ADAPTIVE METHODS TO IMPLEMENT THE OPTIMUM RADAR SIGNAL PROCESSOR

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1. ABSTRACT

In this paper two adaptive algorithms to implement the optimum radar signal processor are presented and their performance are evaluated. The first method considered derives from the application of the Gram-Schmidt orthonormalization algorithm to the input. The second one refers to an algorithm for the direct inversion of the clutter covariance matrix.

2. INTRODUCTION

The theory of detecting a useful target echo-embedded in thermal noise and clutter having known statistics is well established in the literature and will be briefly revised in Sect. 3. The optimum processor consists of a linear filter, maximizing the output signal-to-interference ratio, cascaded with an envelope detector and threshold. The coefficients of the linear filter, which allow to shape the frequency response, depend on the characteristics of the interference (power, correlation and mean doppler frequency) and of the expected target echo. The performance achieved by the optimum filter in a number of relevant operational conditions are also shown in Sect. 3.

In practice the exploitation of this technique implies the design of processors that automatically adapt their filtering action in response to a changing environment and reach to the optimum filter in the steady-state condition. Any adaptive system should therefore optimize the trade-off between the speed of adaptation and the accuracy of estimation of the disturbance characteristics which determine the steady-state performance.

The first adaptive method described in Sect. 4, is based on the Gram-Schmidt transformation which resolves the input signals into a set of mutually decorrelated samples with the same power. This processing, which performs a whitening action on the clutter is cascaded with a filter matched to the modified expected target echo. The processor that orthonormalizes N input samples consists of a modular structure of N (N-1)/2 blocks. Each block operates a transformation on a couple of input samples giving rise to a new couple of uncorrelated data. The main feature of the proposed algorithm is that the transient response time is independent of the clutter characteristics and is nearly proportional to N. The steady-state performance, evaluated by means of computer simulation, are very close to the optimum.

The second method, described in Sect 5, is based on the direct estimate, from the input data, of the inverse of the clutter covariance matrix. A recursive algorithm is employed which combines in a non-linear way the input data and the estimate at the previous step. The speed of adaptation and the steady-state performance loss (due to the limited accuracy of the estimate) depend on a smoothing coefficient which is a-priori selected. Computer simulation results are shown for an environment consisting of one non-stationary clutter. The steady-state performance are achieved with limited losses comparable with those of the Gram-Schmidt algorithm. However, the speed of adaptation is faster at the expense of a more complex processing architecture.

3. OPTIMUM RADAR SIGNAL PROCESSOR

In this Section, the problem of detecting a ra-

dar target echo, embedded in a Gaussian distributed clutter and thermal noise is briefly revised. Assume that a coherent train of N pulses, T second apart, is transmitted and the corresponding echoes pertaining to a some range-cell are processed in order to detect a target having a doppler frequency f_D.
Denote with $\underline{Z}$, $\underline{S}$, $\underline{C}$, the N-dimensional vectors representing (in complex notation) the received samples, the expected target samples $s_i = \exp(-j2\pi(i-1)f_D T)$ and the disturbance samples, respectively. The zero-mean disturbance $\underline{C}$ is completely described through its covariance matrix $\underline{M} = \{\underline{C}^* \underline{C}^T\}$.

In Ref. 1 is shown that the processor maximizing the probability of detection (for a given false alarm rate) is that represented in FIG. 1.
It consists of a linear transversal filter providing the maximum signal-to-disturbance ratio at the output, an envelope detector and a threshold.
The filter coefficients, represented by the N-vector $\underline{W}$, allow to reject the clutter and to enhance the target component (if present) at frequency f_D. The optimum weights depend on the clutter statistics and are "tuned" to the expected target echo:

$$\underline{W}_{opt} = \underline{M}^{-1} \underline{S}^* \qquad (1)$$

The performance of the linear filter can be described in terms of the Improvement (IF) in the signal-to-overall-disturbance power ratio. It can be shown that the following equation holds:

$$IF = \underline{S}^T \underline{M}^{-1} \underline{S}^* \qquad (2)$$

In particular, in Fig. 2 the optimum IF is drawn vs. the number of processed samples N, for a clutter having mean doppler frequency $f_c = 0$, a correlation coefficient ρ between two consecutive samples and a clutter-to-noise ratio CNR of 40 dB. The target is assumed having a doppler frequency $f_D = 0.5/T$.
The filtering action of the optimum processor is well understood from FIG. 3, where the frequency response of the filter is shown together with the input disturbance spectrum. The parameters pertaining to the analysed case are also shown in the same Figure.

