Poularikas A. D. “Windows ”
The Handbook of Formulas and Tables for Signal Processing.
Ed. Alexander D. Poularikas
Boca Raton: CRC Press LLC, 1999

7
Windows

7.1 Introductory Material

7.1.1 Introduction

N = number of samples

T = interval between samples

NT = total time duration of the signal

$\frac{1}{NT}$ = minimum spectral resolution (s^{-1})

DFT = discrete Fourier transform

Leakage = Spectral leakage takes place when the signal has frequencies other than those of the basis set. These other frequencies will exhibit non-zero properties on the entire basis set known as leakage.

7.2 Figures of Merit (see Table 7.1)

7.2.1 Equivalent Noise Bandwidth

$$ENBW = \frac{\sum_n w^2(nT)}{\left|\sum_n w(nT)\right|^2} = \text{equivalent noise bandwidth} =$$

the width of an equivalent ideal rectangular spectral response that will pass the same noise power as the window (filter) under test.

$w(nT)$ = window samples

TABLE 7.1 Figures of Merit for Shaped DFT Filters

Weighting		Highest sidelobe level (dB)	Sidelobe falloff (dB/octave)	Coherent gain	Equivalent noise BW (bins)	3.0-dB BW (bins)	Scallop loss (dB)	Worst-case process loss (dB)	6.0-dB BW (bins)	Overlap correlation (%) 75% OL	Overlap correlation (%) 50% OL
Rectangle		–13	–6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
Triangle		–27	–12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0
	$\alpha = 1.0$	–23	–12	0.64	1.23	1.20	2.10	3.01	1.65	75.5	31.8
$\cos^{\alpha}(x)$	$\alpha = 2.0$	–32	–18	0.50	1.50	1.44	1.42	3.18	2.00	65.9	16.7
Hann	$\alpha = 3.0$	–39	–24	0.42	1.73	1.66	1.08	3.47	2.32	56.7	8.5
	$\alpha = 4.0$	–47	–30	0.38	1.94	1.86	0.86	3.75	2.59	48.6	4.3
Hamming		–43	–6	0.54	1.36	1.30	1.78	3.10	1.81	70.7	23.5
Parabolic		–21	–12	0.67	1.20	1.16	2.22	3.01	1.59	76.5	34.4
Riemann		–26	–12	0.59	1.30	1.26	1.89	3.03	1.74	73.4	27.4
Cubic		–53	–24	0.38	1.92	1.82	0.90	3.72	2.55	49.3	5.0
Tukey	$\alpha = 0.25$	–14	–18	0.88	1.10	1.01	2.96	3.39	1.38	74.1	44.4
	$\alpha = 0.50$	–15	–18	0.75	1.22	1.15	2.24	3.11	1.57	72.7	36.4
	$\alpha = 0.75$	–19	–18	0.63	1.36	1.31	1.73	3.07	1.80	70.5	25.1
Bohman		–46	–24	0.41	1.79	1.71	1.02	3.54	2.38	54.5	7.4
Poisson	$\alpha = 2.0$	–19	–6	0.44	1.30	1.21	2.09	3.23	1.69	69.9	27.8
	$\alpha = 3.0$	–24	–6	0.32	1.65	1.45	1.46	3.64	2.08	54.8	15.1
	$\alpha = 4.0$	–31	–6	0.25	2.08	1.75	1.03	4.21	2.58	40.4	7.4
Hamming Poisson	$\alpha = 0.5$	–35	–18	0.43	1.61	1.54	1.26	3.33	2.14	61.3	12.6
	$\alpha = 1.0$	–39	–18	0.38	1.73	1.64	1.11	3.50	2.30	56.0	9.2
	$\alpha = 2.0$	none	–18	0.29	2.02	1.87	0.87	3.94	2.65	44.6	4.7
Cauchy	$\alpha = 3.0$	–31	–6	0.42	1.48	1.34	1.71	3.40	1.90	61.6	20.2
	$\alpha = 4.0$	–35	–6	0.33	1.76	1.50	1.36	3.83	2.20	48.8	13.2
	$\alpha = 5.0$	–30	–6	0.28	2.06	1.68	1.13	4.28	2.53	38.3	9.0

Taylor	α = 2.0	–40	–6	0.57	1.30	1.25	1.91	3.06	1.74	75.7	28.3
	α = 2.5	–50	–6	0.51	1.43	1.36	1.60	3.15	1.90	71.3	21.4
	α = 3.0	–60	–6	0.47	1.55	1.47	1.37	3.26	2.06	67.0	16.1
	α = 3.5	–70	–6	0.44	1.66	1.58	1.20	3.40	2.21	62.9	12.1
	α = 4.0	–80	–6	0.41	1.76	1.67	1.06	3.52	2.35	59.1	9.1
Gaussian	α = 2.5	–42	–6	0.51	1.39	1.33	1.69	3.14	1.86	67.7	20.0
	α = 3.0	–55	–6	0.43	1.64	1.55	1.25	3.40	2.18	57.5	10.6
	α = 3.5	–69	–6	0.37	1.90	1.79	0.94	3.73	2.52	47.2	4.9
Dolph-Cheby-shev	α = 2.5	–50	0	0.53	1.39	1.33	1.70	3.12	1.85	69.6	22.3
	α = 3.0	–60	0	0.48	1.51	1.44	1.44	3.23	2.01	64.7	16.3
	α = 3.5	–70	0	0.45	1.62	1.55	1.25	3.35	2.17	60.2	11.9
	α = 4.0	–80	0	0.42	1.73	1.65	1.10	3.48	2.31	55.9	8.7
Kaiser-Bessel	α = 2.0	–46	–6	0.49	1.50	1.43	1.46	3.20	1.99	65.7	16.9
	α = 2.5	–57	–6	0.44	1.65	1.57	1.20	3.38	2.20	59.5	11.2
	α = 3.0	–69	–6	0.40	1.80	1.71	1.02	3.56	2.39	53.9	7.4
	α = 3.5	–82	–6	0.37	1.93	1.83	0.89	3.74	2.57	48.8	4.8
Barcilon Temes	α = 3.0	–53	–6	0.47	1.56	1.49	1.34	3.27	2.07	63.0	14.2
	α = 3.5	–58	–6	0.43	1.67	1.59	1.18	3.40	2.23	58.6	10.4
	α = 4.0	–68	–6	0.41	1.77	1.69	1.05	3.52	2.36	54.4	7.6
Exact Blackman		–68	–6	0.46	1.57	1.52	1.33	3.29	2.13	62.7	14.0
Blackman		–58	–18	0.42	1.73	1.68	1.10	3.47	2.35	56.7	9.0
Minimum 3-sample Blackman-Harris		–71	–6	0.42	1.71	1.66	1.13	3.45	1.81	57.2	9.6
Minimum 4-sample Blackman-Harris		–92	–6	0.36	2.00	1.90	0.83	3.85	2.72	46.0	3.8
62-dB 3-sample Blackman-Harris		–62	–6	0.45	1.61	1.56	1.27	3.34	2.19	61.0	12.6
74-dB 4-sample Blackman-Harris		–74	–6	0.40	1.79	1.74	1.03	3.56	2.44	53.9	7.4
4-sample Kaiser–Bessel	α = 3.0	–69	–6	0.40	1.80	1.74	1.02	3.56	2.44	53.9	7.4

