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Permittivity Measurement

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Dielectric materials possess relatively few free charge carriers. Most of the charge carriers are bound and cannot participate in conduction. However, these bound charges can be displaced by applying an external electric field. In such cases, the atom or molecule forms an electric dipole that maintains an electric field. Consequently, each volume element of the material behaves as an electric dipole. The dipole field tends to oppose the applied field. Dielectric materials that exhibit nonzero distribution of such bound charge separations are said to be *polarized*. The volume density of polarization $\vec{P}$ describes the volume density of those electric dipoles. When a material is linear and isotropic in nature, the polarization density is related to applied electric field intensity, $\vec{E}$, as follows:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \tag{46.1}$$

where ε_0 (= 8.854×10^{-12} F m^{-1}) is the permittivity of free-space and χ_e is called the electric susceptibility of the material.

The electric flux density, or displacement, $\vec{D}$ is defined as follows:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \left(1 + \chi_o\right) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E} \tag{46.2}$$

where ε is called the permittivity of the material and ε_r is its relative permittivity or dielectric constant. Electric flux density is expressed in coulombs per meter (C m^{-1}).

Equation 46.2 represents a relation between the electric flux density and electric field intensity in frequency domain. It will hold well in time-domain only if the permittivity is independent of frequency. A material is called *dispersive* if its characteristics are frequency dependent. The product of Equation 46.2 in frequency domain will be replaced by a convolution integral for the time-domain fields.

Assuming that the fields are time-harmonic as $e^{j\omega t}$, the generalized Ampere's law can be expressed in phasor form as follows:

$$\nabla \times \vec{H} = \vec{J}^e + \vec{J} + j\omega \vec{D} \tag{46.3}$$

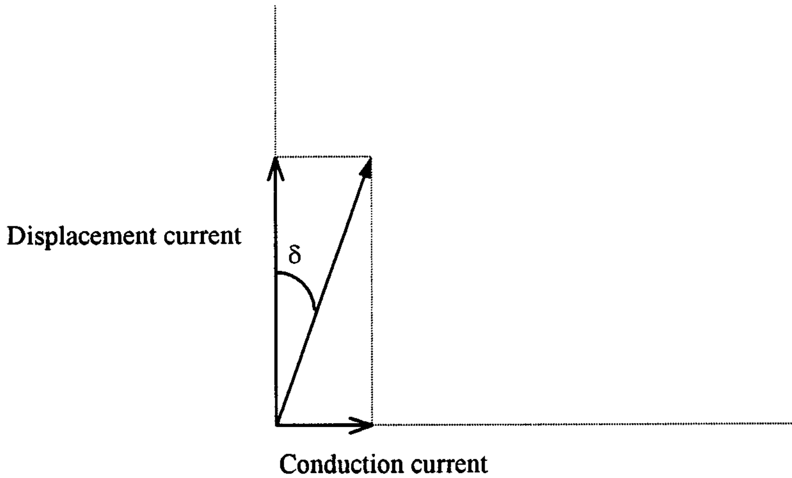

FIGURE 46.1 A phasor diagram representing displacement and loss currents.

where H is the magnetic field intensity in A m^{-1} and J^e is current-source density in A m^{-2}. J is the conduction current density in A m^{-2} and the last term represents the displacement current density. J^e will be zero for a source-free region.

The conduction current density is related to the electric field intensity through Ohm's law as follows:

$$\vec{J} = \sigma \vec{E} \tag{46.4}$$

where σ is the conductivity of material in S m^{-1}.

