signal: impulse
· · ·	response and transfer function, respectively
$h_{or}(t, u)$	optimum realizable linear filter for estimating $s(t)$
$h_{ou}(\tau), H_{ou}(j\omega)$	optimum unrealizable filter (impulse response and
- 1	transfer function)
$h_{ m sub}( au)$	suboptimum filter

$h_w(t, u)$	whitening filter
$h_{w_0}(t, u)$	filter whose output is white on $H_0$
$h_{w_0}(t, u)$	filter corresponding to difference between inverse
$n_{\Delta}(i, u)$	kernels on two hypotheses (3.31)
$\tilde{h}(t, u)$	complex envelope of impulse response of bandpass
n(i, u)	filter
$\tilde{h}$ $(t, r)$	complex realizable whitening filter
$ ilde{h}_{wr}(t,z)$ H	linear matrix transformation
$\mathbf{h}_{o}(t, u)$	optimum linear matrix filter
$I_o(\cdot)$	modified Bessel function of 1st kind and order zero
$I_k(\cdot)$	integrals involved in Edgeworth series expansion
, T T	(defined by (2.160))
$I_1, I_2$	integrals
	incomplete Gamma function
I	identity matrix
J(A)	function in variance bound
$J^{ij}$	elements in $J^{-1}$
$J^{-1}(t, u)$	inverse information kernel elements in information matrix
$J_{ij} \\ J_k(t, u)$	kth term approximation to information kernel
$\mathbf{J}_{k}(t, u)$	information matrix (Fisher's)
-	covariance function of composite signal (3.59)
$K_{\rm com}(t, u:s) K_s(t, u)$	covariance function of signal
	covariance of $r(t)$ on <i>i</i> th hypothesis
$K_{H_i}(t, u) \\ K_{H_0}^{[-1/2]}(t, u)$	covariance of $r(t)$ on <i>i</i> th hypothesis functional square root of $K_{H_0}^{[-1]}(t, u)$
$ \begin{array}{c} K_{H_0} \\ K_x(t, u) \end{array} $	covariance function of $x(t)$
$\widetilde{K}_D( au)$	correlation function of Doppler process
$\widetilde{K}_{DR}( au,\lambda)$	target correlation function
$\widetilde{K}_R\{v\}$	two-frequency correlation function
K	covariance matrix
$\mathbf{\tilde{K}}_{\widetilde{\mathbf{x}}}(t, u)$	covariance function of $\tilde{\mathbf{x}}(t)$
$\mathbf{k}_{d}(t)$	linear transformation of $\mathbf{x}(t)$
$L_n(x)$	nth order Laguerre polynomial
$l(\mathbf{R}), l$	sufficient statistic
l(A)	likelihood function
$l_B$	bias term in log likelihood ratio
$l_D$	term in log likelihood ratio due to deterministic input
$l_R^D$	term on log likelihood ratio due to random input
$l_a$	actual sufficient statistic (sensitivity problem)
$l_c, l_s$	sufficient statistics corresponds to cosine and sine
-	components
$l_v$	correlator output in suboptimum test