7.2.2 Coherent Gain

$$CG = \text{coherent gain} = \frac{1}{N}\sum_{n=0}^{N-1} w(nT) = \text{zero frequency gain (dc gain) of the window}$$

7.2.3 Processing Gain

$$PG = \frac{1}{ENBW} = \frac{\text{output signal-to-noise ratio}}{\text{input signal-to-noise ratio}}$$

7.2.4 Scalloping Loss

$$\text{scalloping loss} = \frac{\left|\sum_n w(nT)\exp\left(-j\frac{\pi}{N}n\right)\right|}{\sum_n w(nT)} =$$

$$= \frac{\text{coherent gain for a tone located half a bin from DFT sample point}}{\text{coherent gain for a tone located at a DFT sample point}}$$

$= \;$ maximum reduction in PG due to signal frequency

7.2.5 Mainlobe Spectral Response

mainlobe spectral response = spectral interval between the peak gain and the –3.0 dB and –6.0 dB response level

7.2.6 Overlap Correlation

Correlation coefficients represent the degree of correlation of filter output points that are separated by 25% and 50% of the filter length. These terms are useful in quantifying the estimation uncertainty (or variance reduction) related to incoherent averaging of filter (window) data.

7.3 Window (Filter) Descriptions

7.3.1 Introduction

- $T = 1$
- $-\pi \le \omega \le \pi$ or $0 \le \omega \le 2\pi$
- *DFT bin* $= 2\pi/N$
- Windows are even (about the origin) sequences with an odd number of points.
- The right-most point of the window will be discarded.
- N will be taken to be even, and the total points will be *odd*, and hence

$$N = 2 \times (\text{total points}) = \text{even}$$

7.3.2 Rectangle (Dirichlet) Window

$$w(n) = 1.0 \qquad\qquad n = -\tfrac{N}{2},\cdots,-1,0,1,\cdots,\tfrac{N}{2}$$

$$W(\omega) = \sum_{n=-N/2}^{N/2} w(n) e^{-jn\omega}$$

To make it realizable shift the sequence by $N/2$ to the right. Hence we obtain

$$w(n) = 1 \qquad n = 0, 1, \cdots, N-1$$

$$W(\omega) = \sum_{n=0}^{N-1} e^{-jn\omega} = e^{-j\frac{N-1}{2}\omega} \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\frac{\omega}{2}}$$

Figure 7.1 shows the rectangular window and its amplitude spectrum $|W(\omega)|$.

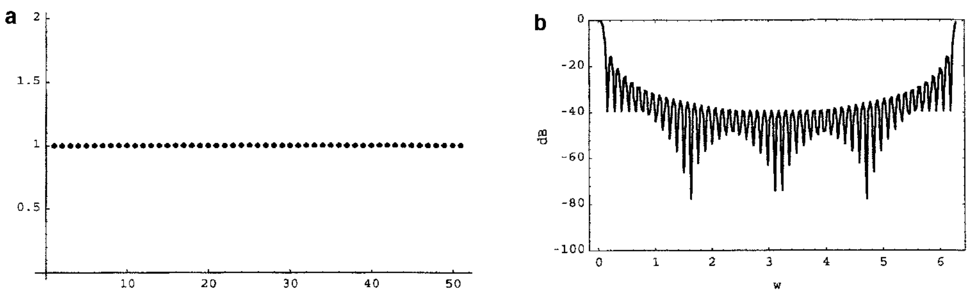

FIGURE 7.1 a) Rectangular window. b) Amplitude spectrum of rectangular window.

7.3.3 Triangle (Fejer, Bartlet) Window

$$w(n) = 1.0 - \frac{|n|}{N/2} \qquad n = -\tfrac{N}{2}, \cdots, -1, 0, 1, \cdots, \tfrac{N}{2}$$

For DFT the window is

$$w(n) = \begin{cases} \dfrac{n}{N/2} & n = 0, 1, \cdots, \tfrac{N}{2} \\ \dfrac{N-n}{N/2} & n = \tfrac{N}{2} + 1, \cdots, N-1 \end{cases}$$

and its DFT is

$$W(\omega) = e^{-j(\frac{N}{2}-1)\omega} \left[\frac{\sin\left(\frac{N}{4}\omega\right)}{\sin\frac{\omega}{2}} \right]^2$$

since the symmetric function $w(n)$ is shifted by $\frac{N}{2} - 1$ positions to produce the DFT sequence. Figure 7.2 shows the triangular window and its DFT.

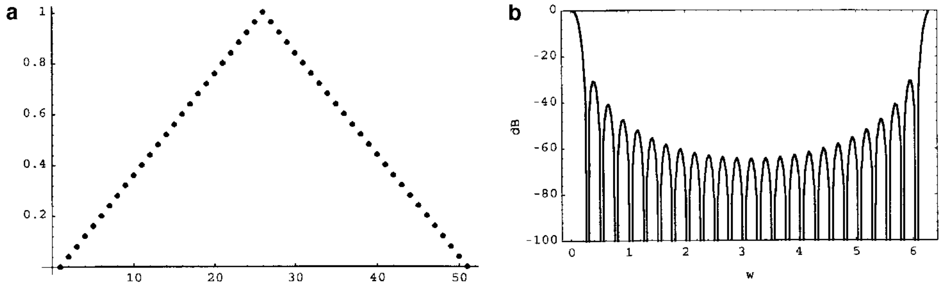

FIGURE 7.2 a) Triangular window. b) Amplitude spectrum of triangular window.

7.3.4 cos$^\alpha$ (t) Windows

$$w(n) = \cos^{\alpha}\left[\left(\frac{n}{N}\right)\pi\right] \qquad n = -\tfrac{N}{2},\cdots,-1,0,1,\cdots,\tfrac{N}{2}$$

$$w(n) = \sin^{\alpha}\left[\left(\frac{n}{N}\right)\pi\right] \qquad n = 0,1,\cdots,N-1$$

Common values of $\alpha : 1 \le \alpha \le 4$
Figures 7.3 through 7.5 show the $\cos^2\left(\frac{n\pi}{N}\right)$, $\cos^3\left(\frac{n\pi}{N}\right)$, and $\cos^4\left(\frac{n\pi}{N}\right)$ windows and their Fourier transform.

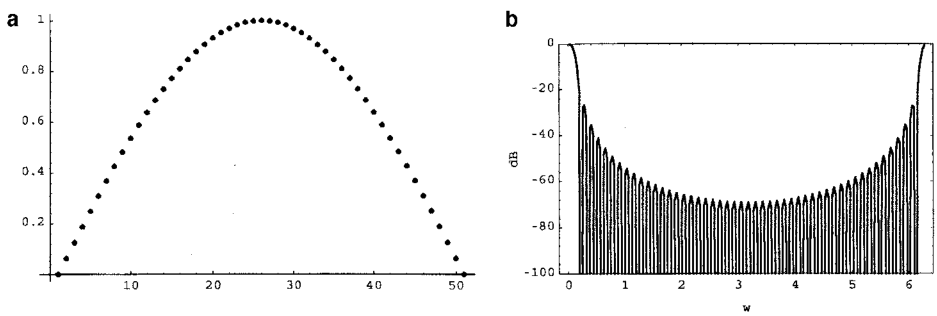

FIGURE 7.3 a) Cosine window with α = 1. b) Amplitude spectrum of cosine window with α = 1.

