

In Chapter 1, the term Radar Cross Section (RCS) was used to describe the amount of scattered power from a target towards the radar, when the target is illuminated by RF energy. At that time, RCS was referred to as a target-specific constant. This was only a simplification and, in practice, it is rarely the case. In this chapter, the phenomenon of target scattering and methods of RCS calculation are examined. Target RCS fluctuations due to aspect angle, frequency, and polarization are presented. Radar cross section characteristics of some simple and complex targets are also introduced. The analysis of extended RCS due to volume and surface clutter will be explored in a later chapter.

2.1. RCS Definition

Electromagnetic waves, with any specified polarization, are normally diffracted or scattered in all directions when incident on a target. These scattered waves are broken down into two parts. The first part is made of waves that have the same polarization as the receiving antenna. The other portion of the scattered waves will have a different polarization to which the receiving antenna does not respond. The two polarizations are orthogonal and are referred to as the Principle Polarization (PP) and Orthogonal Polarization (OP), respectively. The intensity of the *backscattered* energy that has the same polarization as the radar's receiving antenna is used to define the target RCS. When a target is illuminated by RF energy, it acts like an antenna, and will have near and far fields. Waves reflected and measured in the near field are, in general, spherical. Alternatively, in the far field the wavefronts are decomposed into a linear combination of plane waves.

Assume the power density of a wave incident on a target located at range R away from the radar is P_{Di} . The amount of reflected power from the target is

$$P_r = \sigma P_{Di} \quad (2.1)$$

σ denotes the target cross section. Define P_{Dr} as the power density of the scattered waves at the receiving antenna. It follows that

$$P_{Dr} = P_r / (4\pi R^2) \quad (2.2)$$

Equating Eqs. (2.1) and (2.2) yields

$$\sigma = 4\pi R^2 \left(\frac{P_{Dr}}{P_{Di}} \right) \quad (2.3)$$

and in order to ensure that the radar receiving antenna is in the far field (i.e., scattered waves received by the antenna are planar), Eq. (2.3) is modified

$$\sigma = 4\pi R^2 \lim_{R \rightarrow \infty} \left(\frac{P_{Dr}}{P_{Di}} \right) \quad (2.4)$$

The RCS defined by Eq. (2.4) is often referred to as either the monostatic RCS, the backscattered RCS, or simply target RCS.

The backscattered RCS is measured from all waves scattered in the direction of the radar and has the same polarization as the receiving antenna. It represents a portion of the total scattered target RCS σ_t , where $\sigma_t > \sigma$. Assuming spherical coordinate system defined by (ρ, θ, ϕ) , then at range ρ the target scattered cross section is a function of (θ, ϕ) . Let the angles (θ_i, ϕ_i) define the direction of propagation of the incident waves. Also, let the angles (θ_s, ϕ_s) define the direction of propagation of the scattered waves. The special case, when $\theta_s = \theta_i$ and $\phi_s = \phi_i$, defines the monostatic RCS. The RCS measured by the radar at angles $\theta_s \neq \theta_i$ and $\phi_s \neq \phi_i$ is called the bistatic RCS.

The total target scattered RCS is given by

$$\sigma_t = \frac{1}{4\pi} \int_{\phi_s = 0}^{2\pi} \int_{\theta_s = 0}^{\pi} \sigma(\theta_s, \phi_s) \sin\theta_s \, d\theta \, d\phi_s \quad (2.5)$$

The amount of backscattered waves from a target is proportional to the ratio of the target extent (size) to the wavelength, λ , of the incident waves. In fact, a radar will not be able to detect targets much smaller than its operating wavelength. For example, if weather radars use L-band frequency, rain drops become nearly invisible to the radar since they are much smaller than the

wavelength. RCS measurements in the frequency region, where the target extent and the wavelength are comparable, are referred to as the Rayleigh region. Alternatively, the frequency region where the target extent is much larger than the radar operating wavelength is referred to as the optical region. In practice, the majority of radar applications falls within the optical region.

The analysis presented in this book assumes far field monostatic RCS measurements in the optical region. Near field RCS, bistatic RCS, and RCS measurements in the Rayleigh region will not be considered since their treatment falls beyond this book's intended scope. Additionally, RCS treatment in this chapter is mainly concerned with Narrow Band (NB) cases. In other words, the extent of the target under consideration falls within a single range bin of the radar. Wide Band (WB) RCS measurements will be briefly addressed in a later section. Wide band radar range bins are small (typically 10 - 50 cm), hence, the target under consideration may cover many range bins. The RCS value in an individual range bin corresponds to the portion of the target falling within that bin.

2.2. RCS Prediction Methods

Before presenting the different RCS calculation methods, it is important to understand the significance of RCS prediction. Most radar systems use RCS as a means of discrimination. Therefore, accurate prediction of target RCS is critical in order to design and develop robust discrimination algorithms. Additionally, measuring and identifying the scattering centers (sources) for a given target aid in developing RCS reduction techniques. Another reason of lesser importance is that RCS calculations require broad and extensive technical knowledge, thus many scientists and scholars find the subject challenging and intellectually motivating. Two categories of RCS prediction methods are available: exact and approximate.

Exact methods of RCS prediction are very complex even for simple shape objects. This is because they require solving either differential or integral equations that describe the scattered waves from an object under the proper set of boundary conditions. Such boundary conditions are governed by Maxwell's equations. Even when exact solutions are achievable, they are often difficult to interpret and to program using digital computers.

Due to the difficulties associated with the exact RCS prediction, approximate methods become the viable alternative. The majority of the approximate methods are valid in the optical region, and each has its own strengths and limitations. Most approximate methods can predict RCS within few dBs of the truth. In general, such a variation is quite acceptable by radar engineers and designers. Approximate methods are usually the main source for predicting

RCS of complex and extended targets such as aircrafts, ships, and missiles. When experimental results are available, they can be used to validate and verify the approximations.

Some of the most commonly used approximate methods are Geometrical Optics (GO), Physical Optics (PO), Geometrical Theory of Diffraction (GTD), Physical Theory of Diffraction (PTD), and Method of Equivalent Currents (MEC). Interested readers may consult Knott or Ruck (see bibliography) for more details on these and other approximate methods.

2.3. RCS Dependency on Aspect Angle and Frequency

Radar cross section fluctuates as a function of radar aspect angle and frequency. For the purpose of illustration, isotropic point scatterers are considered. An isotropic scatterer is one that scatters incident waves equally in all directions. Consider the geometry shown in Fig. 2.1. In this case, two unity ($1m^2$) isotropic scatterers are aligned and placed along the radar line of sight (zero aspect angle) at a far field range R . The spacing between the two scatterers is 1 meter. The radar aspect angle is then changed from zero to 180 degrees, and the composite RCS of the two scatterers measured by the radar is computed.

This composite RCS consists of the superposition of the two individual radar cross sections. At zero aspect angle, the composite RCS is $2m^2$. Taking scatterer-1 as a phase reference, when the aspect angle is varied, the composite RCS is modified by the phase that corresponds to the electrical spacing between the two scatterers. For example, at aspect angle 10° , the electrical spacing between the two scatterers is

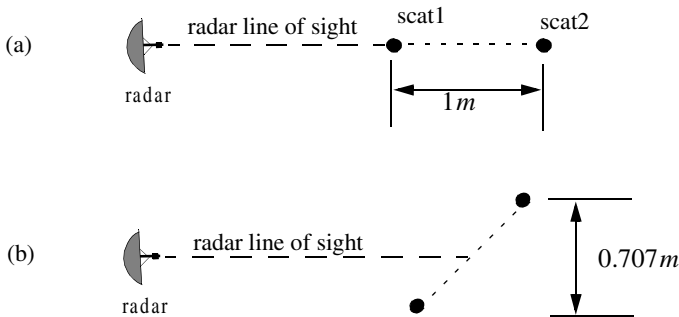


Figure 2.1. RCS dependency on aspect angle. (a) Zero aspect angle, zero electrical spacing. (b) 45° aspect angle, 1.414λ electrical spacing.

$$elec\text{-}spacing = \frac{2 \times (1.0 \times \cos(10))}{\lambda} \quad (2.6)$$

λ is the radar operating wavelength.

Fig. 2.2 shows the composite RCS corresponding to this experiment. This plot can be reproduced using MATLAB function “*rcs_aspect.m*” given in Listing 2.1 in Section 2.8. As indicated by Fig. 2.1, RCS is dependent on the radar aspect angle. Knowledge of this constructive and destructive interference between the individual scatterers can be very critical when a radar tries to extract RCS of complex or maneuvering targets. This is true because of two reasons. First, the aspect angle may be continuously changing. Second, complex target RCS can be viewed to be made up from contributions of many individual scattering points distributed on the target surface. These scattering points are often called scattering centers. Many approximate RCS prediction methods generate a set of scattering centers that define the back-scattering characteristics of such complex targets.

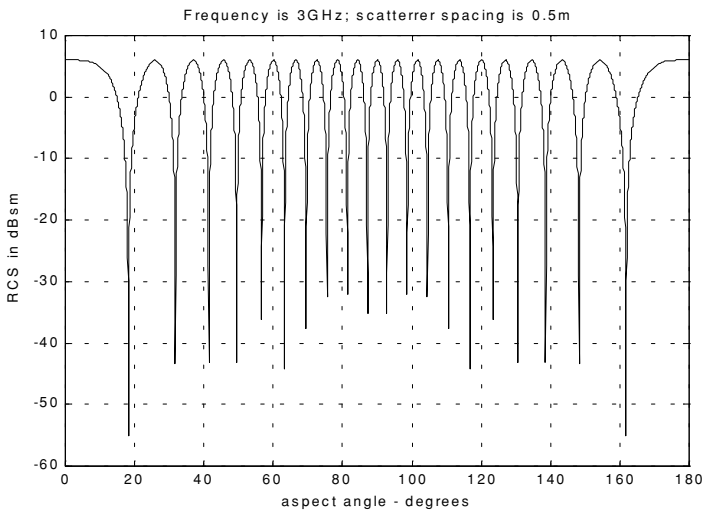


Figure 2.2. Illustration of RCS dependency on aspect angle.

MATLAB Function “*rcs_aspect.m*”

The function “*rcs_aspect.m*” computes and plots the RCS dependency on aspect angle. Its syntax is as follows:

$$[rcs] = rcs_aspect(scatter_spacing, freq)$$

Symbol	Description	Units	Status
<i>scat_spacing</i>	<i>scatterer spacing</i>	<i>meters</i>	<i>input</i>
<i>freq</i>	<i>radar frequency</i>	<i>Hz</i>	<i>input</i>
<i>rsc</i>	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

Next, to demonstrate RCS dependency on frequency, consider the experiment shown in Fig. 2.3. In this case, two far field unity isotropic scatterers are aligned with radar line of sight, and the composite RCS is measured by the radar as the frequency is varied from 8 GHz to 12.5 GHz (X-band). Figs. 2.4 and 2.5 show the composite RCS versus frequency for scatterer spacing of 0.1 and 0.7 meters.

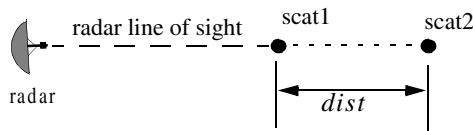


Figure 2.3. Experiment setup which demonstrates RCS dependency on frequency; $dist = 0.1$, or 0.7 m.

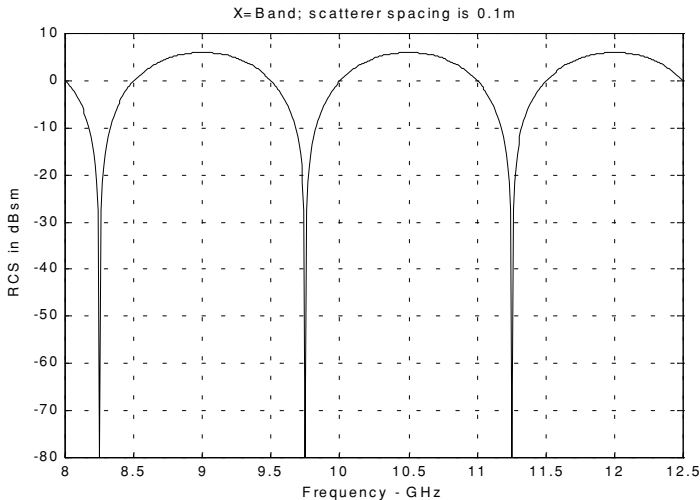


Figure 2.4. Illustration of RCS dependency on frequency.