612 Glossary	
Ĩwo	output of correlator in "white-optimum" receiver
$\Lambda$	a parameter which frequently corresponds to a signal-
	to-noise ratio in message ERB
$\Lambda(\mathbf{R})$	likelihood ratio
$\Lambda(r_k(t))$	likelihood ratio
$\Lambda(r_k(t), A)$	likelihood function
$\Lambda_B$	signal-to-noise ratio in reference bandwidth for
	Butterworth spectra
$\Lambda_{ ext{ef}}$	effective signal-to-noise ratio
$\Lambda_{g}$	generalized likelihood ratio
$\Lambda_m$	parameter in phase probability density
$\Lambda_{3db}$	signal-to-noise ratio in 3-db bandwidth
$\Lambda_{x}$	covariance matrix of vector <b>x</b>
$\Lambda_{\mathbf{x}}(t)$	covariance matrix of state vector $(= \mathbf{K}_{\mathbf{x}}(t, t))$
λ	Lagrange multiplier
$\lambda_{\max}$	maximum eigenvalue
$\lambda_i$	eigenvalue of matrix or integral equation
$\lambda_i(A)$	ith eigenvalue, given A
$egin{aligned} &\lambda_i(A)\ &\lambda_i^{\mathrm{ch}}\ &\lambda_i^s\ &\lambda_i^T\ &\lambda_i^s\ &\lambda_i^s \end{aligned}$	eigenvalues of channel quadratic form
$\lambda_i^{\circ}$	eigenvalue of signal process
$\lambda_i^-$	total eigenvalue
$\Lambda_i$	eigenvalues of $r_*(t)$
$\ln \ln \Lambda(A)$	natural logarithm
$\ln \Lambda(A)$	log likelihood function logarithm to the base <i>a</i>
$\log_a M_x(jv), M_x(jv)$	characteristic function of random variable $x$ (or <b>x</b> )
$M_x(f^{U}), M_x(f^{U})$ $M_{l H_i}(s)$	generating function of $l$ on $H_i$
$m_{l H_i(S)}$ $m_D$	mean Doppler shift
$m_i$	<i>i</i> th coefficient in expansion of $m(t)$
$m_R$	mean delay
$m_{x}(t)$	mean-value function of process
$m_{\Delta}(t)$	difference between mean-value functions
M	matrix used in colored noise derivation
m	mean vector
$\mu(s)$	logarithm of $\phi_{l(\mathbf{R}) H_0}(s)$
$\mu_{BP}(s)$	$\mu(s)$ for bandpass problem
$\mu_{BS}(s)$	$\mu(s)$ for binary symmetric problem
$\mu_D(s)$	component of $\mu(s)$ due to deterministic signal
$\mu_{LEC}(s)$	$\mu(s)$ for low energy coherence case
$\mu_{LP}(s)$	$\mu(s)$ for low-pass problem
$\mu_R(s)$	component of $\mu(s)$ due to random signal
$\mu_{SIB}(s)$	$\mu(s)$ for simple binary problem

<i>u</i> (n)	$\mu(s)$ for separable kernel case
$\mu_{SK}(s)$	
$\mu_{\infty}(s)$	asymptotic form of $\mu(s)$
$\tilde{\mu}(s)$	complex version of $\mu(s)$
N	dimension of observation space
N	number of coefficients in series expansion
$N(m, \sigma)$	Gaussian (or Normal) density with mean $m$ and standard deviation $\sigma$
$N(\omega^2)$	numerator of spectrum
No	spectral height (joules)
n(t)	noise random process
$n_c(t)$	colored noise (does not contain white noise)
n <sub>i</sub>	<i>i</i> th noise component
$n_{*}(t)$	noise component at output of whitening filter
$\hat{n}_{c_r}(t)$	MMSE realizable estimate of colored noise component
$\hat{n}_{c_u}(t)$	MMSE unrealizable estimate of colored noise com-
2(4)	ponent
$\tilde{n}(t)$	complex envelope of noise process
N	noise correlation (matrix numbers)
n, <b>n</b>	noise random variable (or vector variable) Cramér-Rao bound
$\xi_{CR}$	elements in error covariance matrix
$\xi_{ij}(t)$	variance of ML interval estimate
$\xi_{ml}$	
$\xi_P(t)$	expected value of <i>realizable</i> point estimation error
$\xi_P\left(t \mid s(\cdot), \frac{N_0}{2}\right)$	minimum mean-square realizable filtering error of $s(t)$
$S_P(r   S(r), \frac{1}{2})$	in the presence of white noise with spectral height $N_0/2$
$\xi_{Pi}(t)$	variance of error of point estimate of <i>i</i> th signal
$\xi_{Pn}(t)$	normalized realizable point estimation error
$\xi_{P\infty}$	expected value of point estimation error, statistical
۶P∞	steady state
$\xi_{u}$	optimum unrealizable error
	normalized optimum unrealizable error
$\xi_{un}$ $\mathbf{\xi}_{d}(t)$	covariance matrix in estimating $d(t)$
$\xi_{P^{\infty}}$	steady-state error covariance matrix
$\widetilde{\xi}^{P\infty}(t)$	function in optimum receiver equations (9.90)
$\widetilde{\xi}(t:\lambda,\lambda')$	distributed error covariance function matrix
P	power
$Pr(\epsilon)$	probability of error
$Pr_{FSK}(\varepsilon)$	probability of error for binary FSK system
$Pr_{PSK}(\varepsilon)$	probability of error for binary PSK system
$P_{BP}$	power in bandpass problem
$P_D$	probability of detection (a conditional probability)

