ADVANCED MODELS OF TARGETS AND DISTURBANCES
AND RELATED RADAR SIGNAL PROCESSORS

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ABSTRACT

The first part of the paper provides flexible and reliable stochastic models for the radar signals scattered by target and clutter sources. The models allow to consider any shape of autocorrelation function between consecutive pulse echoes and any probability density function for their in-phase and quadrature components. The second part of the paper revises the theory of detecting targets, with any type of probability density and autocorrelation function, embedded in a disturbance having any type of probability density and autocorrelation function. In the third part of the paper, the theory is applied to the cases in which target and/or disturbance may have a log-normal probability density for the amplitudes. Several processing schemes are suggested and corresponding detection performances evaluated. Finally, adaptive implementation schematics are suggested for some of the processors presented.

INTRODUCTION

High performance radar require more and more accurate models of targets to be detected and disturbance to be suppressed. This paper provides flexible and reliable stochastic models which can be tailored to accomodate experimental results at disposal. The proposed models allow to consider any shape of autocorrelation function between successive radar echoes and any probability density function for their in-phase and quadrature components. Non-stationary target and disturbance processes may be also considered. Swerling target models and Gaussian noise intereference are dealt with as special cases of the proposed models.

Current radar signal processing techniques /1/, based on a linear filter (for disturbance cancellation and useful signal enhancement) cascaded with an evelope detector and a comparison with a threshold, suffer poor performance especially when in-phase and quadrature components of the target and/or the disturbances have non-Gaussian probability density function. This observation motivates a deep revision of the radar signal processing techniques which is afforded in the second part of the paper. In particular, the problem of detecting non-Gaussian distributed target echoes in non-Gaussian distributed clutter is tackled according to the theory developed by Kailath /2/. This theory, which is essentially available only for continuous-time, real-valued stochastic processes, is extended to discrete-time complex-valued stochastic processes as in the radar case. Roughly speaking, the rationale of the proposed approach is to estimate the disturbance and the target signal in the two alternative detection hypotheses H_1 and H_o. The detection of the useful signal, if present, is provided by first making the difference between the estimates, then correlating the result with the received echoes and finally comparing the output with a threshold. The estimates are in general complicated nonlinear functions of the received radar echoes. However, relatively simple solutions are conceived in this paper in the special case of target and disturbance having log-normal probability density for the amplitudes.

The third part of the paper deals with the application of the theory to a number of operational cases of interest. For each case considered the processor is derived, its detection performance evaluated and, in some cases, compared with the current signal processor schemes. The following cases are afforded:

a) Gaussian distributed target signal, having any shape of autocorrelation function, embedded in Gaussian distributed clutter;
b) White Gaussian distributed target in log-normal clutter;
c) A-priori known target signal embedded in log-normal clutter;
d) Gaussian distributed and correlated target in log-normal clutter;
e) Log-normal distributed target signal in thermal noise;
f) Log-normal distributed target in log-normal

clutter.

Finally, the problem of real-time implementation of the proposed processors is briefly considered. In particular, actractive adaptive procedures are described for the cases (b) and (c).

ADVANCED MODELS FOR TARGET AND DISTURBANCES

Generally, radar echoes scattered by a useful target are assumed to be generated by a very simple stochastic model. The in-phase and quadrature components are usually modelled as Gaussian distributed random variables and the pulse-to-pulse fluctuations are assumed to be completely correlated or totally uncorrelated (Swerling models). Now, some experimental evidence show that the target may have a non-Rayleigh amplitude density function and a fluctuation between pulses intermediate among the two extreme cases. A chi-squared and log-normal amplitude distributions was hypothesized in /3/. As far as the time correlation of consecutive radar echoes is concerned, the approach followed /4/ was to model the signal as a mixture of non fluctuating and fluctuating components. The correlation was related to the power ratio between the two signal components. The approach, however, assumed the hypothesis of stationary target and did not take in consideration any shape of autocorrelation function. A more general observation refers to the lack, in the open litèrature, of a unified approach to target and disturbance modelling. A relevant exception is /5/ which, however, refers to the continuous time real-valued signal case.

All the above mentioned limitations are overcome by the model proposed in this paper. A white Gaussian distributed complex-valued process feeds the cascade of a linear dynamical filter followed by a nonlinear zero-memory device. The shape of the nonlinearity modifies the probability density of the signal, while the poles of the linear filter and the shape of nonlinearity tailor the autocorrelation function of the process.