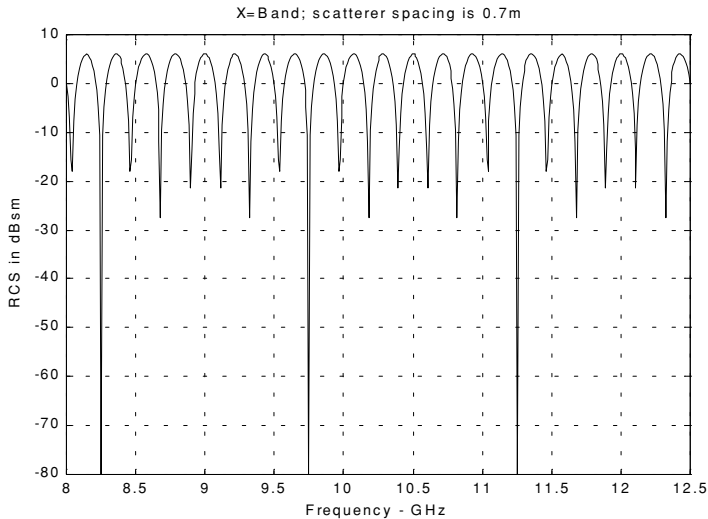


Figure 2.5. Illustration of RCS dependency on frequency.

The plots shown in Figs. 2.4 and 2.5 can be reproduced using MATLAB function “*rcs_frequency.m*” given in Listing 2.2 in Section 2.8. From those two figures, RCS fluctuation as a function of frequency is evident. Little frequency change can cause serious RCS fluctuation when the scatterer spacing is large. Alternatively, when scattering centers are relatively close, it requires more frequency variation to produce significant RCS fluctuation.

MATLAB Function “*rcs_frequency.m*”

The function “*rcs_frequency.m*” computes and plots the RCS dependency on frequency. Its syntax is as follows:

$$[rcs] = rcs_frequency(scatter_spacing, frequ, freql)$$

where

Symbol	Description	Units	Status
<i>scatter_spacing</i>	<i>scatterer spacing</i>	<i>meters</i>	<i>input</i>
<i>freql</i>	<i>start of frequency band</i>	<i>Hz</i>	<i>input</i>
<i>frequ</i>	<i>end of frequency band</i>	<i>Hz</i>	<i>input</i>
<i>rcs</i>	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

2.4. RCS Dependency on Polarization

The material in this section covers two topics. First, a review of polarization fundamentals is presented. Second, the concept of target scattering matrix is introduced.

2.4.1. Polarization

The x and y electric field components for a wave traveling along the positive z direction are given by

$$E_x = E_1 \sin(\omega t - kz) \quad (2.7)$$

$$E_y = E_2 \sin(\omega t - kz + \delta) \quad (2.8)$$

where $k = 2\pi/\lambda$, ω is the wave frequency, the angle δ is the time phase angle which E_y leads E_x , and finally, E_1 and E_2 are, respectively, the wave amplitudes along the x and y directions. When two or more electromagnetic waves combine, their electric fields are integrated vectorially at each point in space for any specified time. In general, the combined vector traces an ellipse when observed in the x-y plane. This is illustrated in [Fig. 2.6](#).

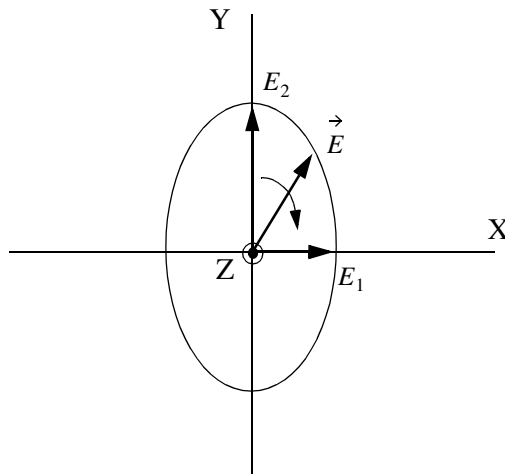


Figure 2.6. Electric field components along the x and y directions.
The positive z direction is out of the page.

The ratio of the major to the minor axes of the polarization ellipse is called the Axial Ratio (AR). When AR is unity, the polarization ellipse becomes a circle, and the resultant wave is then called circularly polarized. Alternatively, when $E_1 = 0$ and $AR = \infty$ the wave becomes linearly polarized.

Eqs. (2.7) and (2.8) can be combined to give the instantaneous total electric field,

$$\vec{E} = \hat{a}_x E_1 \sin(\omega t - kz) + \hat{a}_y E_2 \sin(\omega t - kz + \delta) \quad (2.9)$$

where \hat{a}_x and \hat{a}_y are unit vectors along the x and y directions, respectively. At $z = 0$, $E_x = E_1 \sin(\omega t)$ and $E_y = E_2 \sin(\omega t + \delta)$, then by replacing $\sin(\omega t)$ by the ratio E_x/E_1 and by using trigonometry properties Eq. (2.9) can be rewritten as

$$\frac{E_x^2}{E_1^2} - \frac{2E_x E_y \cos \delta}{E_1 E_2} + \frac{E_y^2}{E_2^2} = (\sin \delta)^2 \quad (2.10)$$

Note that Eq. (2.10) has no dependency on ωt .

In the most general case, the polarization ellipse may have any orientation, as illustrated in Fig. 2.7. The angle ξ is called the tilt angle of the ellipse. In this case, AR is given by

$$AR = \frac{OA}{OB} \quad (1 \leq AR \leq \infty) \quad (2.11)$$

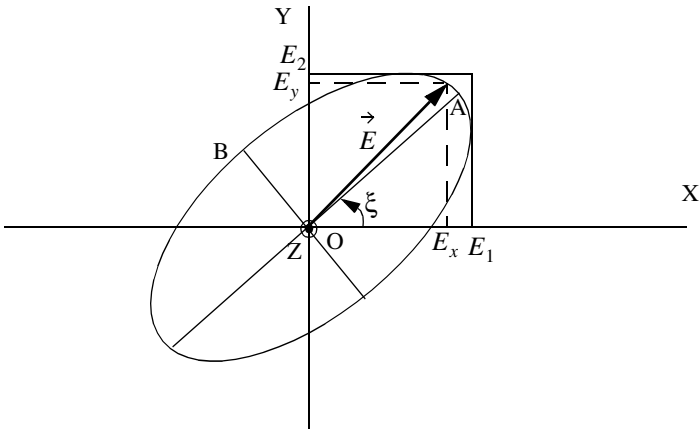


Figure 2.7. Polarization ellipse in the general case.

When $E_1 = 0$, the wave is said to be linearly polarized in the y direction, while if $E_2 = 0$ the wave is said to be linearly polarized in the x direction. Polarization can also be linear at an angle of 45° when $E_1 = E_2$ and $\xi = 45^\circ$. When $E_1 = E_2$ and $\delta = 90^\circ$, the wave is said to be Left Circularly Polarized (LCP), while if $\delta = -90^\circ$ the wave is said to be Right Circularly Polarized (RCP). It is a common notation to call the linear polarizations along the x and y directions by the names horizontal and vertical polarizations, respectively.

In general, an arbitrarily polarized electric field may be written as the sum of two circularly polarized fields. More precisely,

$$\vec{E} = \vec{E}_R + \vec{E}_L \quad (2.12)$$

where \vec{E}_R and \vec{E}_L are the RCP and LCP fields, respectively. Similarly, the RCP and LCP waves can be written as

$$\vec{E}_R = \vec{E}_V + j\vec{E}_H \quad (2.13)$$

$$\vec{E}_L = \vec{E}_V - j\vec{E}_H \quad (2.14)$$

where \vec{E}_V and \vec{E}_H are the fields with vertical and horizontal polarizations, respectively. Combining Eqs. (2.13) and (2.14) yields

$$E_R = \frac{E_H - jE_V}{\sqrt{2}} \quad (2.15)$$

$$E_L = \frac{E_H + jE_V}{\sqrt{2}} \quad (2.16)$$

Using matrix notation Eqs. (2.15) and (2.16) can be rewritten as

$$\begin{bmatrix} E_R \\ E_L \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix} = [T] \begin{bmatrix} E_H \\ E_V \end{bmatrix} \quad (2.17)$$

$$\begin{bmatrix} E_H \\ E_V \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} E_R \\ E_L \end{bmatrix} = [T]^{-1} \begin{bmatrix} E_R \\ E_L \end{bmatrix} \quad (2.18)$$

For many targets the scattered waves will have different polarization than the incident waves. This phenomenon is known as depolarization or cross-polarization. However, perfect reflectors reflect waves in such a fashion that an incident wave with horizontal polarization remains horizontal, and an incident

wave with vertical polarization remains vertical but is phase shifted 180° . Additionally, an incident wave which is RCP becomes LCP when reflected, and a wave which is LCP becomes RCP after reflection from a perfect reflector. Therefore, when a radar uses LCP waves for transmission, the receiving antenna needs to be RCP polarized in order to capture the PP RCS, and LCR to measure the OP RCS.

2.4.2. Target Scattering Matrix

Target backscattered RCS is commonly described by a matrix known as the scattering matrix, and is denoted by $[S]$. When an arbitrarily linearly polarized wave is incident on a target, the backscattered field is then given by

$$\begin{bmatrix} E_1^s \\ E_2^s \end{bmatrix} = [S] \begin{bmatrix} E_1^i \\ E_2^i \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} E_1^i \\ E_2^i \end{bmatrix} \quad (2.19)$$

The superscripts i and s denote incident and scattered fields. The quantities s_{ij} are in general complex and the subscripts 1 and 2 represent any combination of orthogonal polarizations. More precisely, $1 = H, R$, and $2 = V, L$. From Eq. (2.3), the backscattered RCS is related to the scattering matrix components by the following relation:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = 4\pi R^2 \begin{bmatrix} |s_{11}|^2 & |s_{12}|^2 \\ |s_{21}|^2 & |s_{22}|^2 \end{bmatrix} \quad (2.20)$$

It follows that once a scattering matrix is specified, the target backscattered RCS can be computed for any combination of transmitting and receiving polarizations. The reader is advised to see Ruck for ways to calculate the scattering matrix $[S]$.

Rewriting Eq. (2.20) in terms of the different possible orthogonal polarizations yields

$$\begin{bmatrix} E_H^s \\ E_V^s \end{bmatrix} = \begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} \begin{bmatrix} E_H^i \\ E_V^i \end{bmatrix} \quad (2.21)$$

$$\begin{bmatrix} E_R^s \\ E_L^s \end{bmatrix} = \begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} \begin{bmatrix} E_R^i \\ E_L^i \end{bmatrix} \quad (2.22)$$

By using the transformation matrix $[T]$ in Eq. (2.17), the circular scattering elements can be computed from the linear scattering elements

$$\begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} = [T] \begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [T]^{-1} \quad (2.23)$$

and the individual components are

$$\begin{aligned} s_{RR} &= \frac{-s_{VV} + s_{HH} - j(s_{HV} + s_{VH})}{2} \\ s_{RL} &= \frac{s_{VV} + s_{HH} + j(s_{HV} - s_{VH})}{2} \\ s_{LR} &= \frac{s_{VV} + s_{HH} - j(s_{HV} - s_{VH})}{2} \\ s_{LL} &= \frac{-s_{VV} + s_{HH} + j(s_{HV} + s_{VH})}{2} \end{aligned} \quad (2.24)$$

Similarly, the linear scattering elements are given by

$$\begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} = [T]^{-1} \begin{bmatrix} s_{RR} & s_{RL} \\ s_{LR} & s_{LL} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [T] \quad (2.25)$$

and the individual components are

$$\begin{aligned} s_{HH} &= \frac{-s_{RR} + s_{RL} + s_{LR} - s_{LL}}{2} \\ s_{VH} &= \frac{j(s_{RR} - s_{LR} + s_{RL} - s_{LL})}{2} \\ s_{HV} &= \frac{-j(s_{RR} + s_{LR} - s_{RL} - s_{LL})}{2} \\ s_{VV} &= \frac{s_{RR} + s_{LL} + js_{RL} + s_{LR}}{2} \end{aligned} \quad (2.26)$$

2.5. RCS of Simple Objects

This section presents examples of backscattered radar cross section for a number of simple shape objects. In all cases, except for the perfectly conducting sphere, only optical region approximations are presented. Radar designers and RCS engineers consider the perfectly conducting sphere to be the simplest target to examine. Even in this case, the complexity of the exact solution, when

compared to the optical region approximation, is overwhelming. Most formulas presented are Physical Optics (PO) approximation for the backscattered RCS measured by a far field radar in the direction (θ, φ) , as illustrated in Fig. 2.8.