4. GRAM-SCHMIDT (GS) PROCESSOR

A fundamental decomposition property of the covariance matrix $\underline{M}$ is the base of the derivation of this processor Refs (2,3). It can be shown that the following relationships hold:

$$\underline{M}^{-1} = \underline{U}^T \underline{L} \underline{U}^* = \underline{D}^T \underline{D}^* \qquad (3)$$

$$\underline{D} = \underline{L}^{-1/2} \underline{U} \qquad (4)$$

where $\underline{L}$ denotes a diagonal matrix formed of real eigenvalues of $\underline{M}$ and $\underline{U}$ is the matrix of the corresponding eigenvectors. The previous decomposition is exploited to evaluate the filter output $y = \underline{Z}^T \underline{M}^{-1} \underline{S}^*$ without resorting to the inversion of the covariance matrix, implying heavy computational resources.
By means of eqn.3 the filter output becomes:

$$y = (\underline{D}\,\underline{Z})^T (\underline{D}\,\underline{S})^* \qquad (5)$$

from which the scheme of FIG.4 is derived.
To properly understand the working principle of this scheme, evaluate the statistics of the Gaussian-distributed signal $\underline{Z}' = \underline{D}\,\underline{Z}$ which is the output of the left-side block "D". It results:

$$E\{\underline{Z}'/H_0\} = \underline{0};\; E\{\underline{Z}'/H_1\} = \underline{D}\,\underline{S} = \underline{b} \qquad (6)$$

$$\text{cov}\{\underline{Z}'\} = \underline{D}^* \underline{M}\,\underline{D} = \underline{I} \qquad (7)$$

Therefore the transformation $\underline{D}$ corresponds to whiten and equalize in power the input disturbance. The right side block "D" provides the coefficients $\underline{b}^*$ of the matched filter to the modified target echo $\underline{b}$ contained in $\underline{Z}'$ and embedded in white noise.

The problem of the implementation of the transformation $\underline{D}$ in an environment with unknown statistics, possibly time-varying, is now afforded. To this end an adaptive processor is envisaged, which estimates in real time the disturbance characteristics and consequently adapts the processor parameters. The Gram-Schmidt orthonormalization algorithm, applied to the input samples, re presents a suitable mean to perform the transformation D (Ref. 2).

It can be shown that the algorithm is conveniently implemented by means of the scheme of Fig.5, which refers to the case of $N = 5$ samples. The working principle is to separately perform the orthogonalization and power equalization. The former is obtained through (N-1) steps; at each step, one sample is taken as a reference and all the successive samples are decorrelated from it. This is obtained by means of a set of equal blocks "A" each producing an output orthogonal to the reference input of the corresponding step. A modular processing architecture is obtained, consisting of $N(N-1)/2$ blocks for N samples. In the power equalization section the orthogonal samples are scaled through the blocks "SC" in order to have unity power. The scaling coefficients, consisting of an estimate σ_{ii} of the sample rms value, are directly provided by the blocks "A" on the diagonal; this estimate is not available for the N-th samples, for which a complete power equalizer (PE) is needed.

The block "A" can be implemented by a Howells.Applebaum loop (Ref. 3), and estimates the correlation coefficient between the input samples and their power. These estimates, which are obtained averaging over adjacent range cells, are more accurate as the number of cells increases; on the other hand, a faster estimation is suited for a time-varying disturbance.

The performance evaluation of the processor is now considered. Two figures are of interest: the steady-state loss of IF with respect to the optimum filter, due to the limited accuracy in the orthonormalization process; the number of range cells needed to reach the steady state. The operative conditions considered in Sect.3 to evaluate the performance of the optimum processor have been employed again to test the Gram-Schmidt algorithm. The obtained results allow to draw the following remarks:

a) The loss of improvement factor is limited to 1 + 2 dB in all the environments tested;
b) The steady-state condition is reached with a number of range cells roughly equal to ten times the number of processed pulses N, regardless of the assumed clutter conditions.

The previous results refer to the case of $N \leq 5$ processed samples. An import point to be noted is that the speed of adaptation is independent of the clutter parameters (i.e. CNR and correlation coefficient).

5. DIRECT MATRIX INVERSION (DMI) PROCESSOR

In this approach, a recursive estimation of $\underline{M}^{-1}$, directly from the input samples $\underline{Z}$, is attempted to obtain the set of optimum weights (1). Also in this case the matrix inversion, which is hardware costly, is avoided.