From Equations 46.2 through 46.4, one obtains:

$$\nabla \times \vec{H} = \vec{J}^e + \sigma \vec{E} + j\omega\varepsilon\vec{E} \tag{46.5}$$

Conduction current represents the loss of power. There is another source of loss in dielectric materials. When a time-harmonic electric field is applied, the dipoles flip back and forth constantly. Because the charge carriers have finite mass, the field must do work to move them and they might not respond instantaneously. This means that the polarization vector will lag behind the applied electric field. This factor shows up at high frequencies. Therefore, Equation 46.5 is modified as follows:

$$\nabla \times \vec{H} = \vec{J}^e + \sigma\vec{E} + \omega\kappa''\vec{E} + j\omega\varepsilon\vec{E} = \vec{J}^e + j\omega\left(\varepsilon - j\frac{\sigma + \omega\kappa''}{\omega}\right)\vec{E} = \vec{J}^e + j\omega\varepsilon^*\vec{E} \tag{46.6}$$

The complex relative permittivity of a material is defined as follows:

$$\varepsilon_r^* = \frac{\varepsilon^*}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\varepsilon - j\frac{\sigma + \omega\kappa''}{\omega}\right) = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r\left(1 - j\tan\delta\right) \tag{46.7}$$

where ε_r' and ε_r'' represent real and imaginary parts of the complex relative permittivity. The imaginary part is zero for a lossless material. The term $\tan\delta$ is called the *loss tangent*. It represents the tangent of angle between the displacement phasor and total current, as shown in Figure 46.1. Thus, it will be close to zero for a low-loss material.

TABLE 46.1 Dielectric Dispersion Parameters for Some Liquids at Room Temperature

Substance	ε_∞	ε_s	α	τ (picoseconds)
Water	5	78	0	8.0789
Methanol	5.7	33.1	0	53.0516
Ethanol	4.2	24	0	127.8545
Acetone	1.9	21.2	0	3.3423
Ethylene glycol	3	37	0.23	79.5775
Propanol	3.2	19	0	291.7841
Butanol	2.95	17.1	0.08	477.4648
Chlorobenzene	2.35	5.63	0.04	10.2920

TABLE 46.2 Complex Permittivity of Some Substances at Room Temperature

Substance	60 Hz	1 MHz	10 GHz
Nylon	3.60–j 0.06	3.14–j 0.07	2.80–j 0.03
Plexiglas	3.45–j 0.22	2.76–j 0.04	2.5–j 0.02
Polyethylene	2.26–j 0.0005	2.26–j 0.0005	2.26–j 0.0011
Polystyrene	2.55–j 0.0077	2.55–j 0.0077	2.54–j 0.0008
Styrofoam	1.03–j 0.0002	1.03–j 0.0002	1.03–j 0.0001
Teflon	2.1–j 0.01	2.1–j 0.01	2.1–j 0.0008
Glass (lead barium)	6.78–j 0.11	6.73–j 0.06	6.64–j 0.31

Dispersion characteristics of a large class of materials can be represented by the following empirical equation of Cole-Cole.

$$\varepsilon_r^* = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \left(j\omega\tau\right)^{1-\alpha}} \tag{46.8}$$

where ε_∞ and ε_s are the relative permittivities of material at infinite and zero frequencies, respectively. ω is the signal frequency in radians per second, and τ is the characteristic relaxation time in seconds. For α equal to zero, Equation 46.8 reduces to the Debye equation. Dispersion parameters for a few liquids are given in Table 46.1.

Complex permittivity of a material is determined using lumped circuits at low frequencies, and distributed circuits or free-space reflection and transmission of waves at high frequencies. Capacitance and dissipation factor of a lumped capacitor are measured using a bridge or a resonant circuit. The complex permittivity is calculated from this data. Complex permittivities for some substances are presented in Table 46.2.

At high frequencies, the sample is placed inside a transmission line or a resonant cavity. Propagation constants of the transmission line or resonant frequency and quality factor of the cavity resonator are used to calculate the complex permittivity. Propagation characteristics of electromagnetic waves are influenced by the complex permittivity of that medium. Therefore, a material can be characterized by monitoring the reflected and transmitted wave characteristics as well.