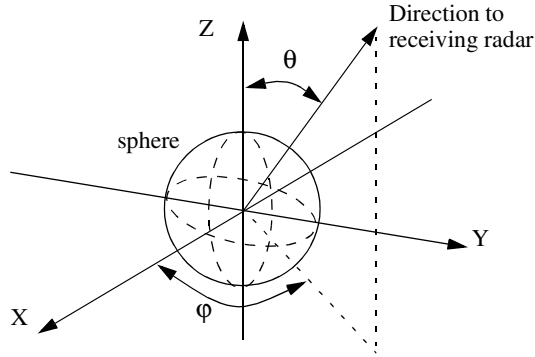


Figure 2.8. Direction of antenna receiving backscattered waves.

2.5.1. Sphere

Due to symmetry, waves scattered from a perfectly conducting sphere are co-polarized (have the same polarization) with the incident waves. This means that the cross-polarized backscattered waves are practically zero. For example, if the incident waves were Left Circularly Polarized (LCP), then the backscattered waves will also be LCP. However, because of the opposite direction of propagation of the backscattered waves, they are considered to be Right Circularly Polarized (RCP) by the receiving antenna. Therefore, the PP backscattered waves from a sphere are LCP, while the OP backscattered waves are negligible.

The normalized exact backscattered RCS for a perfectly conducting sphere is a Mie series given by

$$\frac{\sigma}{\pi r^2} = \left(\frac{j}{kr}\right) \sum_{n=1}^{\infty} (-1)^n (2n+1) \left[\left(\frac{krJ_{n-1}(kr) - nJ_n(kr)}{krH_{n-1}^{(1)}(kr) - nH_n^{(1)}(kr)} \right) - \left(\frac{J_n(kr)}{H_n^{(1)}(kr)} \right) \right] \quad (2.27)$$

where r is the radius of the sphere, $k = 2\pi/\lambda$, λ is the wavelength, J_n is the spherical Bessel of the first kind of order n , and $H_n^{(1)}$ is the Hankel function of order n , and is given by

$$H_n^{(1)}(kr) = J_n(kr) + jY_n(kr) \quad (2.28)$$

Y_n is the spherical Bessel function of the second kind of order n . Plots of the normalized perfectly conducting sphere RCS as a function of its circumference in wavelength units are shown in Figs. 2.9a and 2.9b. These plots can be reproduced using the function “*rsc_sphere.m*” given in Listing 2.3 in Section 2.8.

In Fig. 2.9, three regions are identified. First is the optical region (corresponds to a large sphere). In this case,

$$\sigma = \pi r^2 \quad r \gg \lambda \quad (2.29)$$

Second is the Rayleigh region (small sphere). In this case,

$$\sigma \approx 9\pi r^2 (kr)^4 \quad r \ll \lambda \quad (2.30)$$

The region between the optical and Rayleigh regions is oscillatory in nature and is called the Mie or resonance region.

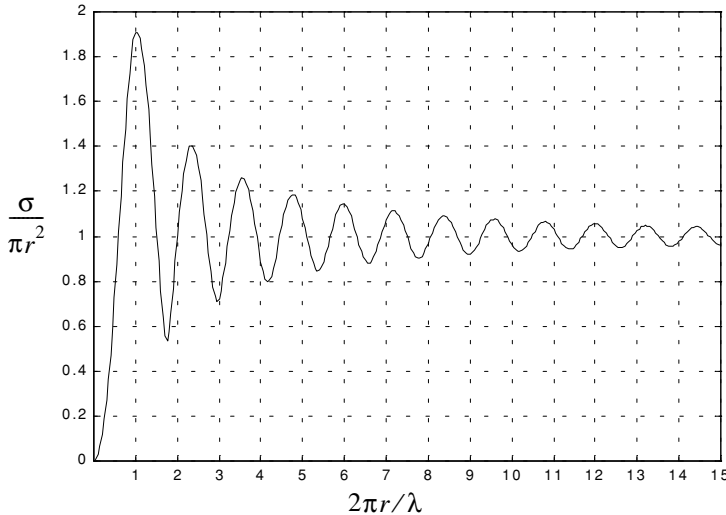


Figure 2.9a. Normalized backscattered RCS for a perfectly conducting sphere.

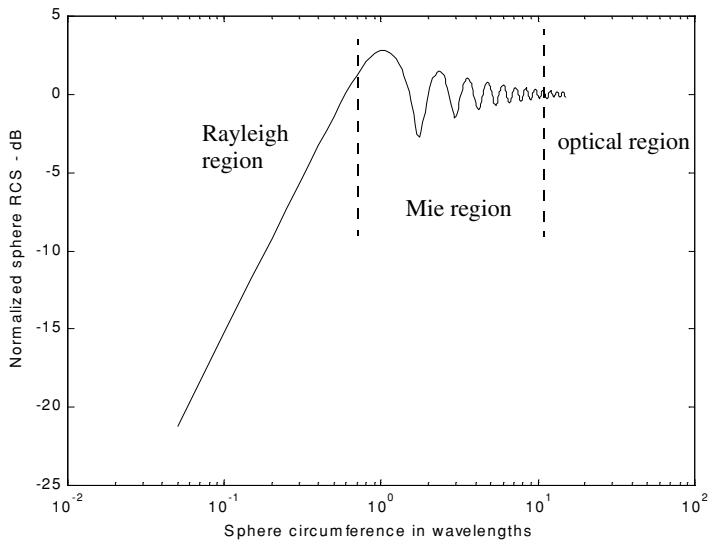


Figure 2.9b. Normalized backscattered RCS for a perfectly conducting sphere using semi-log scale.

The backscattered RCS for a perfectly conducting sphere is constant in the optical region. For this reason, radar designers typically use spheres of known cross sections to experimentally calibrate radar systems. For this purpose, spheres are flown attached to balloons. In order to obtain Doppler shift, spheres of known RCS are dropped out of an airplane and towed behind the airplane whose velocity is known to the radar.

2.5.2. Ellipsoid

An ellipsoid centered at (0,0,0) is shown in Fig. 2.10. It is defined by the following equation:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \quad (2.31)$$

One widely accepted approximation for the ellipsoid backscattered RCS is given by

$$\sigma = \frac{\pi a^2 b^2 c^2}{(a^2 (\sin \theta)^2 (\cos \phi)^2 + b^2 (\sin \theta)^2 (\sin \phi)^2 + c^2 (\cos \theta)^2)^2} \quad (2.32)$$

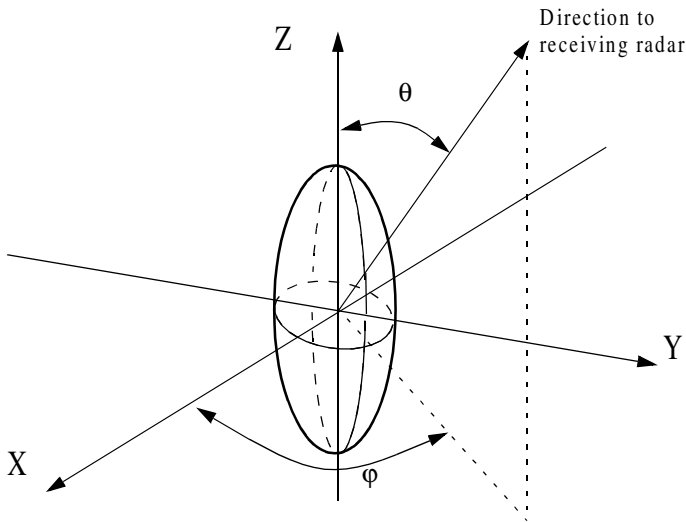


Figure 2.10. Ellipsoid.

When $a = b$, the ellipsoid becomes roll symmetric. Thus, the RCS is independent of ϕ , and Eq. (2.32) is reduced to

$$\sigma = \frac{\pi b^4 c^2}{(a^2 (\sin \theta)^2 + c^2 (\cos \theta)^2)^2} \quad (2.33)$$

and for the case when $a = b = c$,

$$\sigma = \pi c^2 \quad (2.34)$$

Note that Eq. (2.34) defines the backscattered RCS of a sphere. This should be expected, since under the condition $a = b = c$ the ellipsoid becomes a sphere. Fig. 2.11 shows the backscattered RCS for an ellipsoid versus θ for $\phi = 45^\circ$. This plot can be generated using MATLAB function “*rcs_ellipsoid.m*” given in Listing 2.4 in Section 2.8. Note that at normal incidence ($\theta = 90^\circ$) the RCS corresponds to that of a sphere of radius c , and is often referred to as the broadside specular RCS value.

MATLAB Function “*rcs_ellipsoid.m*”

The function “*rcs_ellipsoid.m*” computes and plots the RCS of an ellipsoid versus aspect angle. It utilizes Eq. (2.32) and its syntax is as follows:

$$[rcs] = rcs_ellipsoid(a, b, c, phi)$$

where

Symbol	Description	Units	Status
a	<i>ellipsoid a-radius</i>	<i>meters</i>	<i>input</i>
b	<i>ellipsoid b-radius</i>	<i>meters</i>	<i>input</i>
c	<i>ellipsoid c-radius</i>	<i>meters</i>	<i>input</i>
ϕ	<i>ellipsoid roll angle</i>	<i>degrees</i>	<i>input</i>
r_{cs}	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

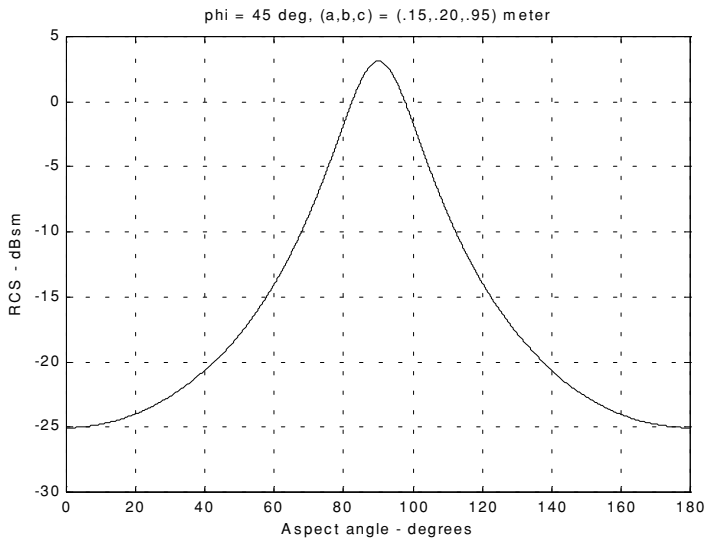


Figure 2.11. Ellipsoid backscattered RCS versus aspect angle, $\phi = 45^\circ$.

2.5.3. Circular Flat Plate

Fig. 2.12 shows a circular flat plate of radius r , centered at the origin. Due to the circular symmetry, the backscattered RCS of a circular flat plate has no dependency on ϕ . The RCS is only aspect angle dependent. For normal incidence (i.e., zero aspect angle) the backscattered RCS for a circular flat plate is

$$\sigma = \frac{4\pi^3 r^4}{\lambda^2} \quad \theta = 0^\circ \quad (2.35)$$

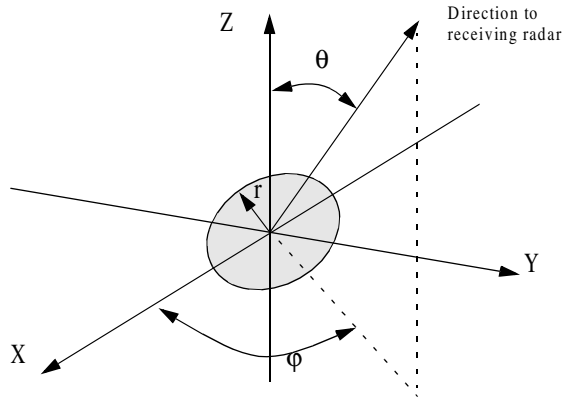


Figure 2.12. Circular flat plate.