The proposed algorithm is the following (Ref.3):

$$\underline{\hat{M}}^{-1}(k)=a\underline{\hat{M}}^{-1}(k-1)-\frac{(1-a)a^{-2}\hat{M}^{-1}(k-1)Z^{*}(k)Z^{T}(k)\hat{M}^{-1}(k-1)}{1+(1-a)a^{-1}\underline{Z}^{T}(k)\underline{\hat{M}}^{-1}(k-1)Z^{*}(k)} \quad (8)$$

It can be noted that the estimate of $\underline{M}^{-1}$ at the k-th step is a non linear combination of the estimate at the previous step and the received samples $\underline{Z}$ pertaining to the k-th range-cell.

The accuracy in the estimation of $\underline{M}^{-1}$ and the time to achieve a steady-state condition are both controlled through the parameter a, ranging in (0,1). The value of a represents the relative importance given to the previous estimate with respect to the current received samples. In other words, when a goes to 1, the adaptation algorithm performs a very narrow-band filtering on the data, and an accurate estimate is achieved after a long transient time; on the opposite, for small a, the speed of adaptation of the algorithm increases at the expence of the steady-state accuracy.

In order to evaluate the performance, the DMI algorithm has been tested in a computer simulation against a time-varying clutter environment.

In particular, a clutter parameter (e.g. CNR, ρ, f_c) has been allowed to switch after the 20-th range cell, from one value to another. The IF has been drawn vs K (range-cell number) in FIGS. 6,7,8, for different values of a and for N = 4 processed pulses. The optimum IF values in the examined conditions are also shown for comparison. As a general conclusion, with a suitable choice of a, it is possible to attain a steady-state loss within 2-3 dBs or less, in a transient time of nearly ten range cells. Also in this case, these performance are not influenced by the disturbance characteristics. It should be investigated the dependence on the number N of processed pulses.

6. CONCLUSIONS

Two different adaptative techhniques have been proposed for the implementation of the optimum radar processor. From a first comparison of performance, it seems that two processors are nearly equivalent from the steady-state loss viewpoint; however, the DMI shows a higher speed of adaptation.
From the viewpoint of hardware complexity, the GS leads to a modular architecture,whereas for the DMI a suitable implementation is not available up to now.

7. REFERENCES

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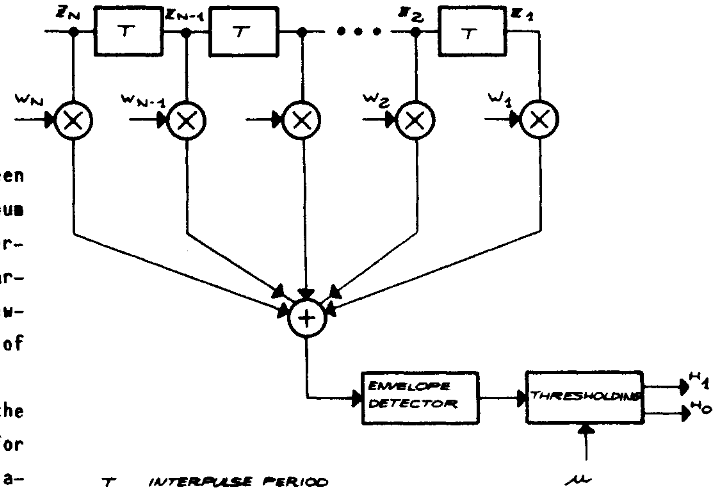


FIG. 1 SCHEME OF THE OPTIMUM PROCESSOR.

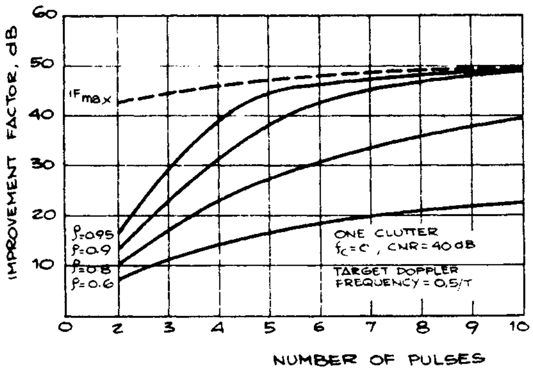


FIG. 2 IMPROVEMENT FACTOR FOR ONE CLUTTER VS. NUMBER OF SAMPLES.