Consider now the case of modelling disturbances such as clutter sources. Usually, a set of N clutter echoes is modelled as a Gaussian distributed, complex-valued process, having a certain covariance matrix. However, in several applications the statistic of the clutter amplitude differs from the Rayleigh one. These situations occur when sea clutter is viewed with a high resolution radar (pulse width less than 0.5 μs) at low grazing angle (less than 5 degrees). They also result when land clutter is viewed at low grazing, regardless of the radar resolution. Measurements of non-Rayleigh clutter statistics indicate that either a log-normal or a Weibull distribution provides a good match for the long term average amplitude distribution of measurement data. A number of papers refer experimental results on non-Rayleigh clutter, see for example the commented collection of papers in /6, Sect. 5/ and the recent papers /7/, /8/ and /9/. Of course, the same model proposed for the target fits to the clutter case. This model has been extensively applied /10/ to generate the in-phase and quadrature components of N samples for a clutter having a log-normal amplitude and any kind of covariance matrix. It is possible to show that in this case the nonlinearity is a complex exponential. Additionally, a suitable mathematical relationship has been derived (see Eqn. 4 of /11/) to design the linear filter from the desired shape of the clutter autocorrelation function (e.g. Gaussian or exponential).

In general, when target, clutter and thermal noise are contemporary present the overall mathematical model to be considered is that shown in Fig. 1. Three independent N-dimensional sequences of white Gaussian noise, namely $\underline{n}_s$, $\underline{n}_d$ and $\underline{n}$, feed three separated branches to provide target ($\underline{s}$), clutter ($\underline{d}$) and thermal noise ($\underline{n}$) samples. The N-dimensional matrices $\underline{F}_s$ and $\underline{F}_d$ perform a linear combination on the incoming samples of $\underline{n}_s$ and $\underline{n}_d$ respectively, providing two correlated sequences which are further processed through the two N-dimensional nonlinearities $\underline{h}_s(.)$ and $\underline{h}_d(.)$.

AN OVERVIEW OF OPTIMUM RADAR DETECTION

The following detection problem should be solved:

$$\underline{z} = \underline{s} + \underline{d} + \underline{n} \quad : H_1$$

$$\underline{z} = \underline{d} + \underline{n} \quad : H_0 \tag{1}$$

where $\underline{z}$ is the set of N received radar echoes; $\underline{s}$ and $\underline{d}$ represent the signal samples scattered by the useful target (if present) and the clutter, respectively. The probability densities $p(\underline{s})$ and $p(\underline{d})$ may be non-Gaussian, while the thermal noise $\underline{n}$ is a zero-mean white Gaussian process with a diagonal covariance matrix $\underline{M}_n$. The sequences $\underline{s}$, $\underline{d}$ and $\underline{n}$ are assumed independent from one another. The optimum detector for this problem /2/ evaluates the log-likelihood ratio:

$$LR(\underline{z}) = \log p\,(\underline{z}/H_1) - \log p\,(\underline{z}/H_0) \tag{2}$$

which is, in general, a complicated nonlinear function of $\underline{z}$.

When $\underline{s}$ and $\underline{d}$ have Gaussian density with zero mean and covariance matrices $\underline{M}_s$ and $\underline{M}_d$, respectively, the LR is the following quadratic form of $\underline{z}$ /12/, /13/:

$$LR(\underline{z}) = \underline{z}^T \left(\underline{M}_r^{-1} - (\underline{M}_r + \underline{M}_s)^{-1}\right) \underline{z}^* \tag{3}$$

with $\underline{M}_r = \underline{M}_d + \underline{M}_n$. It can be shown that Eqn. 3 can be rewritten as follows:

$$LR\ (\underline{z}) = \underline{z}^T\ \underline{M}_r\ \hat{\underline{s}}^* \qquad (4)$$

$$\hat{\underline{s}} = E\ \ (\underline{s}/\underline{z}) = (\underline{M}_s\ (\underline{M}_s + \underline{M}_r)^{-1})^*\ \underline{z} \qquad (5)$$

An equation similar to (4) was found is /1/ for the case of a-priori known target signal. The signal $\underline{s}$ takes the place of the statistical estimate $\hat{s}$ which is actually a linear function of $\underline{z}$. In the literature, Equns. (4) and (5) are referred to as "estimator-correlator", see for example /2/ for the continuous-time real valued process case.