For non-normal incidence, two approximations for the circular flat plate backscattered RCS for any linearly polarized incident wave are

$$\sigma = \frac{\lambda r}{8\pi \sin\theta (\tan(\theta))^2} \quad (2.36)$$

$$\sigma = \pi k^2 r^4 \left(\frac{2J_1(2kr \sin\theta)}{2kr \sin\theta} \right)^2 (\cos\theta)^2 \quad (2.37)$$

where $k = 2\pi/\lambda$, and $J_1(\beta)$ is the first order spherical Bessel function evaluated at β . The RCS corresponding to Eqs. (2.35) through (2.37) is shown in Fig. 2.13. These plots can be reproduced using MATLAB function “*rcs_circ_plate.m*” given in Listing 2.5 in Section 2.8.

MATLAB Function “*rcs_circ_plate.m*”

The function “*rcs_circ_plate.m*” calculates and plots the backscattered RCS from a circular plate. Its syntax is as follows:

$$[rcs] = rcs_circ_plate(r, freq)$$

where

Symbol	Description	Units	Status
<i>r</i>	<i>radius of circular plate</i>	<i>meters</i>	<i>input</i>
<i>freq</i>	<i>frequency</i>	<i>Hz</i>	<i>input</i>
<i>rcs</i>	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

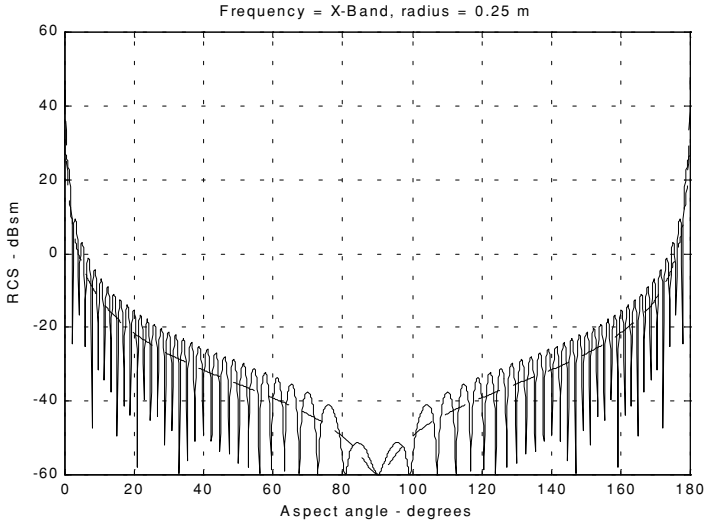


Figure 2.13. Backscattered RCS for a circular flat plate. Solid line corresponds to Eq. (2.37). Dashed line corresponds to Eq. (2.36).

2.5.4. Truncated Cone (Frustum)

Figs. 2.14 and 2.15 show the geometry associated with a frustum. The half cone angle α is given by

$$\tan \alpha = \frac{(r_2 - r_1)}{H} = \frac{r_2}{L} \quad (2.38)$$

Define the aspect angle at normal incidence (broadside) as θ_n . Thus, when a frustum is illuminated by a radar located at the same side as the cone's small end, the angle θ_n is

$$\theta_n = 90^\circ - \alpha \quad (2.39)$$

Alternatively, normal incidence occurs at

$$\theta_n = 90^\circ + \alpha \quad (2.40)$$

At normal incidence, one approximation for the backscattered RCS of a truncated cone due to a linearly polarized incident wave is

$$\sigma_{\theta_n} = \frac{8\pi(z_2^{3/2} - z_1^{3/2})^2}{9\lambda \sin \theta_n} \tan \alpha (\sin \theta_n - \cos \theta_n \tan \alpha)^2 \quad (2.41)$$

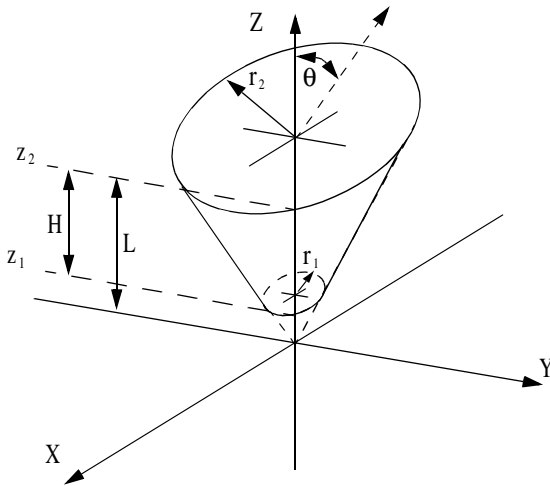


Figure 2.14. Truncated cone (frustum).

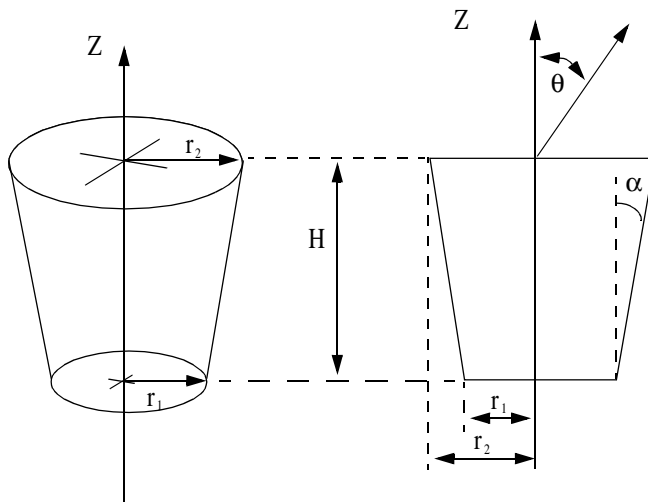


Figure 2.15. Definition of half cone angle.

where λ is the wavelength, and z_1, z_2 are defined in Fig. 2.14. Using trigonometric identities, Eq. (2.41) can be reduced to

$$\sigma_{\theta_n} = \frac{8\pi(z_2^{3/2} - z_1^{3/2})^2}{9\lambda} \frac{\sin\alpha}{(\cos\alpha)^4} \quad (2.42)$$

For non-normal incidence, the backscattered RCS due to a linearly polarized incident wave is

$$\sigma = \frac{\lambda z \tan\alpha}{8\pi \sin\theta} \left(\frac{\sin\theta - \cos\theta \tan\alpha}{\sin\theta \tan\alpha + \cos\theta} \right)^2 \quad (2.43)$$

where z is equal to either z_1 or z_2 depending on whether the RCS contribution is from the small or the large end of the cone. Again, using trigonometric identities Eq. (2.43) (assuming the radar illuminates the frustum starting from the large end) is reduced to

$$\sigma = \frac{\lambda z \tan\alpha}{8\pi \sin\theta} (\tan(\theta - \alpha))^2 \quad (2.44)$$

When the radar illuminates the frustum starting from the small end (i.e., the radar is in the negative z direction in Fig. (2.14)), Eq. (2.44) should be modified to

$$\sigma = \frac{\lambda z \tan\alpha}{8\pi \sin\theta} (\tan(\theta + \alpha))^2 \quad (2.45)$$

For example, consider a frustum defined by $H = 20.945\text{ cm}$, $r_1 = 2.057\text{ cm}$, $r_2 = 5.753\text{ cm}$. It follows that the half cone angle is 10° . Fig. 2.16 (top) shows a plot of its RCS when illuminated by a radar in the positive z direction. Fig. 2.16 (bottom) shows the same thing, except in this case, the radar is in the negative z direction. Note that for the first case, normal incidence occur at 100° , while for the second case it occurs at 80° . These plots can be reproduced using MATLAB function “*rsc_frustum.m*” given in Listing 2.6 in Section 2.8.

MATLAB Function “*rsc_frustum.m*”

The function “*rsc_frustum.m*” computes and plots the backscattered RCS of a truncated conic section. The syntax is as follows:

$$[rsc] = rsc_frustum (r1, r2, freq, indicator)$$

where

Symbol	Description	Units	Status
$r1$	small end radius	meters	input
$r2$	large end radius	meters	input
$freq$	frequency	Hz	input
$indicator$	indicator = 1 when viewing from large end indicator = 0 when viewing from small end	none	input
r_{cs}	array of RCS versus aspect angle	dBsm	output

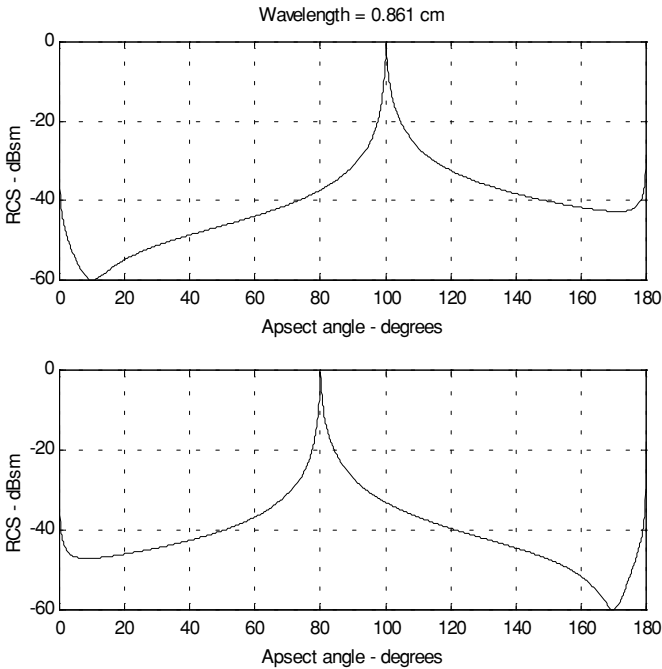


Figure 2.16. Backscattered RCS for a frustum.

2.5.5. Cylinder

Fig. 2.17 shows the geometry associated with a cylinder. Two cases are presented: first, the general case of an elliptical cylinder; second, the case of a circular cylinder. The normal and non-normal incidence backscattered RCS for an

elliptical cylinder due a linearly polarized incident wave are, respectively, given by

$$\sigma_{\theta_n} = \frac{2\pi H^2 r_2^2 r_1^2}{\lambda(r_1^2(\cos\varphi)^2 + r_2^2(\sin\varphi)^2)^{1.5}} \quad (2.46)$$

$$\sigma = \frac{\lambda r_2^2 r_1^2 \sin\theta}{8\pi(\cos\theta)^2(r_1^2(\cos\varphi)^2 + r_2^2(\sin\varphi)^2)^{1.5}} \quad (2.47)$$

For a circular cylinder of radius r , then due to roll symmetry, Eqs. (2.46) and (2.47), respectively, reduce to

$$\sigma_{\theta_n} = \frac{2\pi H^2 r}{\lambda} \quad (2.48)$$

$$\sigma = \frac{\lambda r \sin\theta}{8\pi(\cos\theta)^2} \quad (2.49)$$

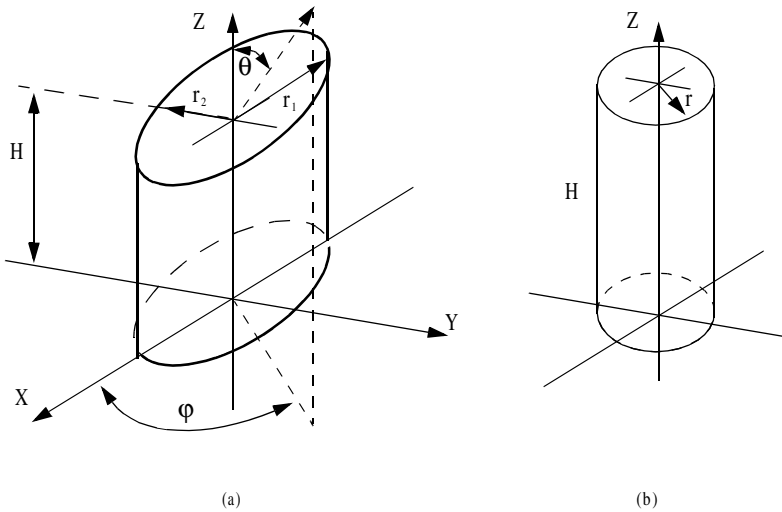


Figure 2.17. (a) Elliptical cylinder; (b) circular cylinder.