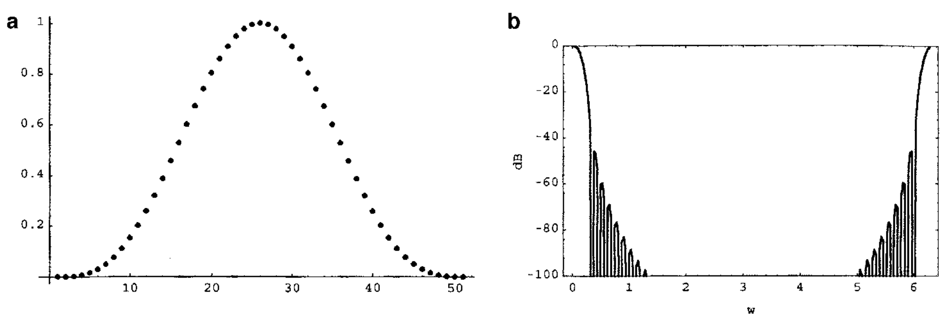

FIGURE 7.4 a) Cosine window with α = 3. b) Amplitude spectrum of cosine window with α = 3.

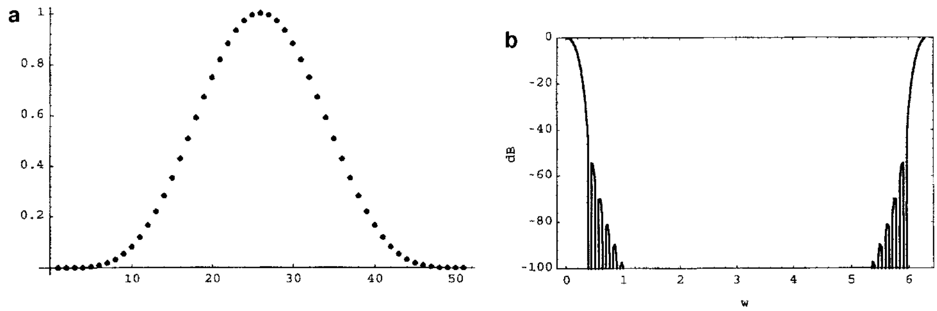

FIGURE 7.5 a) Cosine window with $\alpha = 4$. b) Amplitude spectrum of cosine window with $\alpha = 4$.

7.3.5 Hann Window

$$w(n) = \cos^2\left(\frac{n}{N}\pi\right) = \frac{1}{2}\left[1 + \cos\left(\frac{2n}{N}\pi\right)\right] \qquad n = -\tfrac{N}{2}, \cdots, -1, 0, 1, \cdots, \tfrac{N}{2}$$

$$w(n) = \sin^2\left[\left(\frac{n}{N}\right)\pi\right] = \frac{1}{2}\left[1 - \cos\left[\left(\frac{2n}{N}\right)\pi\right]\right] \qquad n = 0, 1, \cdots, N-1$$

DFT of the window is

$$W(\omega) = \frac{1}{2}D(\omega) + \frac{1}{4}\left[D\left(\omega - \frac{2\pi}{N}\right) + D\left(\omega + \frac{2\pi}{N}\right)\right]$$

$$D(\omega) = e^{j\frac{\omega}{2}} \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\frac{\omega}{2}} = \text{Dirichlet Kernel} \qquad -\pi \le \omega \le \pi$$

See Figure 7.6 for the Hann window.

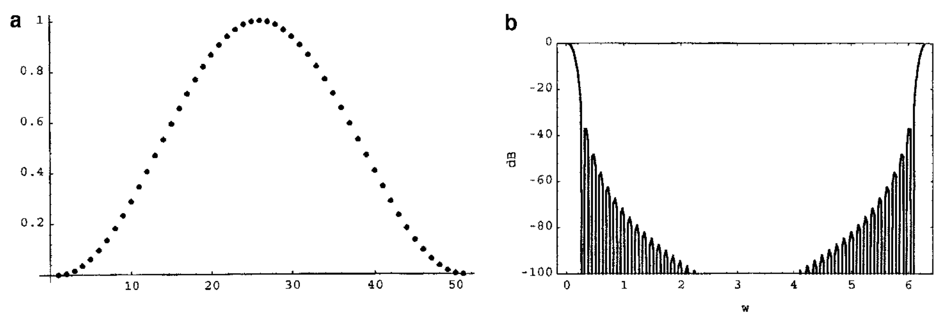

FIGURE 7.6 a) Hann window. b) Amplitude spectrum of Hann window.

7.3.6 Hamming Window

$$w(n) = \alpha + (1-\alpha)\cos\frac{2\pi}{N}n \qquad n = -\tfrac{N}{2}, \cdots, -1, 0, 1, \cdots, \tfrac{N}{2}$$

$$W(\omega) = \alpha D(\omega) + \frac{1}{2}(1-\alpha)\left[D\left(\omega - \frac{2\pi}{N}\right) + D\left(\omega + \frac{2\pi}{N}\right)\right] \qquad -\pi \le \omega \le \pi$$

D = Dirichlet Kernel (see 7.3.5)

$$w(n) = 0.54 + 0.46\cos\frac{2\pi}{N}n \qquad n = -\tfrac{N}{2},\cdots,-1,0,1,\cdots,\tfrac{N}{2}$$

$$w(n) = 0.54 - 0.46\cos\frac{2\pi}{N}n \qquad n = 0,1,\cdots,N-1$$

Figure 7.7 depicts the Hamming window and its amplitude spectrum.

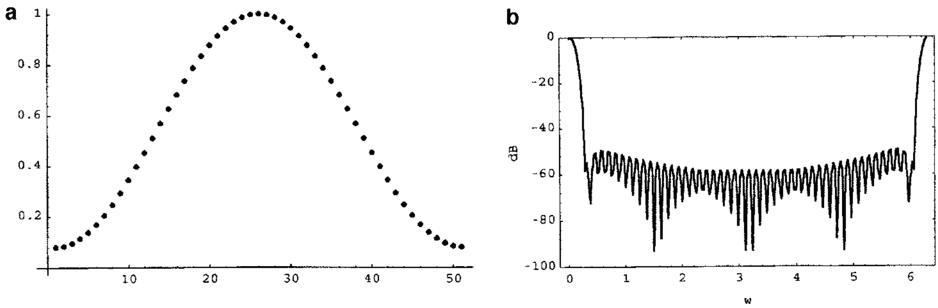

FIGURE 7.7 a) Hamming window. b) Amplitude spectrum of Hamming window.