46.1 Measurement of Complex Permittivity at Low Frequencies [1, 2]

A parallel-plate capacitor is used to determine the complex permittivity of dielectric sheets. For a separation *d* between the plates of area *A* in vacuum, the capacitance is given by:

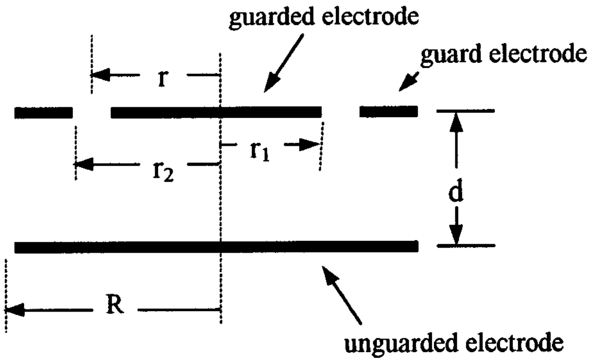

FIGURE 46.2 Geometry of a guarded capacitor.

$$C_0 = 8.854\frac{A}{d}\ \text{pF} \tag{46.9}$$

where all dimensions are measured in meters. If the two plates have different areas, then the smaller one is used to determine C_0. Further, it is assumed that the field distribution is uniform and perpendicular to the plates. Obviously, the fringing fields along the edges do not satisfy this condition. The guard electrodes, as shown in Figure 46.2, are used to ensure that the field distribution is close to the assumed condition. For best results, the width of the guard electrode must be at least $2d$, and the unguarded plate must extend to outer edge of the guard electrode. Further, the gap between the guarded and guard electrodes must be as small as possible.

The radius of guarded electrode is r_1, and the inner radius of guard electrode is r_2. It is assumed that $R - r_2 \geq 2d$. The area A for this parallel plate capacitor is πr^2, where r is defined as follows:

$$r = r_1 + \Delta \tag{46.10}$$

$$\Delta = \frac{1}{2}\left(r_2 - r_1\right) - \frac{2d}{\pi}\ln\left(\cosh\frac{\pi\left(r_2 - r_1\right)}{4d}\right) = \frac{1}{2}\left(r_2 - r_1\right) - 1.4659d\ln\left(\cosh 0.7854\frac{r_2 - r_1}{d}\right) \tag{46.11}$$

Using the Debye model (i.e., $\alpha = 0$ in Equation 46.8), an equivalent circuit for a dielectric-filled parallel plate capacitor can be drawn as shown in Figure 46.3. If a step voltage V is applied to it, then the current I can be found as follows [2].

$$I = \varepsilon_\infty C_0 V\delta\left(t\right) + \frac{VC_0\left(\varepsilon_0 - \varepsilon_\infty\right)}{\tau}\exp\left(-\frac{t}{\tau}\right) \tag{46.12}$$

where $\tau = RC_0(\varepsilon_0 - \varepsilon_\infty)$

The first term in Equation 46.12 represents the charging current of capacitor $\varepsilon_\infty C_0$ in the upper branch. This current is not measured because it disappears instantaneously. In practice, it needs to be bypassed at short times to protect the detector from overloading or burning. The second term of Equation 46.12 represents charging current of the lower branch of an equivalent circuit. The time constant, τ, is determined following the decay characteristics of this current. Further, the resistance R can be found after

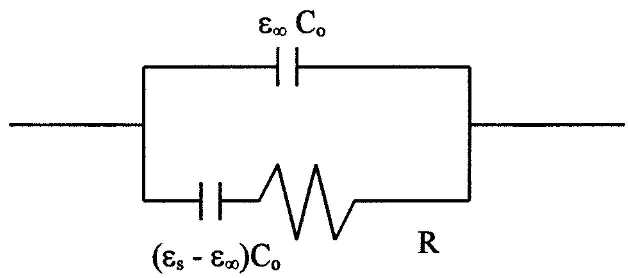

FIGURE 46.3 Equivalent circuit of a parallel-plate capacitor based on the Debye model.