614 Glossary	
$P_{\rm ef}$	effective power
$P_F^{e_1}$	probability of false alarm (a conditional probability)
$P_i$	a priori probability of <i>i</i> th hypothesis
$\dot{P_{LP}}$	power in low-pass problem
	probability of a miss (a conditional probability)
$P_{M} P_{M}^{[1]}$	one-term approximation to $P_M$
P <sub>r</sub>	received power
$P_t$	transmitted power
$ ilde{P}_{DR}\{f,v\}$	transform of $\tilde{S}_{DR}\{f, \lambda\}$
$p_{\mathbf{r}\mid H_i}(\mathbf{R}\mid H_i)$	probability density of <b>r</b> , given that $H_i$ is true
$\phi(t)$	eigenfunction
$\phi(Y)$	Gaussian density, $N(0, 1)$
$\phi_i(t)$	ith coordinate function, ith eigenfunction
$\phi_{\iota(\mathbf{R}) \mathbf{H}_0}(s)$	moment generating function of $l(\mathbf{R})$ , given $H_0$
$\phi_x(s)$	moment generating function of random variable $x$
$\phi(t)$	phase of signal
$\phi(\tau, \omega), \phi(\tau, f)$	time-frequency correlation function
$\phi_{fg}(\tau, \omega)$	time-frequency cross-correlation function
$\psi_L(t)$	low pass phase function
$\psi_{\Omega}\{\lambda, f\}$	spread cross-ambiguity function
$\mathbf{P}(t)$	cross-correlation matrix between input to message
	generator and additive channel noise
$\boldsymbol{\phi}(t,\tau)$	state transition matrix, time-varying system
$ \mathbf{\Phi}(t - t_0) \triangleq \mathbf{\Phi}(\tau) $ Pr [·], Pr (·)	state transition matrix, time-invariant system
	probability of event in brackets or parentheses bandwidth constraint
$\Omega_B$	carrier frequency (radians/second)
$\omega_c$	Doppler shift
ω <sub>D</sub> ῶ	mean frequency
$Q(\alpha, \beta)$	Marcum's Q function
$Q_{H_i}(t, u)$	inverse kernel on <i>i</i> th hypothesis
$Q_n(t, u)$	inverse kernel
<i>q</i>	height of scalar white noise drive
Q ·	covariance matrix of vector white noise drive
$\mathbf{Q}_n(u,z)$	inverse matrix kernel
R	transmission rate
R(t)	target range
$R_x(t, u)$	correlation function
$ ilde{R}_{DR}\{ au,v\}$	two-frequency correlation function
$\mathscr{R}_B$	Bayes risk
r(t)	received waveform (denotes both the random process
	and a sample function of the process)

$r_c(t)$	combined received signal
$r_g(t)$	output when inverse kernel filter operates on $r(t)$
	K term approximation
$r_K(t)$	output of whitening filter
$r_*(t)$	output of winterning inter output of $S_{\rho}(\omega)$ filter (equivalent to cascading two
$r_{**}(t)$	whitening filters) whitening filters)
$\tilde{r}(t)$	complex envelope of signal process
	normalized correlation $s_i(t)$ and $s_i(t)$ (normalized
$ ho_{ij}$	signals)
0	normalized covariance between two random variables
ρ <sub>12</sub> ρ <sub>DR</sub>	target skewness
$\rho_r$	degradation due to interference
$\mathbf{R}(t)$	covariance matrix of vector white noise $\mathbf{w}(t)$
r, R	observation vector
$S(j\omega)$	Fourier transform of $s(t)$
$S_{c}(\omega)$	spectrum of colored noise
$S_Q(\omega)$	Fourier transform of $Q(\tau)$
$S_r(\omega)$	power density spectrum of received signal
$S_x(\omega)$	power density spectrum
$S_{\epsilon_{o}}(j\omega)$	transform of optimum error signal
$ ilde{S}_D\{f\}$	Doppler scattering function
$ ilde{S}_{DR}\{f,\lambda\}$	scattering function
$ ilde{S}_{Du}\{f\}$	uniform Doppler profile
$ ilde{S}_{ ilde{n}_r}\{f\}$	spectrum of reverberation return
$ ilde{S}_R(\lambda)$	range scattering function
s(t)	signal component in $r(t)$ , no subscript when only one
-(4 - 1)	signal
s(t, A)	signal depending on A
s(t, a(t))	modulated signal
$s_a(t)$	actual $s(t)$ (sensitivity context)
$s_{\rm com}(t,s)$	composite signal process $(3.58)$
S <sub>i</sub>	coefficient in expansion of $s(t)$ <i>i</i> th signal component
$\frac{s_i}{s_R(t)}$	random component of signal
$s_R(t)$ $s_r(t)$	received signal
$\hat{s}_r(t)$	realizable MMSE estimate of $s(t)$
$s_t(t)$	signal transmitted
$s_0(t)$	signal on $H_0$
$s_0(t)$ $s_1(t)$	signal on $H_1$
$s_1(t, \boldsymbol{\theta}), s_0(t, \boldsymbol{\theta})$	signal with unwanted parameters
$s_{\Omega}(t)$	random signal
$s_{*}(t)$	signal component at output of whitening filter
	· · · ·