7.3.7 Short Cosine Series Window

$$w(n) = \sum_{k=0}^{K/2} a(k)\cos\left[\left(\frac{2\pi}{N}\right)kn\right] \qquad n = -\tfrac{N}{2},\cdots,-1,0,1,\cdots,\tfrac{N}{2}$$

$$\sum_{k=0}^{K/2} a(k) = 1 \qquad \text{constraint}$$

$$W(\omega) = a(0)D(\omega) + \sum_{k=0}^{K/2}\frac{a(k)}{2}\left[D\left(\omega - k\frac{2\pi}{N}\right) + D\left(\omega + k\frac{2\pi}{N}\right)\right] \qquad -\pi \le \omega \le \pi$$

7.3.8 Blackman Window

$$a(0) = 0.42659071 \cong 0.42, \quad a(1) = 0.49656062 \cong 0.50, \quad a(2) = 0.07684867 \cong 0.08$$

$$w(n) = 0.42 + 0.5\cos\left(\frac{2\pi}{N}n\right) + 0.08\cos\left(\frac{2\pi}{N}2n\right) \qquad n = -\tfrac{N}{2},\cdots,-1,0,1,\cdots,\tfrac{N}{2}$$

$$w(n) = 0.42 + 0.5\cos\left(\frac{2\pi}{N}(n-25)\right) + 0.08\cos\left(\frac{2\pi}{N}2(n-25)\right) \qquad n = 0,1,\cdots,N-1$$

Figure 7.8 shows the characteristics of the Blackman window.

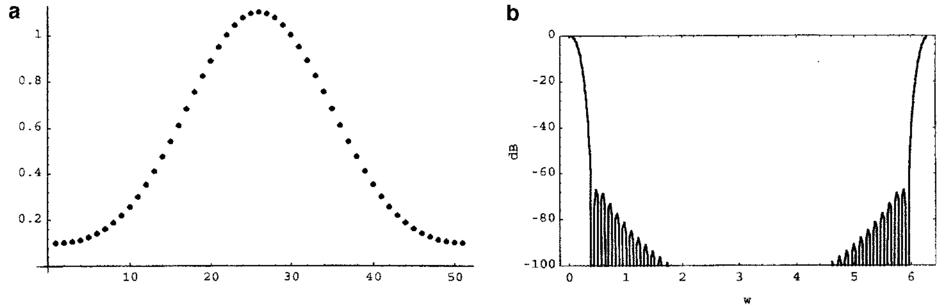

FIGURE 7.8 a) Blackman window. b) Amplitude spectrum of Blackman window.

7.3.9 Harris-Nutall Window

Table 7.2 gives the coefficients for short cosine series windows.

TABLE 7.2 Coefficients of Three- and Four-Term Harris-Nutall Windows

	3-Term (–61 dB)	3-Term (–67 dB)	4-Term (–74 dB)	4-Term (–94 dB)
a(0)	0.44959	0.42323	0.40217	0.35875
a(1)	0.49364	0.49755	0.49703	0.48829
a(2)	0.05677	0.07922	0.09392	0.14128
a(3)	0	0	0.00183	0.01168

Figures 7.9 and 7.10 show the Harris-Nutall window characteristics.

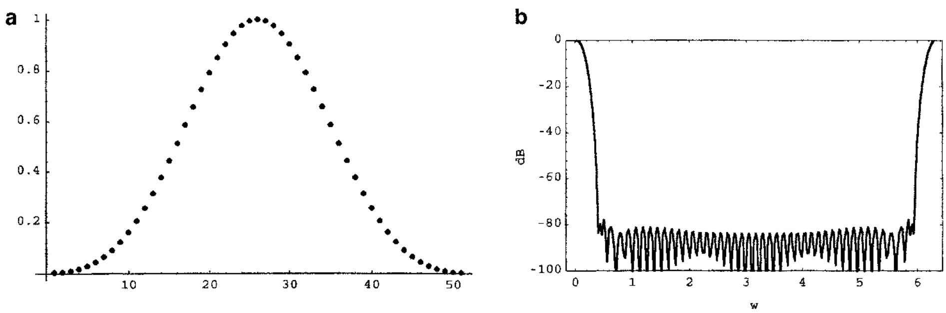

FIGURE 7.9 a) Harris-Nutall window (3-term). b) Amplitude spectrum of Harris-Nutall window (3-term).

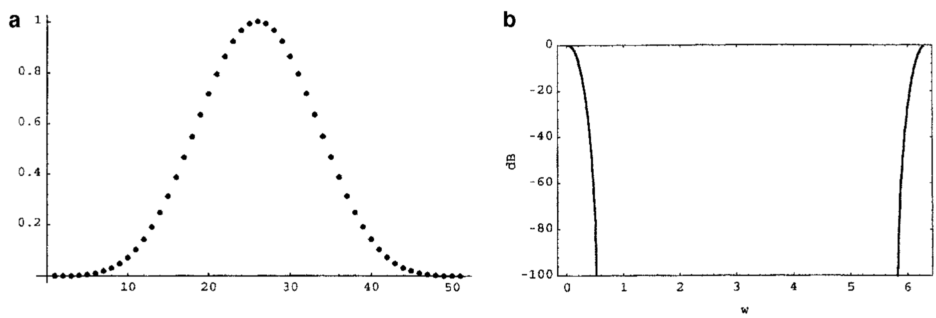

FIGURE 7.10 a) Harris-Nutall window (4-term). b) Amplitude spectrum of Harris-Nutall window (4-term).

7.3.10 Sampled Kaiser-Bessel Window

$$\text{Kaiser-Bessel spectrum window } = W(\omega) = \frac{\sinh\sqrt{\pi^2\alpha^2-(\omega N/2)^2}}{\sqrt{\pi^2\alpha^2-(\omega N/2)^2}} \qquad 0 \le \alpha \le 4$$

$$H_1(m) = \frac{\sinh(\pi\sqrt{\alpha^2-m^2})}{\pi\sqrt{\alpha^2-m^2}} \qquad \omega = m(2\pi/N)$$

$$c = H_1(0)+2H_1(1)+2H_1(2)+[2H_1(3)]$$

$$a(0) = \frac{H_1(0)}{c}, \qquad a(m) = \frac{2H_1(0)}{c}, \qquad m = 1,2,3$$

$$a(0) = 0.40243, \qquad a(1) = 0.49804, \qquad a(2) = 0.09831, \qquad a(3) = 0.00122$$

7.3.11 Parabolic (Riesz, Bochner, Parzen) Window

$$w(n) = 1.0 - \left(\frac{n}{N/2}\right)^2 \qquad 0 \le |n| \le \tfrac{N}{2}$$

$$w(n) = 1.0 - \left(\frac{n-\frac{N}{2}}{N/2}\right)^2 \qquad n = 0,1,2,\cdots,N-1$$

Figure 7.11 shows the parabolic window and its spectrum characteristics.

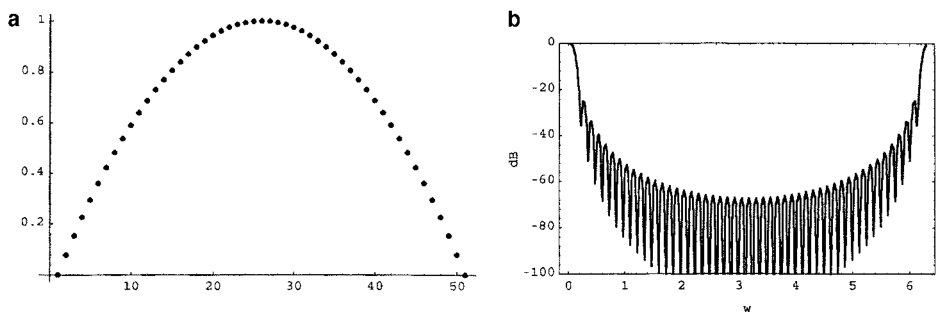

FIGURE 7.11 a) Parabolic window. b) Amplitude spectrum of Parabolic window.