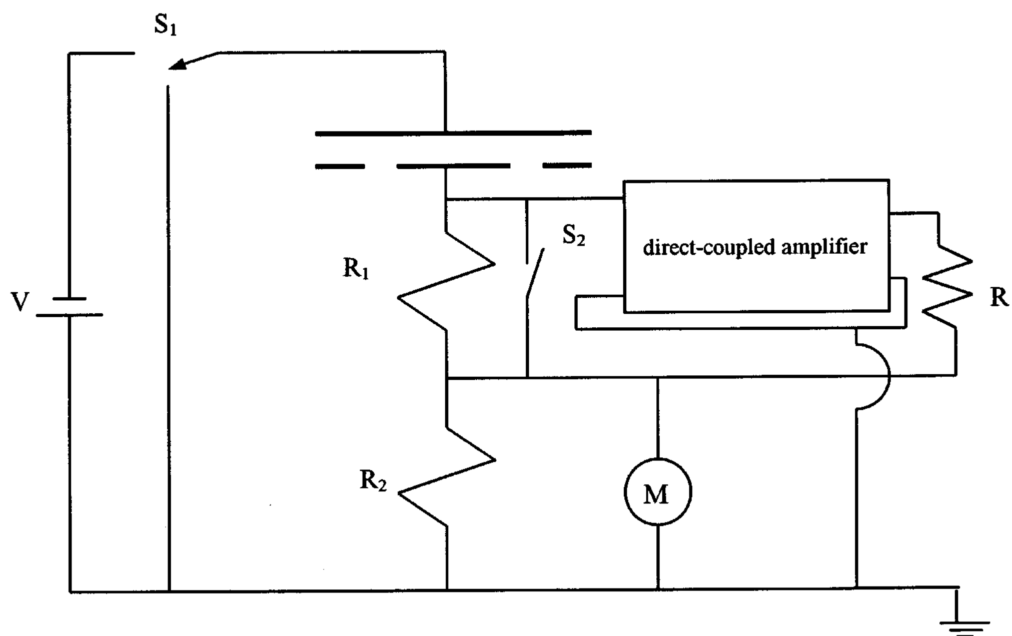

FIGURE 46.4 Circuit arrangement for the characterization of dielectric materials using a step voltage**.**

extrapolating this current-time curve to $t = 0$. The discharging current characteristics are used to remove V at $t = 0$.

A typical circuit arrangement for the characterization of dielectric materials using a step voltage is shown in Figure 46.4. A standard resistor R_1 of either 10 GΩ or 1 TΩ is connected between the guarded electrode and the load resistor R_2. A feedback circuit is used that forces the voltage drop across R_1 equal in magnitude but opposite in polarity to that of across R_2. It works as follows. Suppose that the node between capacitor and R_1 has a voltage V_1 with respect to ground. It is amplified but reversed in polarity by the amplifier. Therefore, the current through R_1 will change. This process continues until the input to the amplifier is zero. The junction between R_1 and the capacitor will then be at the ground potential. Thus, the meter M measures voltage across R_2 that is negative of the voltage across R_1. Since R_1 is known, the current through it can be calculated. This current also flows through the sample. S_1 is used to switch from charging to discharging mode while S_2 is used to provide a path for surge currents.

Capacitance and dissipation factor of the dielectric-loaded parallel-plate capacitor are used in the medium frequency range to determine the complex permittivity of materials. A substitution method is generally employed in a Schering bridge circuit for this measurement.

