A Primer on WAVELETS and their Scientific Applications

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To my wife, and the memory of my mother

Contents

Preface

> "Wavelet theory" is the result of a multidisciplinary effort that brought together mathematicians, physicists and engineers...this connection has created a flow of ideas that goes well beyond the construction of new bases or transforms.
>
> *Stéphane Mallat*[1]

The past decade has witnessed an explosion of activity in wavelet analysis. Thousands of research papers have been published on the theory and applications of wavelets. Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering.

For an idea of the wide range of problems that are being solved using wavelets, here is a list of some of the problems discussed in this book:

- *Audio denoising:* Long distance telephone messages often contain significant amounts of noise. How do we remove this noise in order to clarify the messages?

- *Signal compression:* The efficient transmission of large amounts of data, over the Internet for example, requires some kind of compression. Are there ways we can compress this data as much as possible without losing significant information?

- *Object detection:* What methods can we use to pick out a small image, say of an aircraft, from the midst of a larger more complicated image?

- *Fingerprint compression:* The FBI has 25 million fingerprint records. If these fingerprint records were digitized without any compression,

[1]Mallat's quote is from [MAL].

they would gobble up 250 *trillion* bytes of storage capacity. Is there a way to compress these records to a manageable size, without losing any significant details in the fingerprints?

- *Image denoising:* Images formed by electron microscopes and by optical lasers are often contaminated by large amounts of unwanted clutter (referred to as *noise*). Can this noise be removed in order to clarify the image?

- *Image enhancement:* When an optical microscope image is recorded, it often suffers from blurring. How can the appearance of the objects in these images be sharpened?

- *Image recognition:* How do humans recognize faces? Can we teach machines to do it?

- *Diagnosing heart trouble:* Is there a way to detect abnormal heartbeats, hidden within a complicated electrocardiogram?

- *Speech recognition:* What factors distinguish consonants from vowels? How do humans recognize different voices?

All of these problems can be tackled using wavelets. We will show how during the course of this book.

In Chapter 1 we introduce the simplest wavelets, the Haar wavelets. We also introduce many of the basic concepts—wavelet transforms, energy conservation and compaction, multiresolution analysis, compression and denoising—that will be used in the remainder of the book. For this reason, we devote more pages to the theory of Haar wavelets than perhaps they deserve alone; keep in mind that this material will be amplified and generalized throughout the remainder of the book.

Chapter 2 is the heart of the book. In this chapter we describe the Daubechies wavelets, which have played a key role in the explosion of activity in wavelet analysis. After a simple introduction to their mathematical properties we then describe several applications of these wavelets. First, we explain in detail how they can be used to compress audio signals—this application is vital to the fields of telephony and telecommunications. Second, we describe how a method known as *thresholding* provides a powerful technique for removing random noise (static) from audio signals. Removing random noise is a fundamental necessity when dealing with all kinds of data in science and engineering. The threshold method, which is analogous to how our nervous system responds only to inputs above certain thresholds, provides a nearly optimal method for removing random noise. Besides random noise, Daubechies wavelets can also be used to remove isolated "pop-noise" from audio.

Wavelet analysis can also be applied to images. We shall examine compression of images, including fingerprint compression, and denoising of images. The image denoising examples that we examine include some examples motivated by *magnetic resonance imaging* (MRI) and laser imaging.

Chapter 2 concludes with some examples from image processing. We discuss edge detection, and the sharpening of blurred images, and an example from computer vision where wavelet methods can be used to enormously increase the speed of identification of an image.

Chapter 3 relates wavelet analysis to frequency analysis. Frequency analysis, also known as *Fourier analysis,* has long been one of the cornerstones of the mathematics of science and engineering. We shall briefly describe how wavelets are characterized in terms of their effects on the frequency content of signals. One application that we discuss is object identification—locating a small object within a complicated scene—where wavelet analysis in concert with Fourier analysis provides a powerful approach.

In the final chapter we deal with some extensions which reach beyond the fundamentals of wavelets. We describe a generalization of wavelet transforms known as *wavelet packet transforms.* We apply these wavelet packet transforms to compression of audio signals, images, and fingerprints. Then we turn to the subject of *continuous wavelet transforms*, as they are implemented in a discrete form on a computer. Continuous wavelet transforms are widely used in seismology and have also been used very effectively for analyzing speech and electrocardiograms.

The goal of this primer is to guide the reader through the main ideas of wavelet analysis to facilitate a knowledgeable reading of the present research literature, especially in the applied fields of audio and image processing and biomedicine. Although there are several excellent books on the theory of wavelets, these books are focused on the construction of wavelets and their mathematical properties. Furthermore, they are all written at a graduate school level of mathematical and/or engineering expertise. There is a real need for a simple introduction, a *primer,* which uses only elementary algebra and a smidgen of calculus to explain the underlying ideas behind wavelet analysis, and devotes the majority of its pages to explaining how these underlying ideas can be applied to solve significant problems in audio and image processing and in biology and medicine.

To keep the mathematics simple, we focus on the discrete theory—technically known as *subband coding.* It is in the continuous theory of wavelet analysis where the most difficult mathematics lies; yet when this continuous theory is applied it is almost always converted into the discrete approach that we describe in this primer. Focusing on the discrete case will allow us to concentrate on the applications of wavelet analysis while at the same time keeping the mathematics under control. On the rare occasions when we need to use more advanced mathematics, we shall mark these discussions off from the main text by putting them into subsections that are marked by asterisks in their titles. An effort has been made to ensure that subsequent discussions do not rely on this more advanced material.

Without question the best way, perhaps the only way, to learn about applications of wavelets is to experiment with making such applications. This experimentation is typically done on a computer. In order to simplify this computer experimentation, I have created software, called FAWAV, which can be downloaded over the Internet—see Appendix A. FAWAV runs under WINDOWS™ 95, 98, and NT 4.0, and *requires no programming to use.* Further details about FAWAV can be found in Appendix A; suffice it for now to say that it is designed to allow the reader to duplicate all of the applications described in this primer and to experiment with other ideas.

This primer is only a first introduction to wavelets and their scientific applications. For that reason we limit ourselves to describing what are known technically as periodic orthogonal wavelets. Other types of wavelets—biorthogonal wavelets, spline wavelets, multiwavelets, etc.—are also used in applications, but we feel that these other wavelets can be understood much more easily if the periodic orthogonal ones are studied first. In the Notes and references sections that conclude each chapter, we provide the reader with ample references where further information on these other wavelets and many other topics can be found.

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