7.3.12 Riemann Window

$$w(n) = \frac{\sin\frac{2\pi n}{N}}{\frac{2\pi n}{N}} \qquad 0 \le |n| \le \tfrac{N}{2}$$

$$w(n) = \frac{\sin\left(\frac{2\pi\left(n - \frac{N}{2}\right)}{N}\right)}{\frac{2\pi\left(n - \frac{N}{2}\right)}{N}} \qquad n = 0,1,2,\cdots,N-1$$

Figure 7.12 shows the window's characteristics.

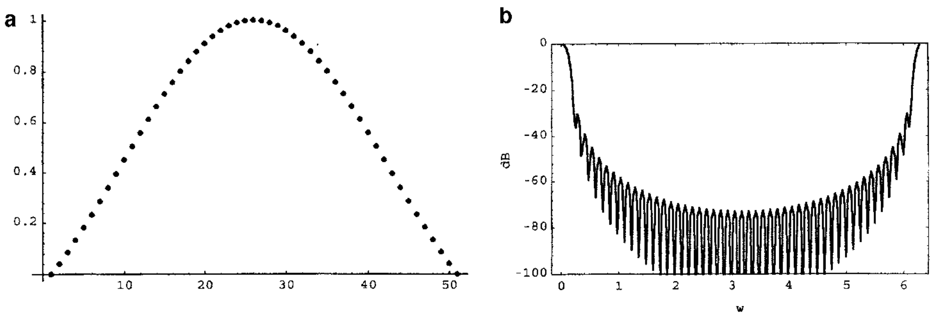

FIGURE 7.12 a) Riemann window. b) Amplitude spectrum of Riemann window.

7.3.13 de la Vallé-Poussin (Jackson, Parzen) Window

$$w(n) = \begin{cases} 1 - 6\left[\frac{n}{N/2}\right]^2\left[1 - \frac{|n|}{N/2}\right] & 0 \le |n| \le \frac{N}{4} \\ 2\left[1 - \frac{|n|}{N/2}\right]^3 & \frac{N}{4} \le |n| \le \frac{N}{2} \end{cases}$$

Figure 7.13 shows the window and its frequency response.

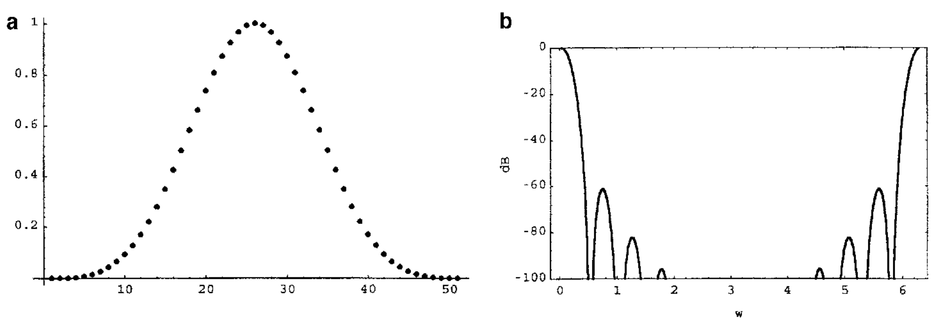

FIGURE 7.13 a) de la Vallé-Poussin window. b) Amplitude spectrum of de la Vallé-Poussin window.

7.3.14 Cosine Taper (Tukey) Window

The Tukey window is equal to one over $(1 - \alpha/2)N$ points, with a cosine taper from one to zero for the remaining points $(\alpha/2)N$.

$$w(n) = \begin{cases} 1 & 0 \le |n| \le \alpha \frac{N}{2} \\ 0.5\left(1 + \cos\left[\pi \dfrac{n - \alpha(N/2)}{(1-\alpha)(N/2)}\right]\right) & \alpha \frac{N}{2} < |n| \le \frac{N}{2} \end{cases}$$

Figures 7.14 and 7.15 show the window and its frequency responses for $\alpha = 8/25$ and $\alpha = 12/25$.

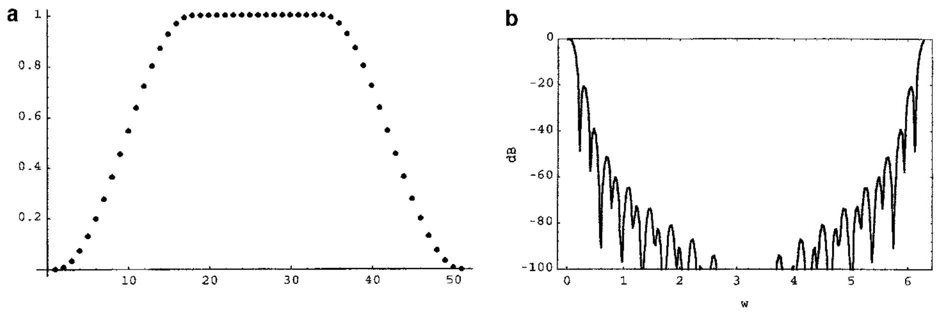

FIGURE 7.14 a) Tukey window with $\alpha = 8/25$. b) Amplitude spectrum of Tukey window with $\alpha = 8/25$.

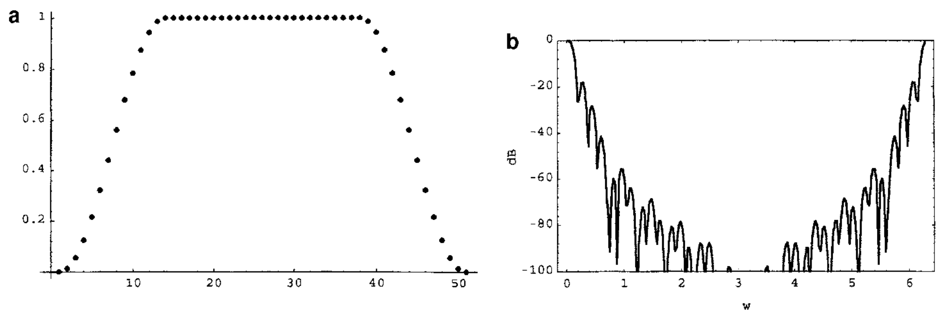

FIGURE 7.15 a) Tukey window with $\alpha = 12/25$. b) Amplitude spectrum of Tukey window with $\alpha = 12/25$.

7.3.15 Bohman Window

$$w(n) = \left(1 - \frac{|n|}{N/2}\right)\cos\left(\pi \frac{|n|}{N/2}\right) + \frac{1}{\pi}\sin\left(\pi \frac{|n|}{N/2}\right), \qquad 0 \le |n| \le \frac{N}{2}$$

Figure 7.16 shows the window and its spectrum.

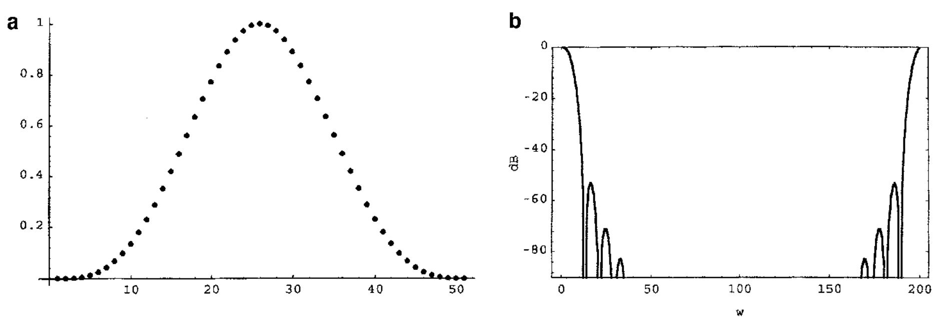

FIGURE 7.16 a) Bohman window. b) Amplitude spectrum of Bohman window.