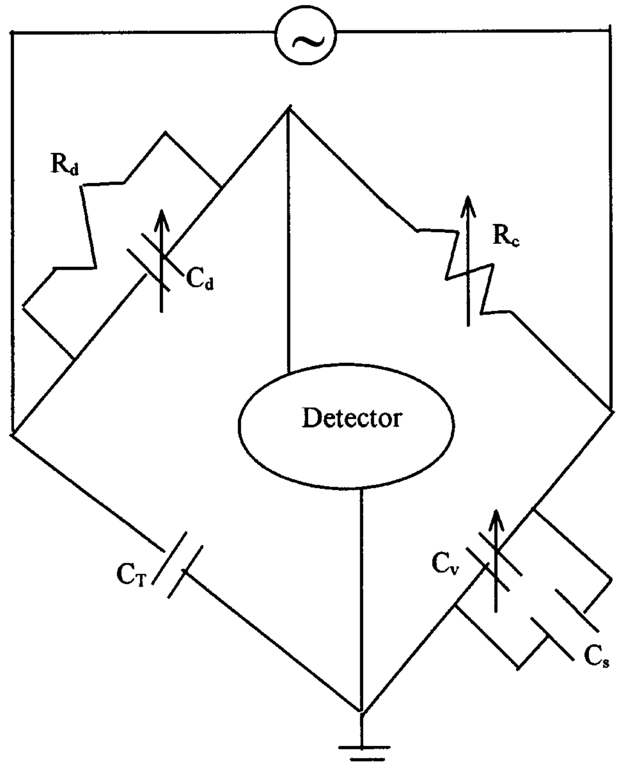

FIGURE 46.5 Schering bridge.

In the Schering bridge shown in Figure 46.5, assume that the capacitor C_v is disconnected for the time being, and the capacitor C_s contains the dielectric sample. In the case of a lossy dielectric sample, it can be modeled as an ideal capacitor C_x in series with a resistor R_x. The bridge is balanced by adjusting C_d and R_c. An analysis of this circuit under the balanced condition produces the following relations.

$$R_x = \frac{C_d R_c}{C_T} \tag{46.13}$$

and

$$C_x = \frac{C_T R_d}{R_c} \tag{46.14}$$

Quality factor Q of a series RC circuit is defined as the tangent of its phase angle, while the inverse of Q is known as the dissipation factor D. Hence,

$$Q = \frac{X_x}{R_x} = \frac{1}{\omega C_x R_x} = \frac{1}{D} \tag{46.15}$$

For a fixed R_d, the capacitor C_d can be calibrated directly in terms of dissipation factor. Similarly, the resistor R_c can be used to determine C_x. However, an adjustable resistor limits the frequency range. A substitution method is preferred for precision measurement of C_x at higher frequencies. In this technique,

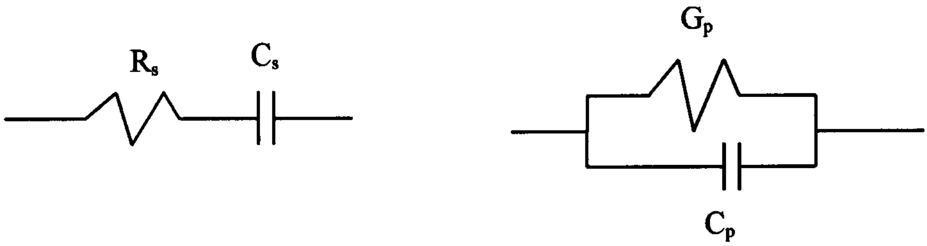

FIGURE 46.6 Series and parallel equivalent circuits of a dielectric loaded capacitor.

a calibrated precision capacitor C_v is connected in parallel with C_s as shown in Figure 46.5 and the bridge is balanced. Assume that the settings of two capacitors at this condition are C_{d1} and C_{v1}. The capacitor C_s is then removed and the bridge is balanced again. Let the new settings of these capacitors be C_{d2} and C_{v2}, respectively. Equivalent circuit parameters of the dielectric loaded capacitor C_s are then found as follows.

$$C_x = C_{v2} - C_{v1} \tag{46.16}$$

$$D_x = \frac{C_{v2}}{C_x}\delta D \tag{46.17}$$

where $\delta D = \omega R_d (C_{d1} - C_{d2})$

Complex permittivity of the specimen is calculated from these data as follows:

$$\varepsilon_r' = \frac{C_x}{C_0} \tag{46.18}$$

and

$$\varepsilon_r'' = \frac{C_x D_x}{C_0} \tag{46.19}$$

Thus far, a series *RC* circuit equivalent model is used for the dielectric-loaded capacitor. As illustrated in Figure 46.6, an equivalent parallel *RC* model can also be obtained for it. The following equations can be used to switch back and forth between these two equivalent models.