An equivalent recursive formulation /12/, in place of the batch approach of Equns. (4), (5), is represented in Fig. 2 where the estimates $\hat{z}_1$ and $\hat{z}_0$ (in the H_1 and H_0 hypotheses respectively) and the covariances P_1 and P_0 are given by recursive Kalman filtering algorithms.

Now it is possible to show that Fig. 2 applies also when the probability densities of $\underline{s}$ and $\underline{d}$ are non Gaussian with the estimators being nonlinear functions of z. This was demonstrated by Kailath /2/ for the continuous-time real valued case. Extensions are also available under suitable hypotheses for the discrete-time case. It is worth noting that the original test of hypotheses of Equn. (1) can be replaced by the following one:

$$\nu_1(n/n-1) \sim (0,\ P_1\ (n/n-1)) \qquad : H_1$$

$$\nu_0(n/n-1) \sim (0,\ P_0\ (n/n-1)) \qquad : H_0 \qquad (6)$$

corresponding to the comparison of the likelihood of ν_1 to be a white Gaussian process with zero mean and variance P_1 versus the likehood of ν_0 to be a white Gaussian process with zero mean and variance P_0. In other words, the original radar measurements z have been transformed in two white Gaussian sequences ν_1 and ν_0 (statistical innovations); the evaluation of the corresponding likelihood ratio follows straightforward.

APPLICATION TO SEVERAL OPERATIONAL CASES

The purpose of this section is to apply the previous theory to practical cases of interest; original results are presented concerning to the derivation of detection architectures and the evaluation of corresponding detection performances.

Gaussian-distributed time-correlated target in Gaussian-distributed time-correlated clutter

This topic has been widely analyzed in /13/. Here a limited set of detection performance are presented; they follow by Montecarlo simulation of Equn. 3. Figure 3 shows the detection performance of a target having a Gaussian shaped autocorrelation function (the parameter being the correlation coefficient between any two consecutive pulses of the train) embedded in white noise. Five pulses have been processed and the threshold was set to maintain $P_{FA} = 1.4\ 10^{-4}$. It can be seen that a small amount of fluctuation helps the detection for relatively high signal-to-noise ratio (SNR). The same conclusion does not apply when clutter is present. An extensive set of detection curves can be found in /13/.

Detection of white Gaussian target (Swerling II) in log normal clutter

Only, the cancellation of clutter can be achieved. For this particular case, a convenient approximation /10/ of the general architecture of Fig. 2 is the nice processor of Fig. 4. Briefly speaking, the portion of clutter present in the current sample z_k is cancelled by substracting the estimated clutter $\hat{z}_k$. The estimate is simply performed in the following three steps: i)transformation of the previous log normal clutter samples (e.g. z_{k-2} and z_{k-1}) in Gaussian distributed samples by means of a complex logarithm; ii)one-step ahead linear prediction of Gaussian distributed clutter sample $\hat{z}'_k$; iii)transformation, with a complex exponential function, of $\hat{z}'_k$ in the log-normal clutter sample $\hat{z}_k$.

The detection performance of this processor have been evaluated in the following operational situation: a) target signal according to Swerling II model; b) one clutter source having a Gaussian shaped autocorrelation function (ρ_c =0.9), a mean Doppler frequency f_c=0 and a clutter-to-noise ratio CNR=30 dB. Fig. 5 indicates an example of detection performance when three pulses are processed and the threshold is set to obtain P_{FA}=1.4 10^{-4}. In the same figure the performance of a conventional linear MTI are also indicated; it is noted a loss of 10 dB on average. A comprehensive set of detection curves can be found elsewhere /10/ together with a detailed analysis of the proposed processor.

A-priori known target signal in log normal clutter

The problem afforded in this case is the clutter cancellation together with the enhancement of useful signal. On the basis of the results of the previous subsection the general architecture of Fig. 2 is approximated by the schematic of Fig. 6. Detection performance are illustrated in the Figs. 7 through 10. Figures 7 through 9 refer to the case of two samples and to a Gaussian shaped autocorrelation function for the clutter source. The doppler frequencies of the target and clutter are 0.5 PRF and 0, respectively. False alarm probability has been set constantly equal to 10^{-4}; different values of ρ_c and CNR have been considered. Figs. 11 and 11 refer to 3 and 5 samples respectively.