7.3.16 Poisson Window

$$w(n) = \exp\left(-\alpha \frac{|n|}{N/2}\right), \qquad 0 \le |n| \le \tfrac{N}{2}$$

Figures 7.17 through 7.19 show the window and its spectrum with α = 1.5, α = 3, and α = 4.

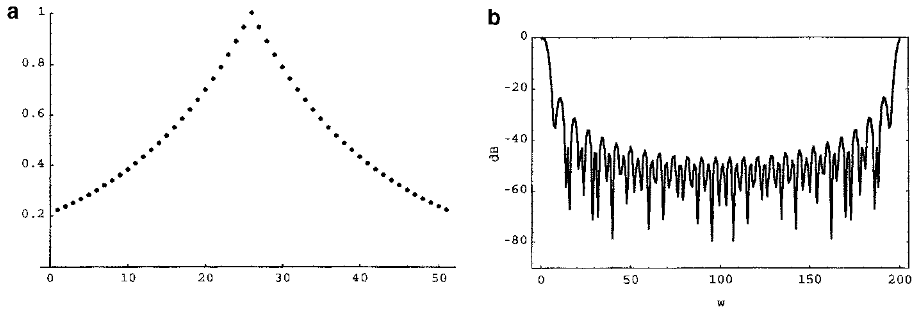

FIGURE 7.17 a) Poisson window with α = 1.5. b) Amplitude spectrum of Poisson window with α = 1.5.

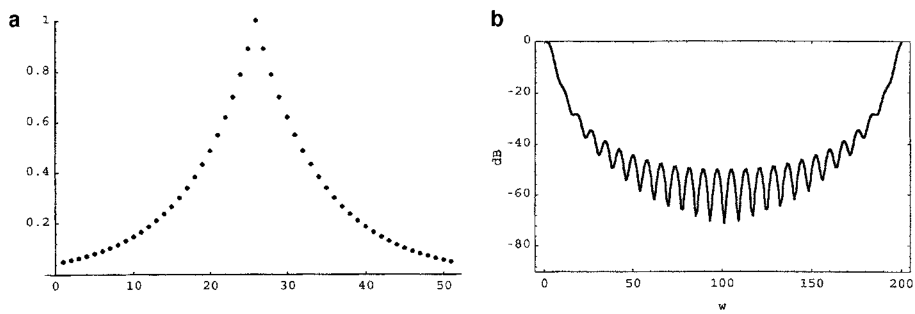

FIGURE 7.18 a) Poisson window with α = 3.0. b) Amplitude spectrum of Poisson window with α = 3.0.

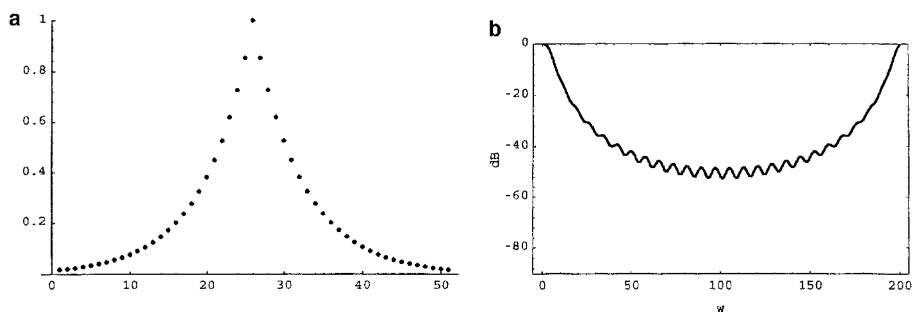

FIGURE 7.19 a) Poisson window with α = 4.0. b) Amplitude spectrum of Poisson window with α = 4.0.

7.3.17 Hann-Poisson Window

$$w(n) = 0.5\left[1 + \cos\left(\pi \frac{n}{N/2}\right)\right]\exp\left(-\alpha \frac{|n|}{N/2}\right), \qquad 0 \le |n| \le \tfrac{N}{2}$$

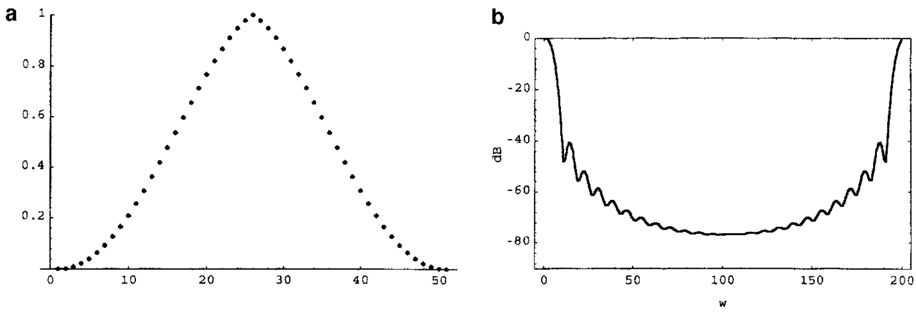

FIGURE 7.20 a) Hann-Poisson window with α = 0.5. b) Amplitude spectrum of Hann-Poisson window with α = 0.5.

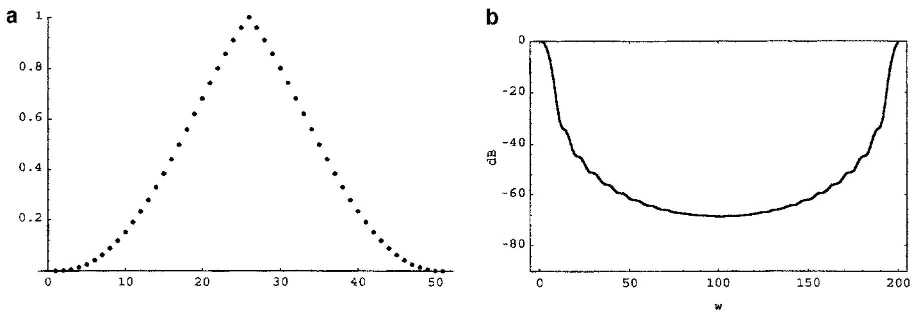

FIGURE 7.21 a) Hann-Poisson window with α = 1.0. b) Amplitude spectrum of Hann-Poisson window with α = 1.0.

Figures 7.20 and 7.21 show the window and its spectrum with α = 0.5 and α = 1.0, respectively.

7.3.18 Cauchy (Abel, Poisson) Window

$$w(n) = \frac{1}{1+\left(\alpha \frac{n}{N/2}\right)^2}, \qquad 0 \le |n| \le \tfrac{N}{2}$$

Figures 7.22 through 7.24 show the window and its spectrum with α = 3.0, α = 4.0, and α = 6.0, respectively.