$$G_p = \frac{R_s}{R_s^2 + \dfrac{1}{\omega^2 C_s^2}} = \frac{1}{R_s}\left(\frac{1}{1+Q^2}\right) \tag{46.20}$$

$$C_p = \frac{C_s}{1+\left(\omega R_s C_s\right)^2} = \frac{C_s}{1+D^2} \tag{46.21}$$

$$R_s = \frac{G_p}{G_p^2 + \omega^2 C_p^2} = \frac{1}{G_p}\left(\frac{1}{1+Q^2}\right) \tag{46.22}$$

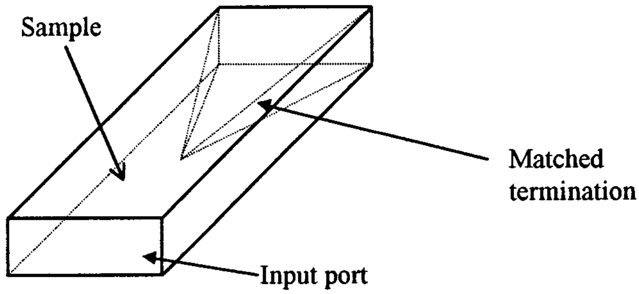

FIGURE 46.7 A waveguide termination filled with liquid or powder sample.

$$C_s = \frac{G_p^2 + \omega^2 C_p^2}{\omega^2 C_p} = C_p\left(1 + D^2\right) \tag{46.23}$$

and

$$Q = \frac{1}{D} = \frac{\omega C_p}{G_p} = \frac{1}{\omega R_s C_s} \tag{46.24}$$

Proper shielding and grounding arrangements are needed for a reliable measurement, especially at higher frequencies. Grounding and edge capacitances of the sample holder need to be taken into account for improved accuracy. Further, a guard point needs to be obtained that may require balancing in some cases. An inductive-ratio-arm capacitance bridge can be another alternative to consider for such application [1].

46.2 Measurement of Complex Permittivity Using Distributed Circuits

Measurement techniques based on the lumped circuits are limited up to the lower end of the VHF band. Characterization of materials at microwave frequencies requires the distributed circuits. A number of techniques have been developed on the basis of wave reflection and transmission characteristics inside a transmission line or in free space. Some other methods employ a resonant cavity that is loaded with the sample. Cavity parameters are measured and the material characteristics are deduced from that. A number of these techniques, described in [3, 4], can be used for a sheet material. These techniques require cutting a piece of sample to be placed inside a transmission line or a cavity. In case of liquid or powder samples, a so-called modified infinite sample method can be used. In this technique, a waveguide termination is filled completely with the sample, as shown in Figure 46.7. Since a tapered termination is embedded in the sample, the wave incident on it will be dissipated with negligible reflection and it will look like the sample is extending to infinity. The impedance at its input port will depend on the electrical properties of filling sample. Its VSWR S and location of first minimum d from the load plane are measured using a slotted line. The complex permittivity of sample is then calculated as follows [5].

$$\varepsilon_r' = \left(\frac{\lambda}{\lambda_c}\right)^2 + \frac{\left[1 - \left(\lambda/\lambda_c\right)^2\right] \times \left[S^2 \sec^4(\beta d) - \left(1 - S^2\right)^2 \tan^2(\beta d)\right]}{\left[1 + S^2 \tan^2(\beta d)\right]^2} \tag{46.25}$$

and

$$\varepsilon_r'' = \frac{\left[1-\left(\lambda/\lambda_c\right)^2\right] \times \left[2S\left(1-S^2\right)^2 \sec^4(\beta d)\tan(\beta d)\right]}{\left[1+S^2\tan^2(\beta d)\right]^2} \tag{46.26}$$

where λ = Free-space wavelength
λ_c = Cut-off wavelength for the mode of propagation in empty guide
β = Propagation constant in the feeding guide

It is assumed that the waveguide supports TE_{10} mode only.

Resonant Cavity Method

A cavity resonator can be used to determine the complex permittivity of materials at microwave frequencies. If a cavity can be filled completely with the sample, then the following procedure can be used.