For comparison purpose the detection performance of a processor, optimized for Gaussian clutter, have been evaluated when it is fed by a log-normal clutter. Remarkable loss in SNR are noted; in the case of N=2, $P_{FA}= 6\ 10^{-5}$, CNR=30 dB and $\rho_c = 0.99$, the loss amounts to 20 dB.

Log-normal distributed and time correlated target in white Gaussian noise

The main problem in this case is to find a suitable approximation of $\hat{z}_1$ (n/n-1) (which is equal to $\hat{s}_1$ (n/n-1)). The algorithm illustrated in Fig. 12 shows the approach followed in this case. The log normal, time-correlated, target signal is first transformed in a Gaussian process, then predicted one step ahead and finally re-transformed in a log normal process. The statistical detection test is now performed on the two new processes ν_1 and ν_0. It has been assumed that P_1 and P_0 are equal (exact prediction of s), therefore they are not required in the likelihood computation. The time integration is performed over a number of N pulses. Fig. 13 shows detection performance for N=2 and SNR=20dB for each pulse. A Gaussian shaped autocorrelation function has been assumed for the target, ρ_s being the one-lag correlation coefficient. Fig. 14 illustrates detection performance for different number of pulses and a same value of $\rho_s = 0.9$.

Gaussian distributed and correlated target echoes in log-normal clutter

In this case a convenient approach corresponds to the replacement of the a-priori known signal $\underline{s}$ of the Fig. 6 with a convenient estimate $\hat{s}_1(n/n-1)=$ E (s (n)/z(k), k=1,2,...(n-1); H_1) of the target signal. The mathematical expression of $\hat{s}_1$ is a complicated nonlinear function of z. An approximation of the signal estimate is represented by a monocromatic signal having the mean Doppler frequency of the original spectrum and an amplitude equal to the square root of SNR which is assumed known a-priori. Fig. 15 illustrates the detection performance for this particular approximation. An on-line estimate of s is obtained by linearization of the cexp(.) function which provides the clutter portion of the scattered signal and applying Equn. 5 to the linearized measurement equation. Fig. 16 shows the detection performance for this more accurated signal estimate; comparison with results of Fig. 15 is straightforward. It is interesting to note that when the autocorrelation of target signal vanishes, the processor reduces to that of Fig. 4.

Log-normal distributed and time correlated target signal in log-normal clutter

This topic is affordable with techniques similar to those applied in the previous subsection. Detection performance for the case of a-priori known SNR are shown in Fig. 17. Comparison with the curve of Fig. 15 shows the difference in detection performance due to the different probability density of the target signal. To obtain a more accurate estimate of target signal is necessary to linearize the cexp(.) functions partaining to target and clutter signals. Corresponding detection performance are not shown here.

IMPLEMENTATION PROBLEMS AND CONCLUDING REMARKS

The real-time implementation of the processors presented in the previous Sections is now afforded. Special attention should be paid to the adaptive implementation which was shown /11/ to be a powerful approach for the case in which target and clutter are Gaussian distributed. In particular, the adaptive methods described in /11/ are extended to the processor of Fig. 4. Fig. 18 gives an idea of a possible adaptive implementation for the processor. The adaptive processing is confined to the evaluation of the weights w_{k-2} and w_{k-1} of the schematic of Fig. 4. The same adaptive procedure can be applied to the other processors of Figs. 6 and 12 to accomplish the clutter and signal estimation. Furthermore, adaptive calculation of the decision threshold is generally needed, since it depends on clutter and target parameters.

As a general conclusion, it can be said that this paper makes a breakthrough in the theory of target and clutter modelling and in the derivation of related optimum and sub-optimum detection processors. However, this area of research is at the first stage of development and needs more investigation. In particular, the approach followed here should be extended to other target and clutter models such as the Weibull one.