Fig. 2.18 shows a plot of the cylinder backscattered RCS using Eqs. (2.48) and (2.49). This plot can be reproduced using MATLAB function “*rcs_cylinder.m*” given in Listing 2.7 in Section 2.8. Note that the broadside specular occurs at aspect angle of 90° .

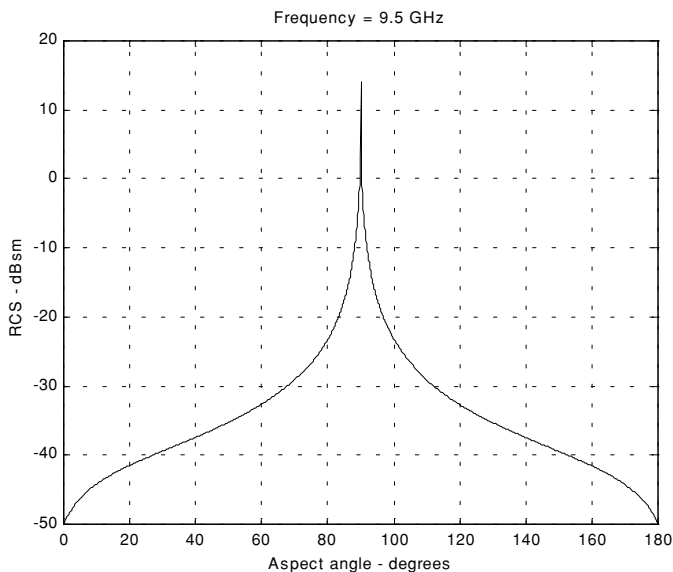


Figure 2.18. Backscattered RCS for a cylinder, $r = 0.125m$ and $H = 1m$.

MATLAB Function “*rcs_cylinder.m*”

The function “*rcs_cylinder.m*” computes and plots the backscattered RCS of a cylinder. The syntax is as follows:

$$[rcs] = rcs_cylinder(r, h, freq)$$

where

Symbol	Description	Units	Status
<i>r</i>	<i>radius</i>	<i>meters</i>	<i>input</i>
<i>h</i>	<i>length of cylinder</i>	<i>meters</i>	<i>input</i>
<i>freq</i>	<i>frequency</i>	<i>Hz</i>	<i>input</i>
<i>rcs</i>	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

2.5.6. Rectangular Flat Plate

Consider a perfectly conducting rectangular thin flat plate in the x-y plane as shown in Fig. 2.19. The two sides of the plate are denoted by $2a$ and $2b$. For a linearly polarized incident wave in the x-z plane, the horizontal and vertical backscattered RCS are, respectively, given by

$$\sigma_V = \frac{b^2}{\pi} \left| \sigma_{1V} - \sigma_{2V} \left[\frac{1}{\cos \theta} + \frac{\sigma_{2V}}{4} (\sigma_{3V} + \sigma_{4V}) \right] \sigma_{5V}^{-1} \right|^2 \quad (2.50)$$

$$\sigma_H = \frac{b^2}{\pi} \left| \sigma_{1H} - \sigma_{2H} \left[\frac{1}{\cos \theta} - \frac{\sigma_{2H}}{4} (\sigma_{3H} + \sigma_{4H}) \right] \sigma_{5H}^{-1} \right|^2 \quad (2.51)$$

where $k = 2\pi/\lambda$ and

$$\sigma_{1V} = \cos(k a \sin \theta) - j \frac{\sin(k a \sin \theta)}{\sin \theta} = (\sigma_{1H})^* \quad (2.52)$$

$$\sigma_{2V} = \frac{e^{j(ka - \pi/4)}}{\sqrt{2\pi}(ka)^{3/2}} \quad (2.53)$$

$$\sigma_{3V} = \frac{(1 + \sin \theta) e^{-jk a \sin \theta}}{(1 - \sin \theta)^2} \quad (2.54)$$

$$\sigma_{4V} = \frac{(1 - \sin \theta) e^{jk a \sin \theta}}{(1 + \sin \theta)^2} \quad (2.55)$$

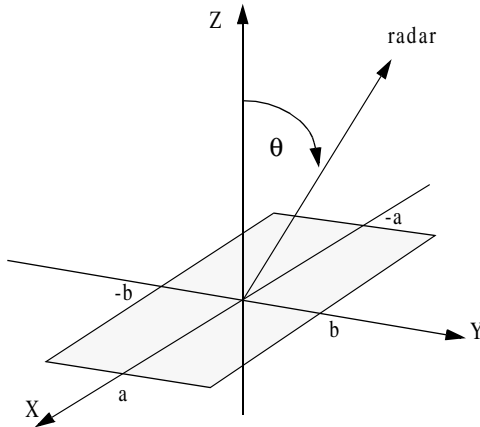


Figure 2.19. Rectangular flat plate.

$$\sigma_{5V} = 1 - \frac{e^{j(2ka - \pi/2)}}{8\pi(ka)^3} \quad (2.56)$$

$$\sigma_{2H} = \frac{4e^{j(ka + \pi/4)}}{\sqrt{2\pi}(ka)^{1/2}} \quad (2.57)$$

$$\sigma_{3H} = \frac{e^{-jk a \sin \theta}}{1 - \sin \theta} \quad (2.58)$$

$$\sigma_{4H} = \frac{e^{jk a \sin \theta}}{1 + \sin \theta} \quad (2.59)$$

$$\sigma_{5H} = 1 - \frac{e^{j(2ka + (\pi/2))}}{2\pi(ka)} \quad (2.60)$$

Eqs. (2.50) and (2.51) are valid and quite accurate for aspect angles $0^\circ \leq \theta \leq 80^\circ$. For aspect angles near 90° , Ross¹ obtained by extensive fitting of measured data an empirical expression for the RCS. It is given by

$$\sigma_H \rightarrow 0$$

$$\sigma_V = \frac{ab^2}{\lambda} \left\{ \left[1 + \frac{\pi}{2(2a/\lambda)^2} \right] + \left[1 - \frac{\pi}{2(2a/\lambda)^2} \right] \cos \left(2ka - \frac{3\pi}{5} \right) \right\} \quad (2.61)$$

The backscattered RCS for a perfectly conducting thin rectangular plate for incident waves at any θ, ϕ can be approximated by

$$\sigma = \frac{4\pi a^2 b^2}{\lambda^2} \left(\frac{\sin(ak \sin \theta \cos \phi)}{ak \sin \theta \cos \phi} \frac{\sin(bk \sin \theta \sin \phi)}{bk \sin \theta \sin \phi} \right)^2 (\cos \theta)^2 \quad (2.62)$$

Eq. (2.62) is independent of the polarization, and is only valid for aspect angles $\theta \leq 20^\circ$. Fig. 2.20, shows an example for the backscattered RCS of a rectangular flat plate, for both vertical (Fig. 2.20a) and horizontal (Fig. 2.20b) polarizations, using Eqs. (2.50), (2.51) and (2.62). In this example, $a = b = 10.16 \text{ cm}$ and wavelength $\lambda = 3.25 \text{ cm}$. This plot can be reproduced using MATLAB function “*rcs_rect_plate*” given in Listing 2.8 in Section 2.8.

MATLAB Function “*rcs_rect_plate.m*”

The function “*rcs_rect_plate.m*” calculates and plots the backscattered RCS of a rectangular flat plate. Its syntax is as follows:

1. Ross, R. A. Radar Cross Section of Rectangular Flat Plate as a Function of Aspect Angle, *IEEE Trans. AP-14:320*, 1966.

$$[rcs] = rcs_rect_plate(a, b, freq)$$

where

Symbol	Description	Units	Status
a	short side of plate	meters	input
b	long side of plate	meters	input
$freq$	frequency	Hz	input
rcs	array of RCS versus aspect angle	dBsm	output

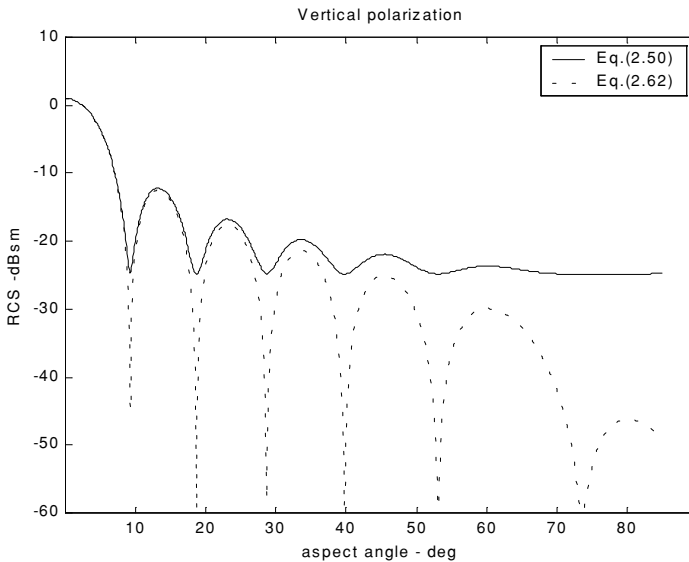


Figure 2.20a. Backscattered RCS for a rectangular flat plate.

2.5.7. Triangular Flat Plate

Consider the triangular flat plate defined by the isosceles triangle as oriented in Fig. 2.21. The backscattered RCS can be approximated for small aspect angles (less than 30°) by

$$\sigma = \frac{4\pi A^2}{\lambda^2} (\cos \theta)^2 \sigma_0 \quad (2.63)$$

$$\sigma_0 = \frac{[(\sin\alpha)^2 - (\sin(\beta/2))^2]^2 + \sigma_{01}}{\alpha^2 - (\beta/2)^2} \quad (2.64)$$

$$\sigma_{01} = 0.25(\sin\phi)^2[(2a/b)\cos\phi\sin\beta - \sin\phi\sin 2\alpha]^2 \quad (2.65)$$

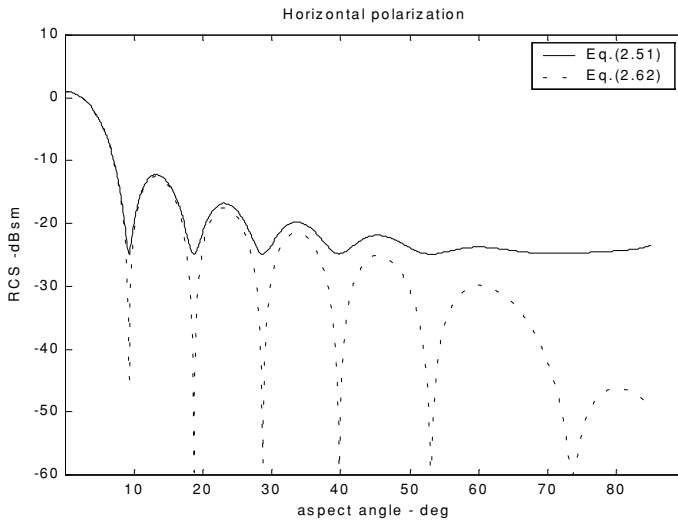


Figure 2.20b. Backscattered RCS for a rectangular flat plate.

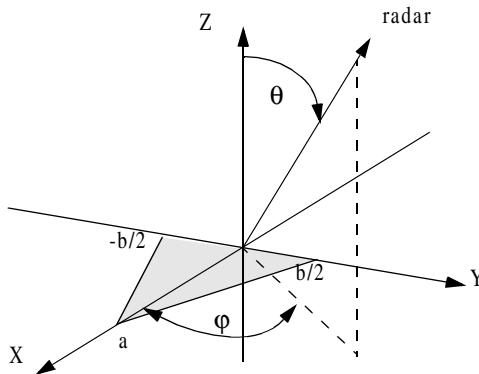


Figure 2.21. Coordinates for a perfectly conducting isosceles triangular plate.

where $\alpha = k a \sin \theta \cos \varphi$, $\beta = k b \sin \theta \sin \varphi$, and $A = ab/2$. For waves incident in the plane $\varphi = 0$, the RCS reduces to

$$\sigma = \frac{4\pi A^2}{\lambda^2} (\cos \theta)^2 \left[\frac{(\sin \alpha)^4}{\alpha^4} + \frac{(\sin 2\alpha - 2\alpha)^2}{4\alpha^4} \right] \quad (2.66)$$

and for incidence in the plane $\varphi = \pi/2$

$$\sigma = \frac{4\pi A^2}{\lambda^2} (\cos \theta)^2 \left[\frac{(\sin(\beta/2))^4}{(\beta/2)^4} \right] \quad (2.67)$$

Fig. 2.22 shows a plot for the normalized backscattered RCS from a perfectly conducting isosceles triangular flat plate. In this example $a = 0.2m$, $b = 0.75m$, and $\varphi = 0, \pi/2$. This plot can be reproduced using MATLAB function “*rsc_isosceles.m*” given in Listing 2.9 in Section 2.8.