Measure the resonant frequency f_1 and the quality factor Q_1 of an empty cavity. Next, fill that cavity with the sample material and measure its new resonant frequency f_2 and quality factor Q_2. The dielectric parameters of the sample are then calculated from the following formulae [3].

$$\varepsilon_r = \left(1 + \frac{f_1 - f_2}{f_2}\right)^2 \tag{46.27}$$

and

$$\tan\delta = \frac{1}{Q_2} - \frac{1}{Q_1}\sqrt{\frac{f_1}{f_2}} \tag{46.28}$$

On the other hand, a cavity perturbation technique will be useful for smaller samples [4]. If the sample is available in a circular cylindrical form, then it may be placed inside a TE_{101} rectangular cavity through the center of its broad face where the electric field is maximum. Its resonant frequency and quality factor with and without sample are then measured. Complex permittivity of sample is calculated as follows.

$$\varepsilon_r' = 1 + \frac{1}{2}\frac{f_1 - f_2}{f_2}\frac{V}{v} \tag{46.29}$$

and

$$\varepsilon_r'' = \frac{V}{4v}\frac{Q_2 - Q_1}{Q_1 Q_2} \tag{46.30}$$

where V and v are cavity and sample volumes, respectively.

Similarly, for a small spherical sample of radius r that is placed in a uniform field at the center of the rectangular cavity, the dielectric parameters are as follows.

$$\varepsilon_r' = \frac{abd}{8\pi r^3}\frac{f_1 - f_2}{f_2} \tag{46.31}$$

and

$$\varepsilon_r'' = \frac{abd}{16\pi r^3}\left(\frac{Q_2 - Q_1}{Q_1 Q_2}\right) \quad (46.32)$$

Where *a*, *b*, and *d* are the width, height, and length of the rectangular cavity, respectively. For best accuracy in cavity perturbation method, the shift in frequency ($f_1 - f_2$) must be very small.

Free-Space Method for Measurement of Complex Permittivity

When a plane electromagnetic wave is incident on a dielectric interface, its reflection and transmission depend on the contrast in the dielectric parameters. Many researchers have used it for determining the complex permittivity of dielectric materials placed in free space. An automatic network analyzer and phase-corrected horn antennas can be used for such measurements [6]. The system is calibrated using the TRL (through, reflect, and line) technique. A time-domain gating is used to minimize the error due to multiple reflections. The sample of thickness *d* is placed in front of a conducting plane and its reflection coefficient S_{11} is measured. A theoretical expression for this reflection coefficient is found as follows.

$$S_{11} = \frac{jZ_d \tan(\beta_d d) - 1}{jZ_d \tan(\beta_d d) + 1} \quad (46.33)$$

Where:

$$Z_d = \frac{1}{\sqrt{\varepsilon_r^*}} \quad (46.34)$$

$$\beta_d = \frac{2\pi}{\lambda}\sqrt{\varepsilon_r^*} \quad (46.35)$$

λ = Free-space wavelength of electromagnetic signal

Equation 46.33 is solved for ε_r^* after substituting the measured S_{11}. Since it represents a nonlinear relation, an iterative numerical procedure can be used.

A Nondestructive Method for Measuring the Complex Permittivity of Materials

Most of the techniques described thus far require cutting and placing a part of sample in the test fixture. Sometimes, it may not be permissible to do so. Further, the dielectric parameters can change in that process. It is especially important in the case of a biological specimen to perform *in vivo* measurements. In one such technique, an open-ended coaxial line is placed in close contact with the sample and its input reflection coefficient is measured using an automatic network analyzer [7, 8]. As recommended by the manufacturers, the network analyzer is calibrated initially using an open-circuit, a short-circuit, and a matched load. The reference plane is then moved to the measuring end of the coaxial line using a short-circuit.