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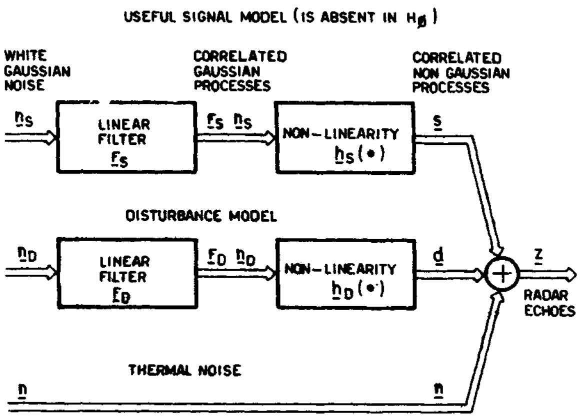

FIG. 1 - OVERALL TARGET AND DISTURBANCE MODEL

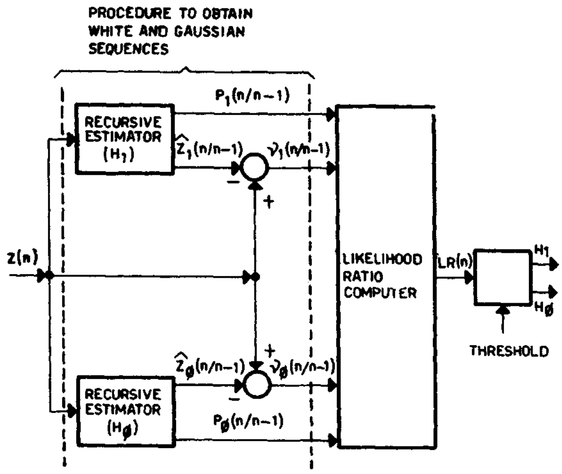

FIG. 2 - RECURSIVE EVALUATION OF LIKELIHOOD RATIO AND DETECTION PROCEDURE

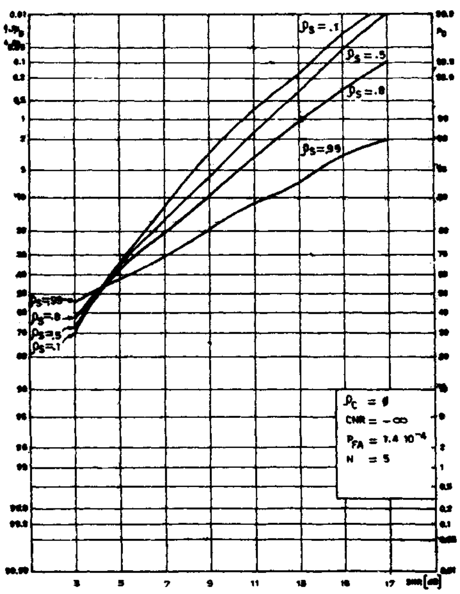

FIG. 3 - DETECTION PERFORMANCE OF A GAUSSIAN DISTRIBUTED AND CORRELATED TARGET IN THERMAL NOISE

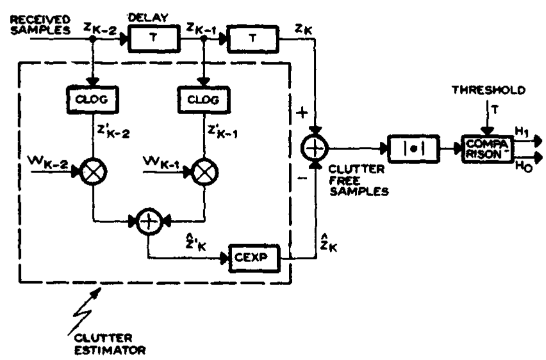

FIG. 4 - NONLINEAR (HOMOMORPHIC) CLUTTER CANCELLER

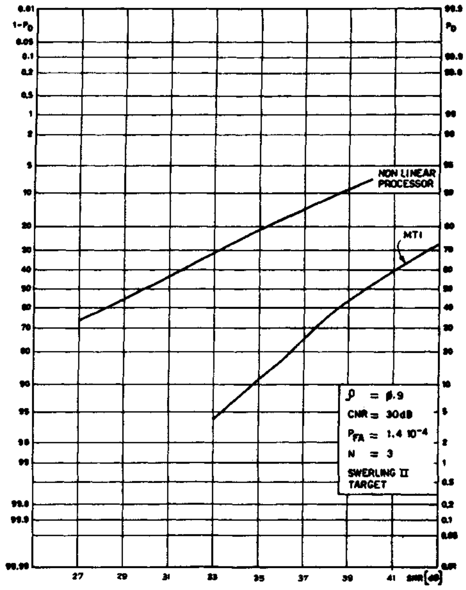

FIG. 5 - PERFORMANCE COMPARISON BETWEEN MTI AND NONLINEAR CANCELLER FOR A LOGNORMAL CLUTTER

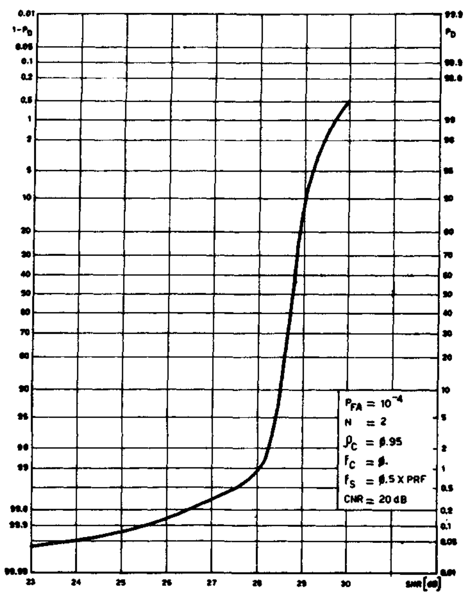

FIG. 7 - DETECTION PERFORMANCE FOR A-PRIORI KNOWN TARGET IN LOGNORMAL CLUTTER (CNR = 20 dB, ρ_c = 0.95)