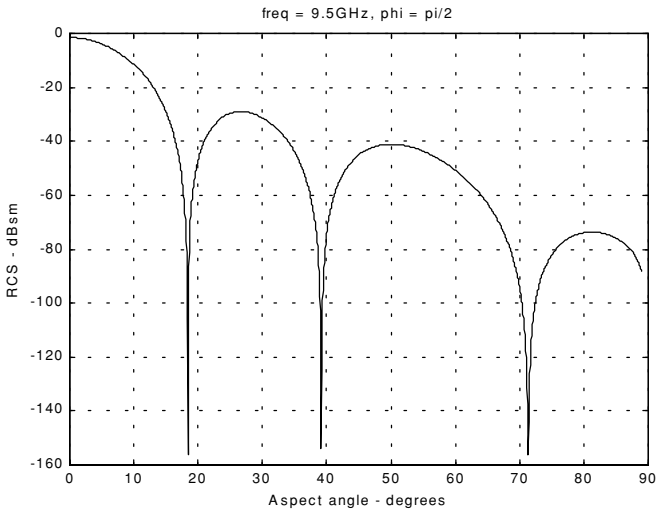


Figure 2.22. Backscattered RCS for a perfectly conducting triangular flat plate, $a = 20cm$ and $b = 75cm$.

MATLAB Function “*rsc_isosceles.m*”

The function “*rsc_isosceles.m*” calculates and plots the backscattered RCS of a triangular flat plate. Its syntax is as follows:

$$[rsc] = rsc_isosceles(a, b, freq, phi)$$

where

Symbol	Description	Units	Status
a	<i>height of plate</i>	<i>meters</i>	<i>input</i>
b	<i>base of plate</i>	<i>meters</i>	<i>input</i>
$freq$	<i>frequency</i>	<i>Hz</i>	<i>input</i>
phi	<i>roll angle</i>	<i>degrees</i>	<i>input</i>
rsc	<i>array of RCS versus aspect angle</i>	<i>dBsm</i>	<i>output</i>

2.6. RCS of Complex Objects

A complex target RCS is normally computed by coherently combining the cross sections of the simple shapes that make that target. In general, a complex target RCS can be modeled as a group of individual scattering centers distributed over the target. The scattering centers can be modeled as isotropic point scatterers (N-point model) or as simple shape scatterers (N-shape model). In any case, knowledge of the scattering centers' locations and strengths is critical in determining complex target RCS. This is true, because as seen in Section 2.3, relative spacing and aspect angles of the individual scattering centers drastically influence the overall target RCS. Complex targets that can be modeled by many equal scattering centers are often called Swerling 1 or 2 targets. Alternatively, targets that have one dominant scattering center and many other smaller scattering centers are known as Swerling 3 or 4 targets.

In NB radar applications, contributions from all scattering centers combine coherently to produce a single value for the target RCS at every aspect angle. However, in WB applications, a target may straddle over many range bins. For each range bin, the average RCS extracted by the radar represents the contributions from all scattering centers that fall within that bin.

As an example, consider a circular cylinder with two perfectly conducting circular flat plates on both ends. Assume linear polarization and let $H = 1m$ and $r = 0.125m$. The backscattered RCS for this object versus aspect angle is shown in Fig. 2.23. Note that at aspect angles close to 0° and 180° the RCS is mainly dominated by the circular plate, while at aspect angles close to normal incidence, the RCS is dominated by the cylinder broadside specular return. This plot can be reproduced using MATLAB program "*rsc_cylinder_complex.m*" given in Listing 2.10 in Section 2.8.

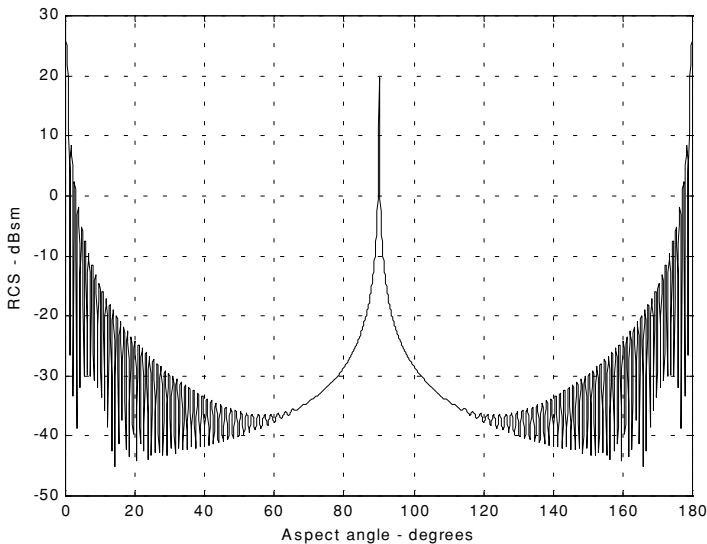


Figure 2.23. Backscattered RCS for a cylinder with flat plates.

2.7. RCS Fluctuations and Statistical Models

In most practical radar systems there is relative motion between the radar and an observed target. Therefore, the RCS measured by the radar fluctuates over a period of time as a function of frequency and the target aspect angle. This observed RCS is referred to as the radar dynamic cross section. Up to this point, all RCS formulas discussed in this chapter assumed stationary target, where in this case, the backscattered RCS is often called static RCS.

Dynamic RCS may fluctuate in amplitude and/or in phase. Phase fluctuation is called glint, while amplitude fluctuation is called scintillation. Glint causes the far field backscattered wavefronts from a target to be non-planar. For most radar applications, glint introduces linear errors in the radar measurements, and thus it is not of a major concern. However, cases where high precision and accuracy are required, glint can be detrimental. Examples include precision instrumentation tracking radar systems, missile seekers, and automated aircraft landing systems. For more details on glint, the reader is advised to visit cited references listed in the bibliography.

Radar cross-section scintillation can vary slowly or rapidly depending on the target size, shape, dynamics, and its relative motion with respect to the radar.

Thus, due to the wide variety of RCS scintillation sources changes in the radar cross section are modeled statistically as random processes. The value of an RCS random process at any given time defines a random variable at that time. Many of the RCS scintillation models were developed and verified by experimental measurements.

2.7.1. RCS Statistical Models - Scintillation Models

This section presents the most commonly used RCS statistical models. Statistical models that apply to sea, land, and volume clutter, such as the Weibull and Log-normal distributions, will be discussed in a later chapter. The choice of a particular model depends heavily on the nature of the target under examination.

Chi-Square of Degree $2m$

The Chi-square distribution applies to a wide range of targets; its *pdf* is given by

$$f(\sigma) = \frac{m}{\Gamma(m)\sigma_{av}} \left(\frac{m\sigma}{\sigma_{av}}\right)^{m-1} e^{-m\sigma/\sigma_{av}} \quad \sigma \geq 0 \quad (2.68)$$

where $\Gamma(m)$ is the gamma function with argument m , and σ_{av} is the average value. As the degree gets larger the distribution corresponds to constrained RCS values (narrow range of values). The limit $m \rightarrow \infty$ corresponds to a constant RCS target (steady-target case).

Swerling I and II (Chi-Square of Degree 2)

In Swerling I, the RCS samples measured by the radar are correlated throughout an entire scan, but are uncorrelated from scan to scan (slow fluctuation). In this case, the *pdf* is

$$f(\sigma) = \frac{1}{\sigma_{av}} \exp\left(-\frac{\sigma}{\sigma_{av}}\right) \quad \sigma \geq 0 \quad (2.69)$$

where σ_{av} denotes the average RCS overall target fluctuation. Swerling II target fluctuation is more rapid than Swerling I, but the measurements are pulse to pulse uncorrelated. This is illustrated in Fig. 2.24. Swerling II RCS distribution is also defined by Eq. (2.69). Swerlings I and II apply to targets consisting of many independent fluctuating point scatterers of approximately equal physical dimensions.

Swerling III and IV (Chi-Square of Degree 4)

Swerlings III and IV have the same *pdf*, and it is given by

$$f(\sigma) = \frac{4\sigma}{\sigma_{av}^2} \exp\left(-\frac{2\sigma}{\sigma_{av}}\right) \quad \sigma \geq 0 \quad (2.70)$$

The fluctuations in Swerling III are similar to Swerling I; while in Swerling IV they are similar to Swerling II fluctuations (see Fig. 2.24). Swerlings III and IV are more applicable to targets that can be represented by one dominant scatterer and many other small reflectors. Fig. 2.25 shows a typical plot of the *pdfs* for Swerling cases. This plot can be reproduced using MATLAB program “*Swerling_models.m*” given in Listing 2.11 in Section 2.8.

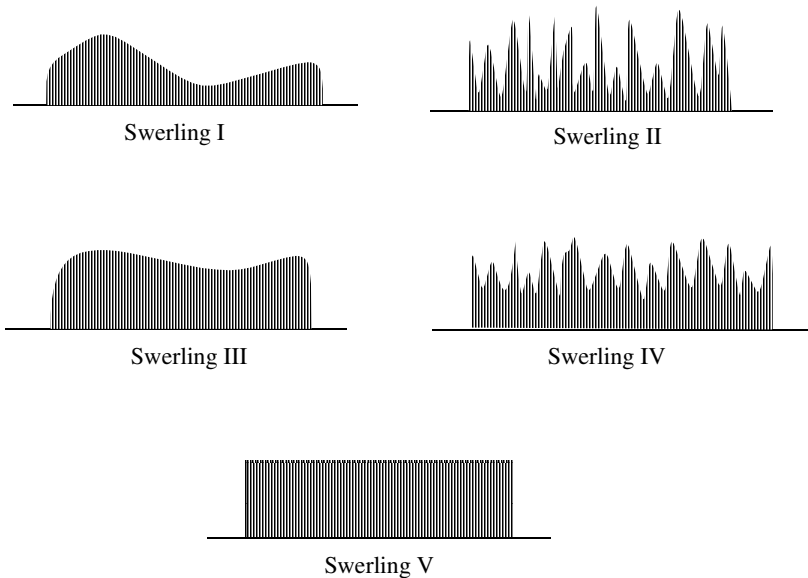


Figure 2.24. Radar returns from targets with different Swerling fluctuations. Swerling V corresponds to a steady RCS target case.

2.8. MATLAB Program/Function Listings

This section presents listings for all MATLAB programs/functions used in this chapter. The user is advised to rerun these programs with different input parameters. All functions have companion MATLAB “*filename_driver.m*” files that utilize MATLAB Graphical User Interface (GUI). Figure 2.26 shows a typical GUI screen capture associated with the cylinder case.

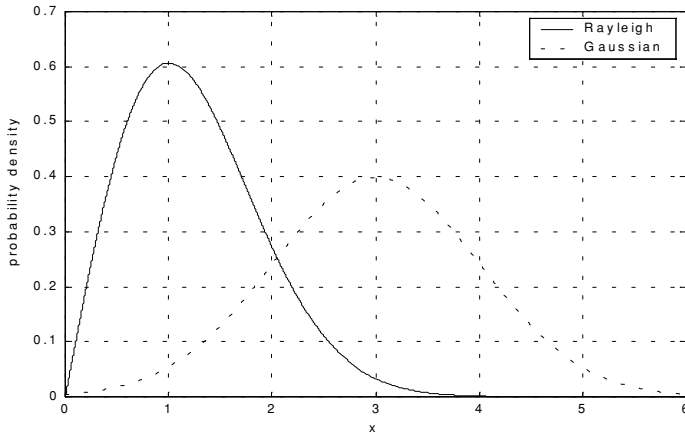


Figure 2.25. Probability densities for Swerling targets.