Assume that *a* and *b* are inner and outer radii of the coaxial line, respectively. ω is the angular frequency; μ_0 and ε_0 are the permeability and permittivity of the free space, respectively, $k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r^*}$ is the wavenumber in material medium. Admittance of the coaxial aperture in contact with material medium is as follows.

$$Y_L = \frac{2}{\int_a^b E_\rho(\rho',0)\,d\rho'} - \frac{2\pi}{\left[\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_l}}\ln(b/a)\right]} \tag{46.36}$$

where $E_\rho(\rho',0)$ is radial electric field intensity over the aperture. It is evaluated from the following integral equation.

$$\frac{1}{\pi\rho} + j\omega\varepsilon_l\varepsilon_o\int_a^b E_\rho(\rho',0)K_c(\rho,\rho')\rho'\,d\rho' = \frac{j\omega\varepsilon_r^*\varepsilon_0}{\pi}\int_a^b E_\rho(\rho',0)\rho'\,d\rho'\int_0^\pi \cos(\phi')\frac{\exp(-jkr)}{r}\,d\phi' \tag{46.37}$$

Where:

$$r = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi')} \tag{46.38}$$

$$K_c(\rho,\rho') = j\sum_{n=0}^{\infty}\frac{\phi_n(\rho)\phi_n(\rho')}{A_n^2\beta_n} \tag{46.39}$$

$$\phi_n = Y_0(\gamma_n a)J_1(\gamma_n\rho) - J_0(\gamma_n a)Y_1(\gamma_n\rho) \tag{46.40}$$

$$\beta_n = \begin{cases}\sqrt{k_l^2 - \gamma_n^2} & k_l > \gamma_n\\ -j\sqrt{\gamma_n^2 - k_l^2} & k_l < \gamma_n\end{cases} \tag{46.41}$$

$$A_n^2 = \frac{2}{\pi^2\gamma_n^2}\left[\frac{J_0^2(\gamma_n a)}{J_0^2(\gamma_n b)} - 1\right] \qquad n > 0; \qquad A_0^2 = \ln\left(\frac{b}{a}\right) \tag{46.42}$$

The eigenvalues γ_n are solutions to the following characteristic equation:

$$J_0(\gamma_n b)Y_0(\gamma_n a) = J_0(\gamma_n a)Y_0(\gamma_n b) \tag{46.43}$$

J_n and Y_n are Bessel functions of the first and second kind of order n, respectively. ε_l is the dielectric constant of the insulator and k_l is wavenumber inside the coaxial line.

Equation 46.37 is solved numerically using the method of moments. A numerical root-finding procedure, such as the Muller's method, is used to solve Equation 46.36 for the complex wavenumber k. Complex permittivity, in turn, is determined from the following relation.

$$\varepsilon_r^* = \frac{k^2}{\omega^2\mu_0\varepsilon_0} \tag{46.44}$$

Defining Terms

Electric dipole: A pair of equal and opposite electric charges separated by a small distance.

Isotropic material: A material in which the electrical polarization has the same direction as the applied electric field.

Electric polarization density: The average electric dipole moment per unit volume.

Electric susceptibility: A dimensionless parameter that relates the polarization density in a material with electric field intensity.

Electric flux density: A fundamental electric field quantity that is related to volume density of free charges. It is also known as the electric displacement.

Time domain field: A field expressed as a function of time. It is a real function that is dependent on time and space coordinates.

Frequency domain field: A phasor quantity (a complex function in general) that depends on space coordinates. The time dependency is assumed to be sinusoidal.

Displacement current density: It represents the time rate of change of electric flux density.

Conduction current density: Current per unit area caused by conduction of charge carriers.

Dielectric constant: A dimensionless constant that represents the permittivity of a material relative to the permittivity of free space.

Loss tangent: A ratio of the imaginary part to the real part of the complex permittivity of a material.

Relaxation time: It represents the time taken by a charge placed inside a material volume to decay to about 37% of its initial value.

Quality factor: A dimensionless quantity that represents the time average energy stored in an electrical circuit relative to energy dissipated in one period.

Dissipation factor: It is the inverse of the quality factor.

Voltage standing wave ratio (VSWR): Defined as a ratio of maximum voltage to the minimum voltage on a transmission line.

Reflection coefficient: Defined as a ratio of reflected phasor voltage to that of incident phasor voltage at a point in the circuit.

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