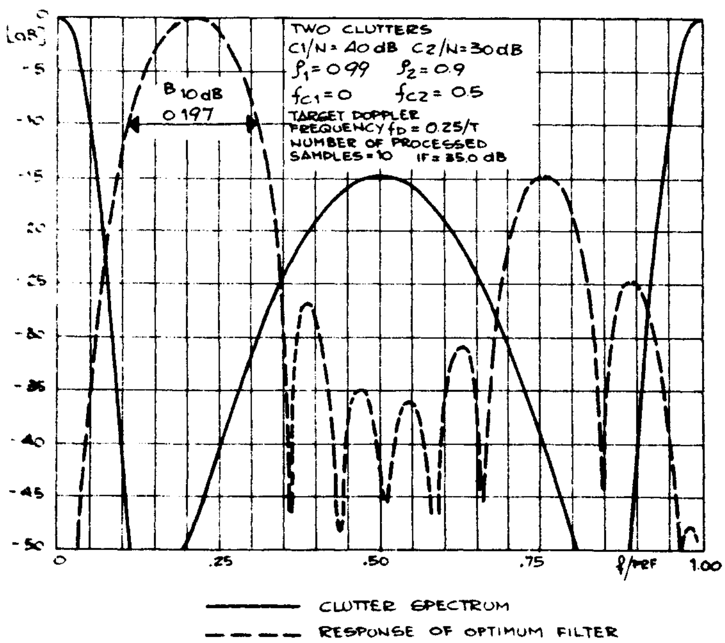


FIG 3 FREQUENCY RESPONCE OF THE OPTIMUM FILTER AGAINST TWO CLUTTERS.

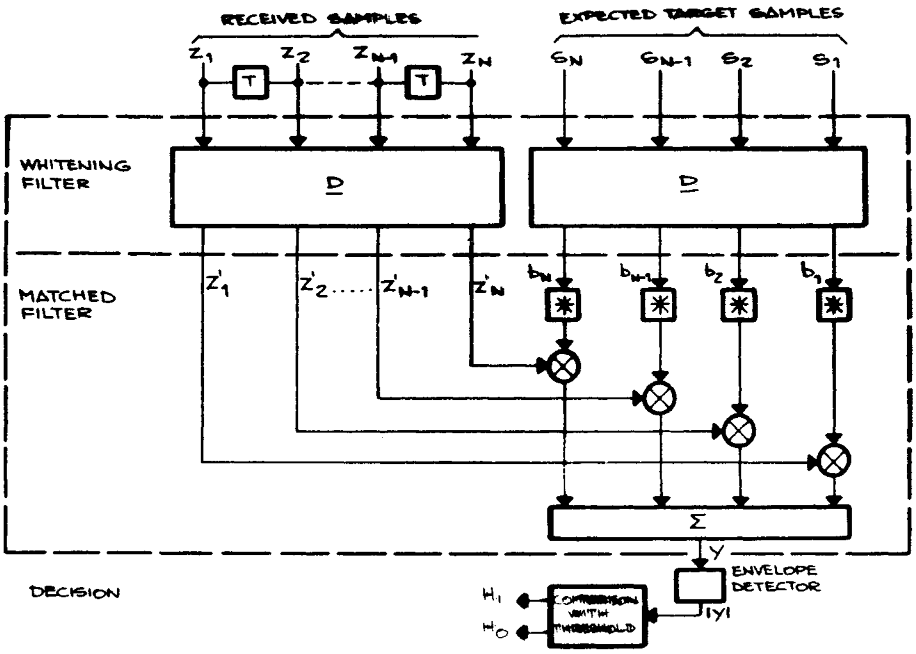
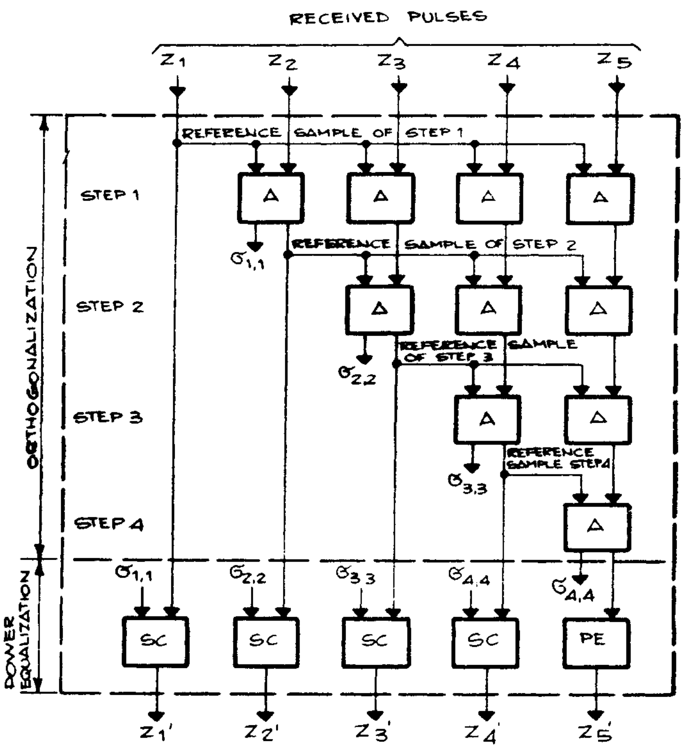