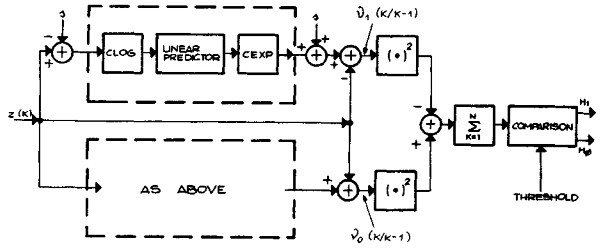

FIG. 6 - DETECTOR FOR A-PRIORI KNOWN TARGET SIGNAL IN LOGNORMAL CLUTTER

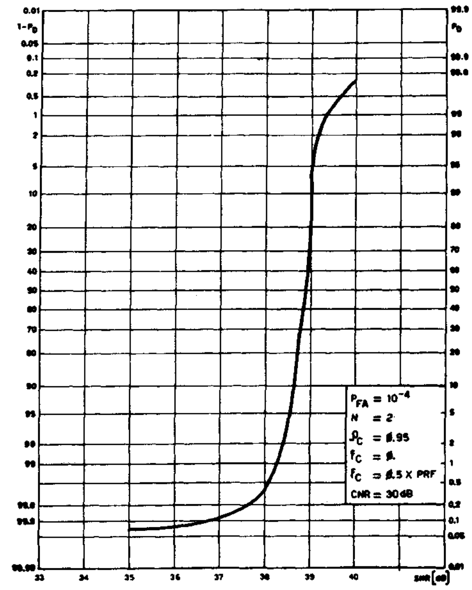

FIG. 8 - SAME AS FIG. 7 EXCEPT FOR CNR = 30 dB

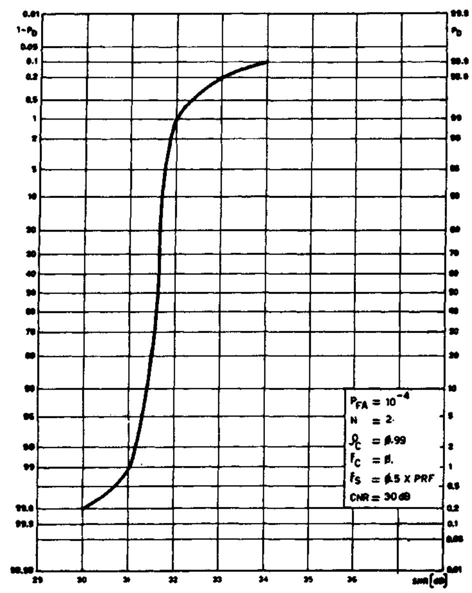

FIG. 9 - SAME AS FIG. 8 EXCEPT FOR ρ_c = 0.99

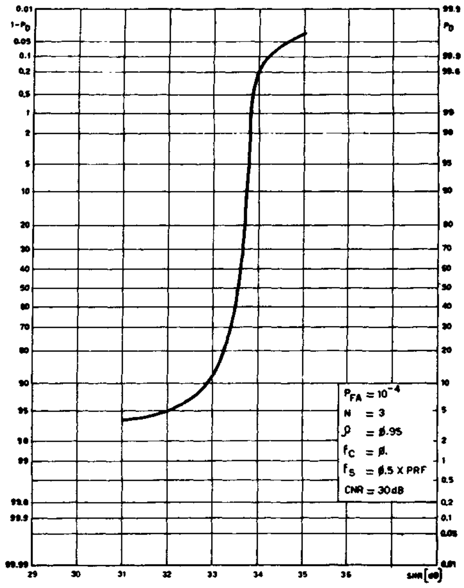