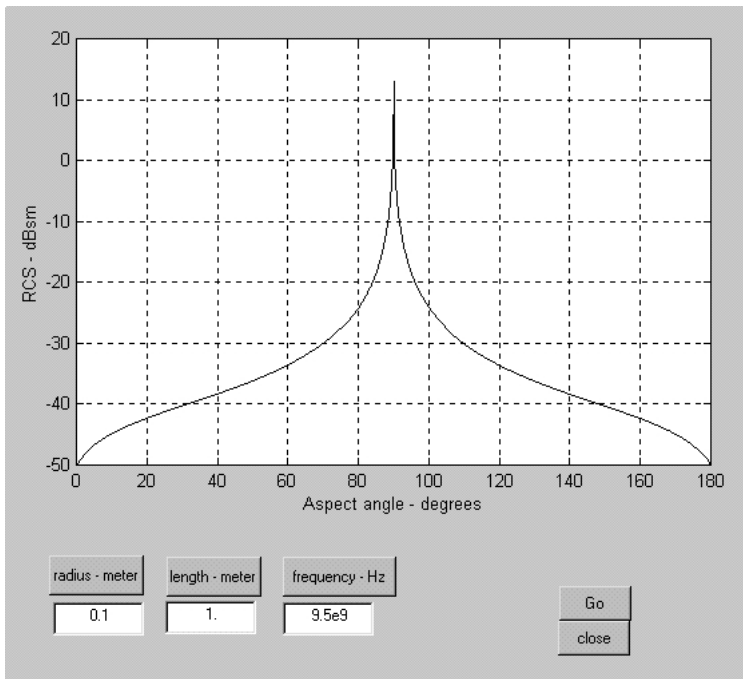


Figure 2.26. GUI work space associated with the function “*rsc_cylinder.m*”.

Listing 2.1. MATLAB Function “rcs_aspect.m”

```
function [rcs] = rcs_aspect (scat_spacing, freq)
% This function demonstrates the effect of aspect angle on RCS.
% Poit scatterers separated by scat_spacing meter. Initially the two scatterers
% are aligned with radar line of sight. The aspect angle is changed from
% 0 degrees to 180 degrees and the equivalent RCS is computed.
% Plot of RCS versus aspect is generated.
eps = 0.00001;
wavelength = 3.0e+8 / freq;
% Compute aspect angle vector
aspect_degrees = 0.:.05:180.;
aspect_radians = (pi/180) .* aspect_degrees;
% Compute electrical scatterer spacing vector in wavelength units
elec_spacing = (2.0 * scat_spacing / wavelength) .* cos(aspect_radians);
% Compute RCS (rcs = RCS_scatter1 + RCS_scatter2)
% Scatter1 is taken as phase reference point
rcs = abs(1.0 + cos((2.0 * pi) .* elec_spacing) ...
        + i * sin((2.0 * pi) .* elec_spacing));
rcs = rcs + eps;
rcs = 20.0*log10(rcs); % RCS in dBsm
% Plot RCS versus aspect angle
figure (1);
plot (aspect_degrees,rcs,'k');
grid;
xlabel ('aspect angle - degrees');
ylabel ('RCS in dBsm');
%title (' Frequency is 3GHz; scatterer spacing is 0.5m');
```

Listing 2.2. MATLAB Function “rcs_frequency.m”

```
function [rcs] = rcs_frequency (scat_spacing, frequ, freql)
% This program demonstrates the dependency of RCS on wavelength
eps = 0.0001;
freq_band = frequ - freql;
delfreq = freq_band / 500.;
index = 0;
for freq = freql:delfreq: frequ
    index = index +1;
    wavelength(index) = 3.0e+8 / freq;
end
elec_spacing = 2.0 * scat_spacing ./ wavelength;
rcs = abs ( 1 + cos((2.0 * pi) .* elec_spacing) ...
        + i * sin((2.0 * pi) .* elec_spacing));
rcs = rcs + eps;
rcs = 20.0*log10(rcs); % RCS ins dBsm
% Plot RCS versus frequency
freq = freql:delfreq:freq;
```

```

plot(freq,rcs);
grid;
xlabel('Frequency');
ylabel('RCS in dBsm');

```

Listing 2.3. MATLAB Program “rsc_sphere.m”.

```

% This program calculates the back-scattered RCS for a perfectly
% conducting sphere using Eq.(2.7), and produce plots similar to Fig.2.9
% Spherical Bessel functions are computed using series approximation and recursion.
clear all
eps = 0.00001;
index = 0;
% kr limits are [0.05 - 15] ==> 300 points
for kr = 0.05:0.05:15
    index = index + 1;
    sphere_rcs = 0. + 0.*i;
    f1 = 0. + 1.*i;
    f2 = 1. + 0.*i;
    m = 1.;
    n = 0.;
    q = -1.;
    % initially set del to huge value
    del = 100000+100000*i;
    while(abs(del) > eps)
        q = -q;
        n = n + 1;
        m = m + 2;
        del = (2.*n-1) * f2 / kr-f1;
        f1 = f2;
        f2 = del;
        del = q * m / (f2 * (kr * f1 - n * f2));
        sphere_rcs = sphere_rcs + del;
    end
    rcs(index) = abs(sphere_rcs);
    sphere_rcsdb(index) = 10. * log10(rcs(index));
end
figure(1);
n=0.05:.05:15;
plot (n,rcs,'k');
set (gca,'xtick',[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15]);
%xlabel ('Sphere circumference in wavelengths');
%ylabel ('Normalized sphere RCS');
grid;
figure (2);
plot (n,sphere_rcsdb,'k');
set (gca,'xtick',[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15]);
xlabel ('Sphere circumference in wavelengths');

```

```

ylabel ('Normalized sphere RCS - dB');
grid;
figure (3);
semilogx (n,sphere_rcsdb,'k');
xlabel ('Sphere circumference in wavelengths');
ylabel ('Normalized sphere RCS - dB');

```

Listing 2.4. MATLAB Function “rcs_ellipsoid.m”

```

function [rcs] = rcs_ellipsoid (a, b, c, phi)
% This function computes and plots the ellipsoid RCS versus aspect angle.
% The roll angle angle phi is fixed,
eps = 0.00001;
sin_phi_s = sin(phi)^2;
cos_phi_s = cos(phi)^2;
% Generate aspect angle vector
theta = 0.:05:180.0;
theta = (theta .* pi) ./ 180.;
if(a ~= b & a ~= c)
    rcs = (pi * a^2 * b^2 * c^2) ./ (a^2 * cos_phi_s .* (sin(theta).^2) + ...
        b^2 * sin_phi_s .* (sin(theta).^2) + ...
        c^2 .* (cos(theta).^2)).^2 ;
else
    if(a == b & a ~= c)
        rcs = (pi * b^4 * c^2) ./ ( b^2 .* (sin(theta).^2) + ...
            c^2 .* (cos(theta).^2)).^2 ;
    else
        if (a == b & a ==c)
            rcs = pi * c^4;
        end
    end
end
rcs_db = 10.0 * log10(rcs);
figure (1);
plot ((theta * 180.0 / pi),rcs_db,'k');
xlabel ('Aspect angle - degrees');
ylabel ('RCS - dBsm');
%title ('phi = 45 deg, (a,b,c) = (.15,.20,.95) meter')
grid;

```

Listing 2.5. MATLAB Function “rcs_circ_plate.m”

```

function [rcs] = rcs_circ_plate (r, freq)
% This function calculates and plots the RCS of a circular flat plate of radius r.
eps = 0.000001;
% Compute wavelength
lambda = 3.e+8 / freq; % X-Band
index = 0;

```

```

for aspect_deg = 0.:1:180
    index = index + 1;
    aspect = (pi / 180.) * aspect_deg;
    % Compute RCS using Eq. (2.35)
    if (aspect == 0 | aspect == pi)
        rcs_po(index) = (4.0 * pi^3 * r^4 / lambda^2) + eps;
        rcs_mu(index) = rcs_po(1);
    else
        % Compute RCS using Eq. (2.36)
        x = (4. * pi * r / lambda) * sin(aspect);
        val1 = 4. * pi^3 * r^4 / lambda^2;
        val2 = 2. * besselj(1,x) / x;
        rcs_po(index) = val1 * (val2 * cos(aspect))^2 + eps;
    % Compute RCS using Eq. (2.36)
        val1m = lambda * r;
        val2m = 8. * pi * sin(aspect) * (tan(aspect)^2);
        rcs_mu(index) = val1m / val2m + eps;
    end
end
rcsdb_po = 10. * log10(rcs_po);
rcsdb_mu = 10 * log10(rcs_mu);
angle = 0.:1:180;
plot(angle,rcsdb_po,'k',angle,rcsdb_mu,'k--')
grid;
xlabel ('Aspect angle - degrees');
ylabel ('RCS - dBsm');
%title ('Frequency = X-Band, radius = 0.25 m');

```

Listing 2.6. MATLAB Function “rcs_frustum.m”

```

function [rcs] = rcs_frustum (r1, r2, h, freq, indicator)
% This program computes the monostatic RCS for a frustum.
% Incident linear Polarization is assumed. To compute RCP or LCP RCS
% one must use Eq. (2.24)
% Normal incidence is according to Eq.s (2.39) and (2.40)
index = 0;
eps = 0.000001;
lambda = 3.0e+8 / freq;
% Comput half cone angle, alpha
alpha = atan(( r2 - r1)/h);
% Compute z1 and z2
z2 = r2 / tan(alpha);
z1 = r1 / tan(alpha);
delta = (z2^1.5 - z1^1.5)^2;
factor = (8. * pi * delta) / (9. * lambda);
large_small_end = indicator;
if (large_small_end == 1)
    % Compute normal incidence, large end

```

```

normal_incidence = (180./pi) * ((pi /2) + alpha)
% Compute RCS from zero aspect to normal incidence
for theta = 0.001:1:1:normal_incidence-.5
    index = index +1;
    theta = theta * pi /180.;
    rcs(index) = (lambda * z1 * tan(alpha) *(tan(theta - alpha))^2) / ...
        (8. * pi *sin(theta)) + eps;
end
%Compute broadside RCS
index = index +1;
rcs_normal = factor * sin(alpha) / ((cos(alpha))^4) + eps;
rcs(index) = rcs_normal;
% Compute RCS from broad side to 180 degrees
for theta = normal_incidence+.5:1:180
    index = index + 1;
    theta = theta * pi / 180. ;
    rcs(index) = (lambda * z2 * tan(alpha) *(tan(theta - alpha))^2) / ...
        (8. * pi *sin(theta)) + eps;
end
else
% Compute normal incidence, small end
normal_incidence = (180./pi) * ((pi /2) - alpha)
% Compute RCS from zero aspect to normal incidence (large end)
for theta = 0.001:1:1:normal_incidence-.5
    index = index +1;
    theta = theta * pi /180.;
    rcs(index) = (lambda * z1 * tan(alpha) *(tan(theta + alpha))^2) / ...
        (8. * pi *sin(theta)) + eps;
end
%Compute broadside RCS
index = index +1;
rcs_normal = factor * sin(alpha) / ((cos(alpha))^4) + eps;
rcs(index) = rcs_normal;
% Compute RCS from broad side to 180 degrees (small end of frustum)
for theta = normal_incidence+.5:1:180
    index = index + 1;
    theta = theta * pi / 180. ;
    rcs(index) = (lambda * z2 * tan(alpha) *(tan(theta + alpha))^2) / ...
        (8. * pi *sin(theta)) + eps;
end
end
% Plot RCS versus aspect angle
delta = 180 /index;
angle = 0.001:delta:180;
plot (angle,10*log10(rcs),'k');
grid;
xlabel ('Aspect angle - degrees');
ylabel ('RCS - dBsm');

```

```
%title ('Wavelength = .861 cm');
```

Listing 2.7. MATLAB Function “rcs_cylinder.m”

```
function [rcs] = rcs_cylinder (r, h, freq)
% This program computes RCS for a cylinder. Circular symmetry is assumed.
% Plot of RCS versus aspect angle is produced
index = 0;
eps =0.00001;
% Compute wavelength
lambda = 3.0e+8 / freq;
% Compute RCS from zero aspect to broadside
for theta = 0.0:1:90-.5
    index = index +1;
    theta = theta * pi /180.;
    rcs(index) = (lambda * r * sin(theta) / ...
        (8. * pi * (cos(theta))^2)) + eps;
end
% Compute RCS for broadside specular
theta = pi/2;
index = index +1;
rcs(index) = (2. * pi * h^2 * r / lambda )+ eps;
% Compute RCS from 90 to 180 degrees
for theta = 90+.5:1:180.
    index = index + 1;
    theta = theta * pi / 180.;
    rcs(index) = ( lambda * r * sin(theta) / ...
        (8. * pi * (cos(theta))^2)) + eps;
end
% Plot results
delta= 180/(index-1)
angle = 0:delta:180;
plot(angle,10*log10(rcs),'k');
grid;
xlabel ('Aspect angle - degrees');
ylabel ('RCS - dBsm');
%title ('Frequency = 9.5 GHz');
```