FIG. 4 DECOMPOSITION OF THE OPTIMUM PROCESSOR.



SC : SCALING OF SIGNAL
PE : POWER EQUALIZER
$\sigma_{i,i}$ = POWER OF THE SIGNAL AT THE i-th LEVEL

FIG. 5 SCHEME OF THE GRAM-SCHMIDT PROCESSOR.

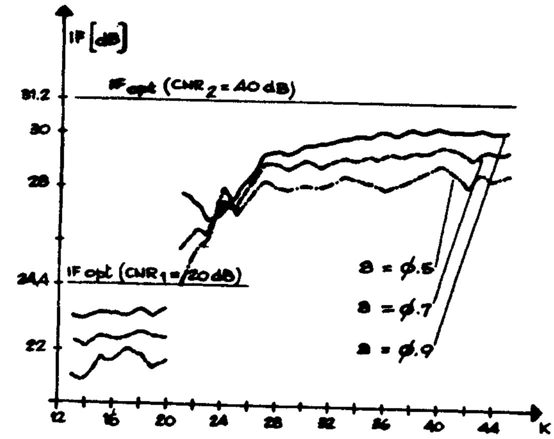


FIG 6 VARIATION OF CNR

CNR_1 = 20 dB

CNR_2 = 40 dB

ρ = 0.9

f_C = 0.

f_D = 0.5/T

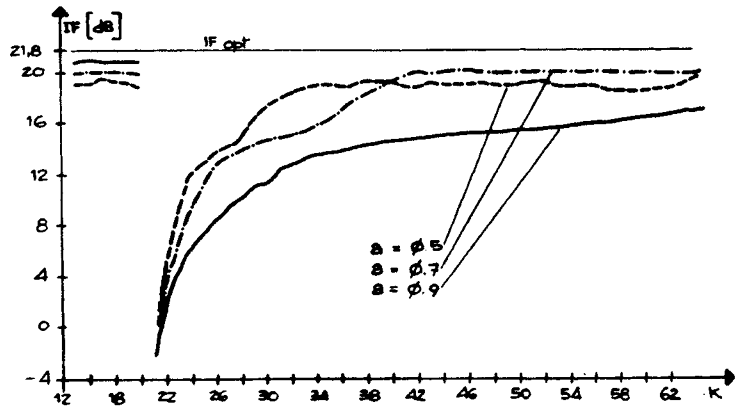


FIG. 7/VARIATION OF f_c

CNR = 40 dB

ρ = 0.9

f_{C1} = 0.

f_{C2} = 0.5/T

f_D = 0.25/T

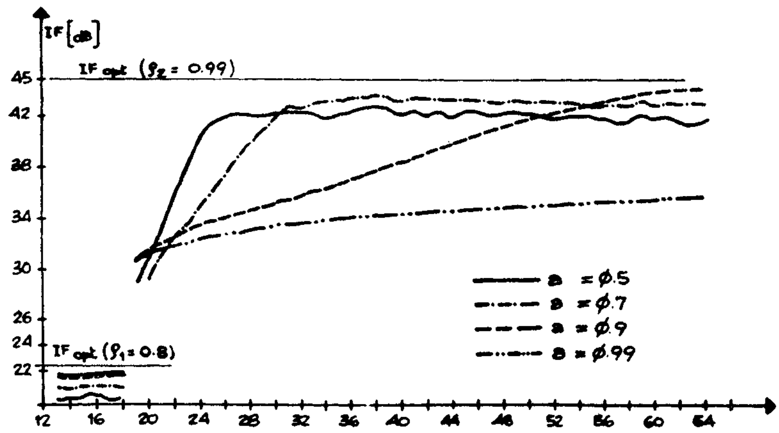


FIG. 8 VARIATION OF ρ

CNR = 40 dB

ρ_1 = 0.8

ρ_2 = 0.99

f_C = 0.

f_D = 0.5/T