FIG. 10 - SAME AS FIG. 8 EXCEPT FOR N = 3

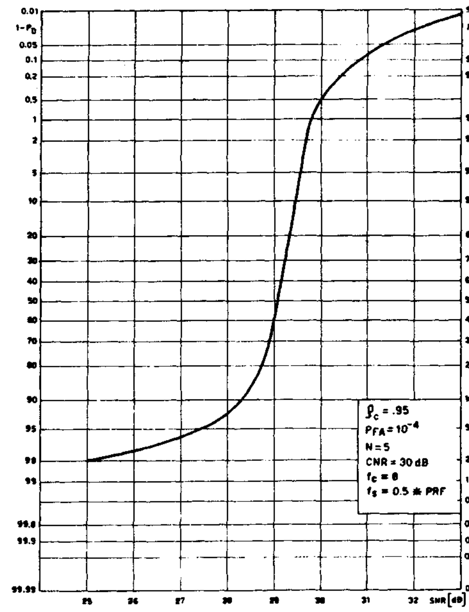

FIG. 11 - SAME AS FIG. 8 EXCEPT FOR N = 5

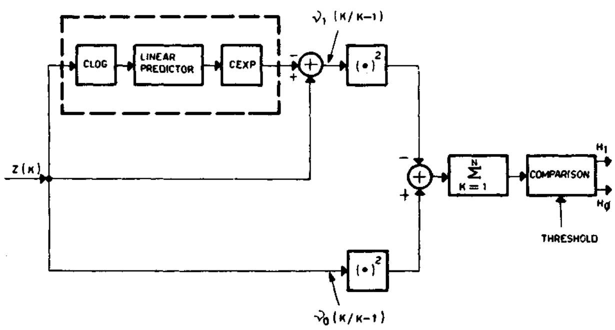

FIG. 12 - DETECTOR FOR LOGNORMAL TARGET IN WGN

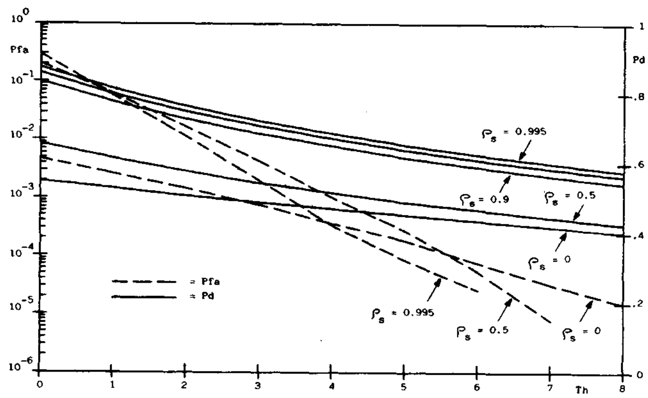

FIG. 13 - DETECTION PERFORMANCE FOR LOGNORMAL TARGET IN WGN
(N = 2, DIFFERENT VALUES OF ρ_S)