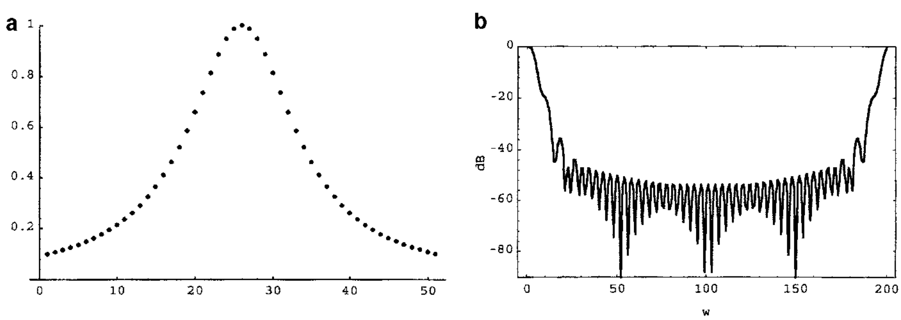

FIGURE 7.22 a) Cauchy window with α = 3.0. b) Amplitude spectrum of Cauchy window with α = 3.0.

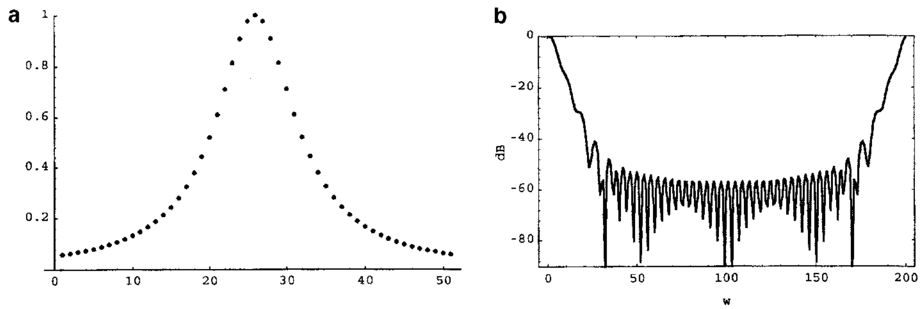

FIGURE 7.23 a) Cauchy window with α = 4.0. b) Amplitude spectrum of Cauchy window with α = 4.0.

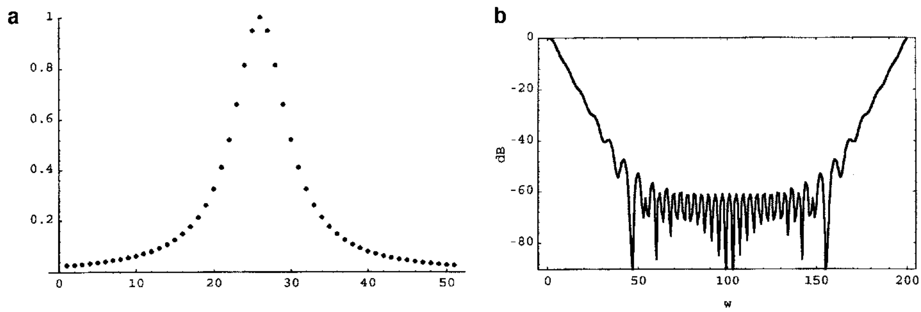

FIGURE 7.24 a) Cauchy window with α = 6.0. b) Amplitude spectrum of Cauchy window with α = 6.0.

7.3.19 Gaussian (Weierstrass) Window

$$w(n) = \exp\left[-\tfrac{1}{2}\left(\alpha \frac{n}{N/2}\right)^2\right], \qquad 0 \le |n| \le \tfrac{N}{2}$$

Figures 7.25 and 7.26 show the window and its spectrum for α = 2.5 and α = 3.5.

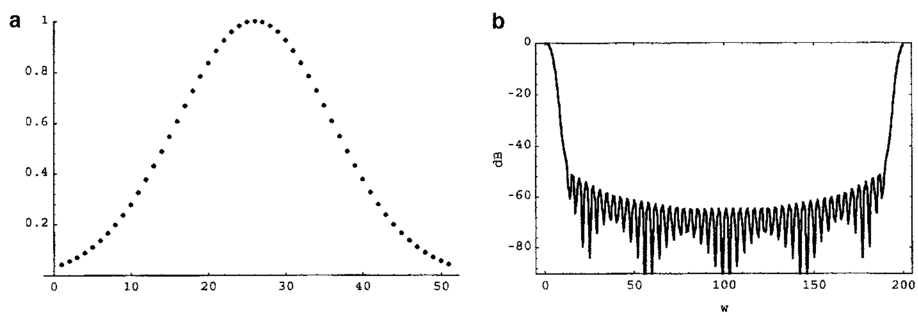

FIGURE 7.25 a) Gaussian window with α = 2.5. b) Amplitude spectrum of Gaussian window with α = 2.5.

7.3.20 Dolph-Chebyshev Window

$$W(k) = \frac{\cosh(N\cosh^{-1}(\beta\cosh(\pi k/N)))}{\cosh(N\cosh^{-1}(\beta))}$$

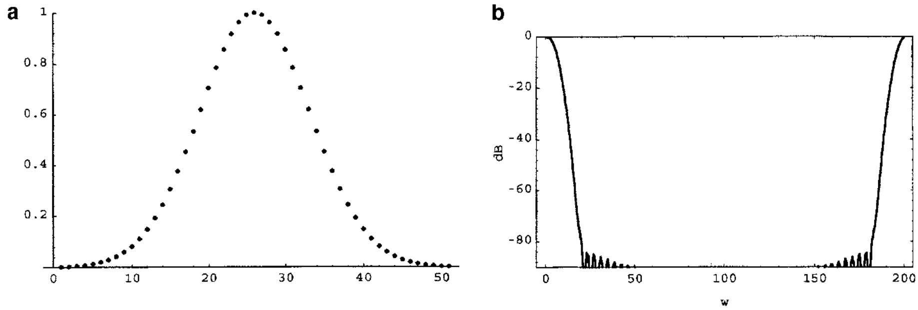

FIGURE 7.26 a) Gaussian window with α = 3.5. b) Amplitude spectrum of Gaussian window with α = 3.5.

where

$$\cosh^{-1}(X) = \ln\left(X + \sqrt{X^2 - 1.0}\right), \qquad |X| > 1.0$$

$$W(k) = \frac{\cos(N\cos^{-1}(\beta\cos(\pi k/N)))}{\cosh(N\cosh^{-1}(\beta))}$$

where

$$\cos^{-1}(X) = \frac{\pi}{2} - \tan^{-1}\left(\frac{X}{\sqrt{X^2 - 1.0}}\right), \qquad |X| \le 1.0$$

where β satisfies

$$\beta = \cosh\left(\frac{1}{N}\cosh^{-1} 10^{\alpha}\right)$$

and

$$w(n) = \sum_{k=0}^{N-1} W(k)\exp\left(j\frac{2\pi}{N}nk\right)$$

7.3.21 Kaiser-Bessel Window

$$w(n) = \frac{I_o\left[\pi\alpha\sqrt{1.0 - \left(\frac{n}{N/2}\right)^2}\right]}{I_o[\pi\alpha]} \qquad 0 \le |n| \le \tfrac{N}{2}$$

$$I_o(x) = \sum_{k=0}^{\infty}\left[\frac{\left(\frac{x}{2}\right)^k}{k!}\right]^2 = \text{zero-order modified Bessel function}$$

Figures 7.27 and 7.28 show the window and its spectrum for α = 2 and α = 3.