Listing 2.8. MATLAB Function “rcs_rect_plate.m”

```
function [rcs] = rcs_rect_plate (a, b, freq)
% This function computes the backscattered RCS for a rectangular flat plate.
% The RCS is computed for vertical and horizontal polarization based on
% Eq.s(2.50)through (2.60). Also Physical Optics approximation Eq.(2.62)
% is computed.
eps = 0.000001;
lambda = 3.0e+8 / freq;
ka = 2. * pi * a / lambda;
```



```

% Compute aspect angle vector
theta_deg = 0.05:0.1:85;
theta = (pi/180.) .* theta_deg;
sigma1v = cos(ka .* sin(theta)) - i .* sin(ka .* sin(theta)) ./ sin(theta);
sigma2v = exp(i * ka - (pi / 4)) / (sqrt(2 * pi) *(ka)^1.5);
sigma3v = (1. + sin(theta)) .* exp(-i * ka .* sin(theta)) ./ ...
    (1. - sin(theta)).^2;
sigma4v = (1. - sin(theta)) .* exp(i * ka .* sin(theta)) ./ ...
    (1. + sin(theta)).^2;
sigma5v = 1. - (exp(i * 2. * ka - (pi / 2)) / (8. * pi * (ka)^3));
sigma1h = cos(ka .* sin(theta)) + i .* sin(ka .* sin(theta)) ./ sin(theta);
sigma2h = 4. * exp(i * ka * (pi / 4.)) / (sqrt(2 * pi * ka));
sigma3h = exp(-i * ka .* sin(theta)) ./ (1. - sin(theta));
sigma4h = exp(i * ka * sin(theta)) ./ (1. + sin(theta));
sigma5h = 1. - (exp(j * 2. * ka + (pi / 4.)) / 2. * pi * ka);
% Compute vertical polarization RCS
rcs_v = (b^2 / pi) .* (abs(sigma1v - sigma2v .*((1. / cos(theta)) ...
    + .25 * sigma2v .* (sigma3v + sigma4v)) .* (sigma5v).^-1)).^2 + eps;
% compute horizontal polarization RCS
rcs_h = (b^2 / pi) .* (abs(sigma1h - sigma2h .*((1. / cos(theta)) ...
    - .25 * sigma2h .* (sigma3h + sigma4h)) .* (sigma5h).^-1)).^2 + eps;
% Compute RCS from Physical Optics, Eq.(2.62)
angle = ka .* sin(theta);
rcs_po = (4. * pi* a^2 * b^2 / lambda^2) .* (cos(theta)).^2 .* ...
    ((sin(angle) ./ angle).^2) + eps;
rcsdb_v = 10. .*log10(rcs_v);
rcsdb_h = 10. .*log10(rcs_h);
rcsdb_po = 10. .*log10(rcs_po);
subplot(1,2,1)
plot(theta_deg, rcsdb_v,'k',theta_deg,rcsdb_po,'k --');
set(gca,'xtick',[10:10:85]);
title('Vertical polarization');
ylabel('RCS -dBsm');
xlabel('aspect angle - deg');
legend('Solid Eq.(2.51)', 'Dashed Eq.(2.62)');
subplot(1,2,2)
plot(theta_deg, rcsdb_h,'k',theta_deg,rcsdb_po,'k --');
set(gca,'xtick',[10:10:85]);
title('Horizontal polarization');
ylabel('RCS -dBsm');
xlabel('aspect angle - deg');
xlabel('aspect angle - deg');
legend('Solid eq.(2.50)', 'Dashed eq.(2.62)');

```

Listing 2.9. MATLAB Function “rcs_isosceles.m”

```

function [rcs] = rcs_isosceles (a, b, freq, phi)
% This program calculates the backscattered RCS for a perfectly

```

```

% conducting triangular flat plate, using Eq.s (2.63) through (2.65)
% The default case is to assume phi = pi/2. These equations are
% valid for aspect angles less than 30 degrees
% compute area of plate
A = a * b / 2.;
lambda = 3.e+8 / 9.5e+8;
phi = pi / 2.;
ka = 2. * pi / lambda;
kb = 2. * pi / lambda;
% Compute theta vector
theta_deg = 0.01:.05:89;
theta = (pi / 180.) .* theta_deg;
alpha = ka * cos(phi) .* sin(theta);
beta = kb * sin(phi) .* sin(theta);
if (phi == pi / 2)
    rcs = (4. * pi * A^2 / lambda^2) .* cos(theta).^2 .* (sin(beta ./ 2)).^4 ...
        ./ (beta./2).^4 + eps;
end
if (phi == 0)
    rcs = (4. * pi * A^2 / lambda^2) .* cos(theta).^2 .* ...
        ((sin(alpha).^4 ./ alpha.^4) + (sin(2 .* alpha) - 2.*alpha).^2 ...
        ./ (4 .* alpha.^4)) + eps;
end
if (phi ~= 0 & phi ~= pi/2)
    sigmao1 = 0.25 * sin(phi)^2 .* ((2. * a / b) * cos(phi) .* ...
        sin(beta) - sin(phi) .* sin(2. .* alpha)).^2;
    fact1 = (alpha).^2 - (.5 .* beta).^2;
    fact2 = (sin(alpha).^2 - sin(.5 .* beta).^2).^2;
    sigmao = (fact2 + sigmao1) ./ fact1;
    rcs = (4. * pi * A^2 / lambda^2) .* cos(theta).^2 .* sigmao + eps;
end
rcsdb = 10. * log10(rcs);
plot(theta_deg,rcsdb,'k')
xlabel ('Aspect angle - degrees');
ylabel ('RCS - dBsm')
%title ('freq = 9.5GHz, phi = pi/2');
grid;

```

Listing 2.10. MATLAB Program “rcs_cylinder_complex.m”

% This program computes the backscattered RCS for a cylinder
% with flat plates.

```

clear all
index = 0;
eps = 0.00001;
a1 = 125;
h = 1.;

```

```

lambda = 3.0e+8 / 9.5e+9;
lambda = 0.00861;
index = 0;
for theta = 0.0:1:90-.1
    index = index + 1;
    theta = theta * pi / 180.;
    rcs(index) = (lambda * a1 * sin(theta) / ...
        (8 * pi * (cos(theta))^2)) + eps;
end
theta*180/pi;
theta = pi/2;
index = index + 1;
rcs(index) = (2 * pi * h^2 * a1 / lambda) + eps;
for theta = 90+.1:1:180.
    index = index + 1;
    theta = theta * pi / 180.;
    rcs(index) = ( lambda * a1 * sin(theta) / ...
        (8 * pi * (cos(theta))^2)) + eps;
end
r = a1;
index = 0;
for aspect_deg = 0.:1:180
    index = index + 1;
    aspect = (pi / 180.) * aspect_deg;
% Compute RCS using Eq. (2.37)
if (aspect == 0 | aspect == pi)
    rcs_po(index) = (4.0 * pi^3 * r^4 / lambda^2) + eps;
    rcs_mu(index) = rcs_po(1);
else
    x = (4. * pi * r / lambda) * sin(aspect);
    val1 = 4. * pi^3 * r^4 / lambda^2;
    val2 = 2. * besselj(1,x) / x;
    rcs_po(index) = val1 * (val2 * cos(aspect))^2 + eps;
end
end
rcs_t=(rcs_po + rcs);
angle = 0:1:180;
plot(angle,10*log10(rcs_t(1:1801)), 'k');
grid;
xlabel ('Aspect angle -degrees');
ylabel ('RCS -dBsm');

```

Listing 2.11. MATLAB Program “Swerling_models.m”

```

% This program computes and plots Swerling statistical models
% sigma_bar = 1.5;
clear all

```

```

sigma = 0:0.001:6;
sigma_bar = 1.5;
swer_3_4 = (4. / sigma_bar^2) .* sigma .* ...
    exp(-2. * (sigma ./ sigma_bar));
%t.*exp(-(t.^2)/2.
swer_1_2 = (1. / sigma_bar) .* exp(-sigma ./ sigma_bar);
plot(sigma,swer_1_2,'k',sigma,swer_3_4,'k');
grid;
gtext ('Swerling I,II');
gtext ('Swerling III,IV');
xlabel ('sigma');
ylabel ('Probability density');
title ('sigma-bar = 1.5');

```

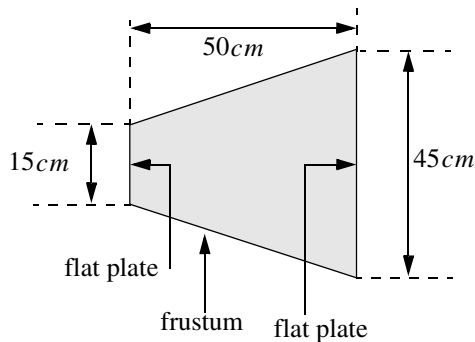
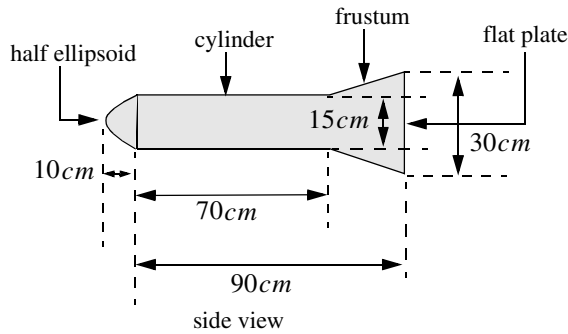
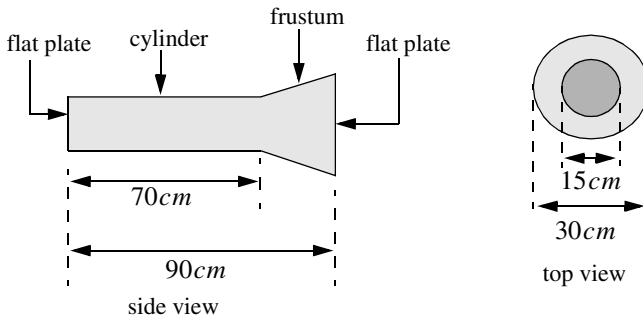
Problems

2.1. Design a cylindrical RCS calibration target such that its broadside RCS (cylinder) and end (flat plate) RCS are equal to $10m^2$ at $f = 9.5GHz$. The RCS for a flat plate of area A is $\sigma_{fp} = 4\pi f^2 A^2 / c^2$.

2.2. The following table is constructed from a radar cross-section measurement experiment. Calculate the mean and standard deviation of the radar cross section.

Number of samples	RCS, m²
2	55
6	67
12	73
16	90
20	98
24	110
26	117
19	126
13	133
8	139
5	144
3	150

2.3. Develop a MATLAB simulation to compute and plot the backscattered RCS for the following objects. Utilize the simple shape MATLAB functions developed in this chapter. Assume that the radar is located on the left side of the page and that its line of sight is aligned with the target body axis. Assume an X-band radar.

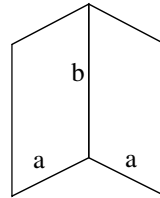
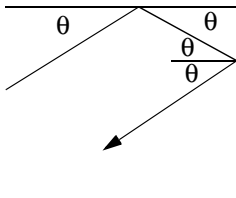


2.4. The backscattered RCS for a corner reflector is given by

$$\sigma = \left[\sqrt{\frac{16\pi a^4}{\lambda^2} (\sin \theta)^2} + \sqrt{\frac{4\pi a^4}{\lambda^2} \left(\frac{\sin\left(\frac{2\pi a}{\lambda} \sin \theta\right)}{\frac{2\pi a}{\lambda} \sin \theta} \right)^2} \right]^2 \quad 0^\circ \leq \theta \leq 45^\circ$$

This RCS is symmetric about the angle $\theta = 45^\circ$. Develop a MATLAB program to compute and plot the RCS for a corner reflector. The RCS at the $\theta = 45^\circ$ is

$$\sigma = \frac{8\pi a^2 b^2}{\lambda^2}$$



corner reflector