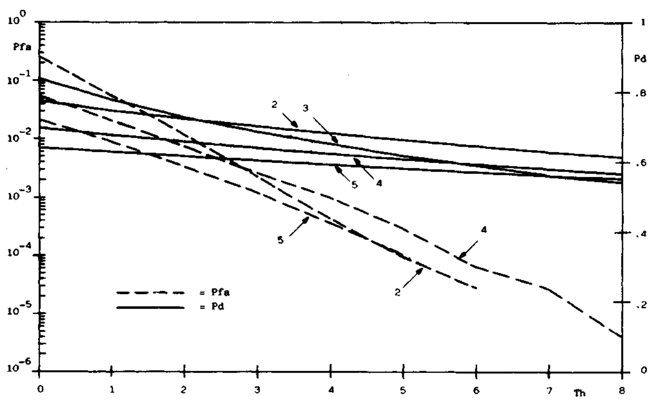

FIG. 14 - SAME AS 13 EXCEPT FOR ρ_S = 0.9, VARIABLE NUMBER OF PULSES N

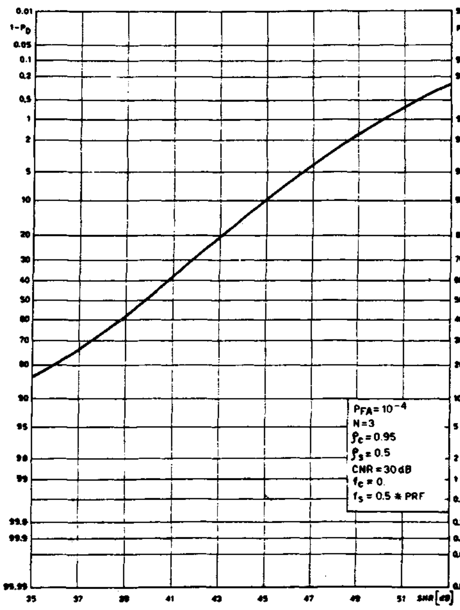

FIG. 15 - DETECTION PERFORMANCE FOR GAUSSIAN DISTRIBUTED AND CORRELATED TARGET IN LOGNORMAL CLUTTER (SNR KNOWN A-PRIORI)

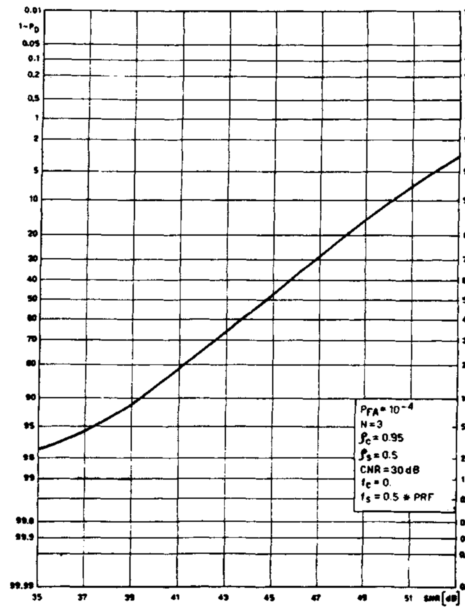

FIG. 16 - SAME AS 15 EXCEPT FOR LINEARIZED ESTIMATE OF TARGET SIGNAL

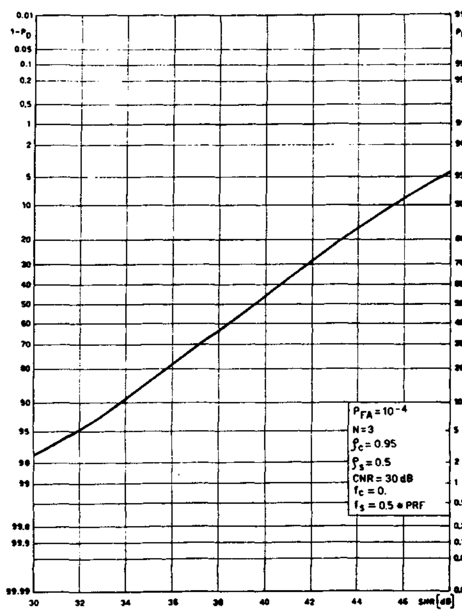

FIG. 17 - DETECTION PERFORMANCE FOR LOGNORMAL DISTRIBUTED AND CORRELATED TARGET IN LOGNORMAL CLUTTER

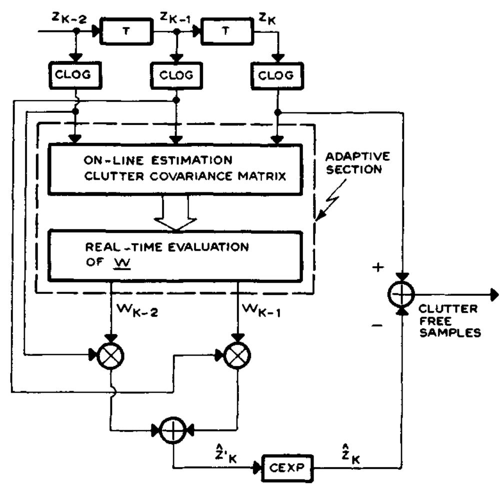

FIG. 18 - ADAPTIVE APPROACH TO LOGNORMAL CLUTTER CANCELLATION