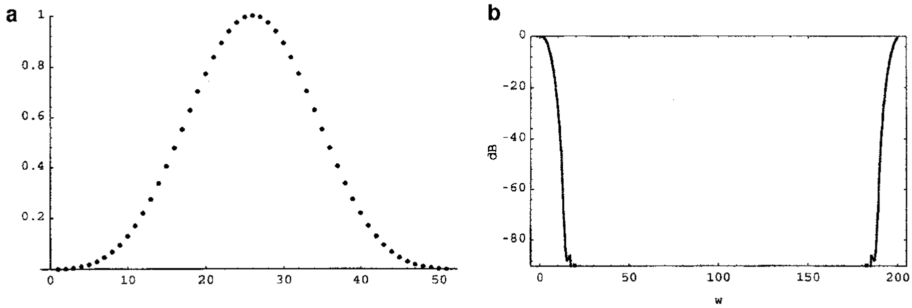

FIGURE 7.27 a) Kaiser-Bessel window with α = 3.0. b) Amplitude spectrum of Kaiser-Bessel window with α = 3.0.

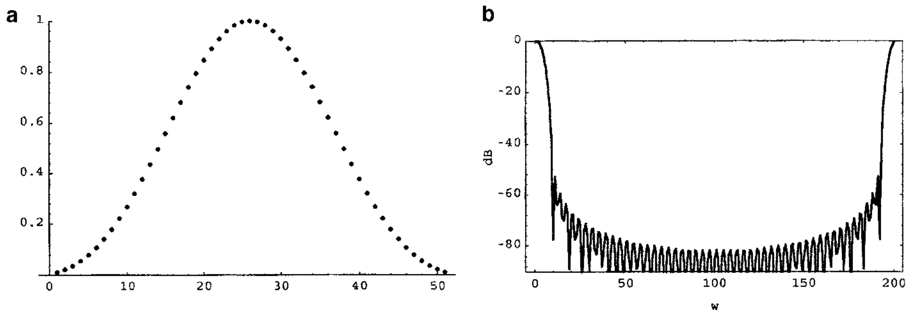

FIGURE 7.28 a) Kaiser-Bessel window with α = 2.0. b) Amplitude spectrum of Kaiser-Bessel window with α = 2.0.

7.3.22 Barcilon-Themes Window

$$W(k) = \frac{A\cos[y(k)] + B[y(k)/C]\sin[y(k)]}{(C + AB)[(y(k)/C)^2 + 1.0]}$$

where

$$A = \sinh C = \sqrt{10^{2\alpha} - 1.0}$$

$$B = \cosh C = 10^{\alpha}$$

$$C = \cosh^{-1}(10^{\alpha})$$

$$\beta = \cosh(C/N)$$

$$y(k) = N\cos^{-1}\left[\beta\cos\left(\frac{\pi k}{N}\right)\right]$$

7.3.23 Highest Sidelobe Level versus Worst-Case Processing Loss

Figure 7.29 shows the highest sidelobe level versus worst-case processing loss. Shaped DFT filters in the lower left tend to perform well.

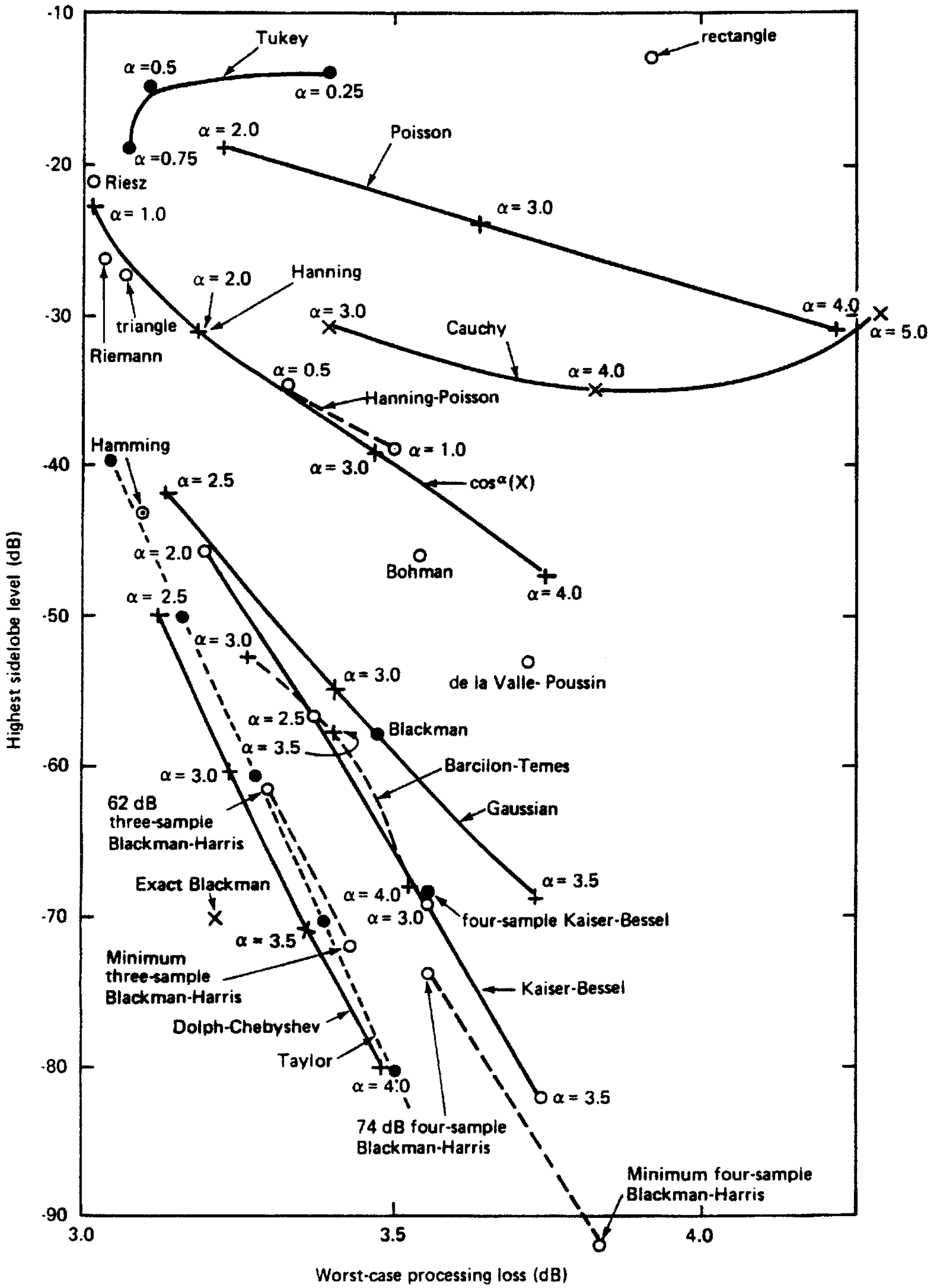

FIGURE 7.29 Highest sidelobe level versus worst-case processing loss. Shaped DFT filters in the lower left tend to perform well.

References

Harris, F. J., On the use of windows for harmonic analysis with the discrete fourier transforms, *Proc. IEEE*, 66, 55-83, January 1978.