616 Glossary	
$ ilde{\Sigma}(t)$	complex covariance matrix $(=\mathbf{\xi}_{P}(t))$
$\sigma^2$	variance
$\sigma_1^2, \sigma_0^2$	variance on $H_1$ , $H_0$
$\sigma_D^2$	mean-square Doppler spread
$\sigma_{\omega}{}^2$	mean-square bandwidth
$\sigma_R^2$	mean-square delay spread
$\sigma_t^2$	mean-square duration
$\mathbf{s}(t)$	vector signal
T T	pulse duration
	initial observation time (same as $T_i$ )
$T_d$	duration of pulse sequence
$T_f$ $T_i$	final observation time initial observation time
	pulse repetition interval
T <sub>p</sub> t	mean (arrival) time
au	round-trip delay time
θ, θ	unwanted parameter
$\theta(\mathbf{A}, \mathbf{A}_a)$	generalized ambiguity function
$\theta(\tau, \omega), \theta(\tau, f)$	signal ambiguity function
$\theta_{fg}(\tau,\omega)$	cross-ambiguity function
$\theta_{\Omega}(\mathbf{A}_a, \mathbf{A})$	generalized spread ambiguity function
$\theta_{\Omega_D}\{\lambda_a, \lambda: m_a, m\}$	Doppler-spread ambiguity function
θĨ	phase estimate
$\theta_{\rm ch}(t)$	phase of channel response
$\hat{\boldsymbol{\theta}}_1$	estimate of $\boldsymbol{\theta}_1$
	transition matrix
	transpose of matrix
[]†	conjugate transpose of matrix
$u_{-1}(t)$	unit step function
u(t), u(t)	input to system
$\widetilde{u}(t)$ V	elementary rectangular signal variable in piecewise approximation to $K_{i}(t)$
	variable in piecewise approximation to $V_{ch}(t)$ envelope of channel response
$V_{ m ch}(t)  v$	target velocity
Ŵ	bandwidth parameter (cps)
 W(jω)	transfer function of whitening filter
W <sub>ch</sub>	channel bandwidth (cps) single-sided
$W^{\mathrm{cn}}(j\omega)$	transform of inverse of whitening filter
w(t)	white noise process
w( au)	impulse response of whitening filter
$\tilde{w}(t)$	complex white noise process

W	a matrix operation whose output vector has a diagonal
	covariance matrix
x(t)	input to modulator
x(t)	random process
$\hat{x}(t)$	estimate of random process
x	random vector
$\mathbf{x}(t)$	state vector
$\mathbf{x}_{a}(t)$	augmented state vector
$\mathbf{x}_{d}(t)$	state vector for desired operation
$\mathbf{x}_{f}(t)$	prefiltered state vector
$\mathbf{x}_{M}(t)$	state vector, message
$\mathbf{x}_{N}(t)$	state vector, noise
$\tilde{\mathbf{x}}(t, \lambda)$	distributed complex state variable
Y(t, u)	kernel in singularity discussion (3.151)
y(t)	output of differential equation
y(t)	transmitted signal
Ζ	observation space
$Z_{1}, Z_{2}$	subspace of observation space
z(t)	output of whitening filter
$\mathbf{z}(t)$	gain matrix in state-variable filter $(\Delta \mathbf{h}_o(t, t))$