Chapter 3

Continuous Wave and Pulsed Radars

Continuous Wave (CW) radars utilize CW waveforms, which may be considered to be a pure sinewave of the form $\cos 2\pi f_0 t$. Spectra of the radar echo from stationary targets and clutter will be concentrated at f_0. The center frequency for the echoes from moving targets will be shifted by f_d, the Doppler frequency. Thus by measuring this frequency difference CW radars can very accurately extract target radial velocity. Because of the continuous nature of CW emission, range measurement is not possible without some modifications to the radar operations and waveforms, which will be discussed later.

3.1. Functional Block Diagram

In order to avoid interruption of the continuous radar energy emission, two antennas are used in CW radars, one for transmission and one for reception. Fig. 3.1 shows a simplified CW radar block diagram. The appropriate values of the signal frequency at different locations are noted on the diagram. The individual Narrow Band Filters (NBF) must be as narrow as possible in bandwidth in order to allow accurate Doppler measurements and minimize the amount of noise power.

In theory, the operating bandwidth of a CW radar is infinitesimal (since it corresponds to an infinite duration continuous sinewave). However, systems with infinitesimal bandwidths cannot physically exist, and thus the bandwidth of CW radars is assumed to correspond to that of a gated CW waveform (see Chapter 5).

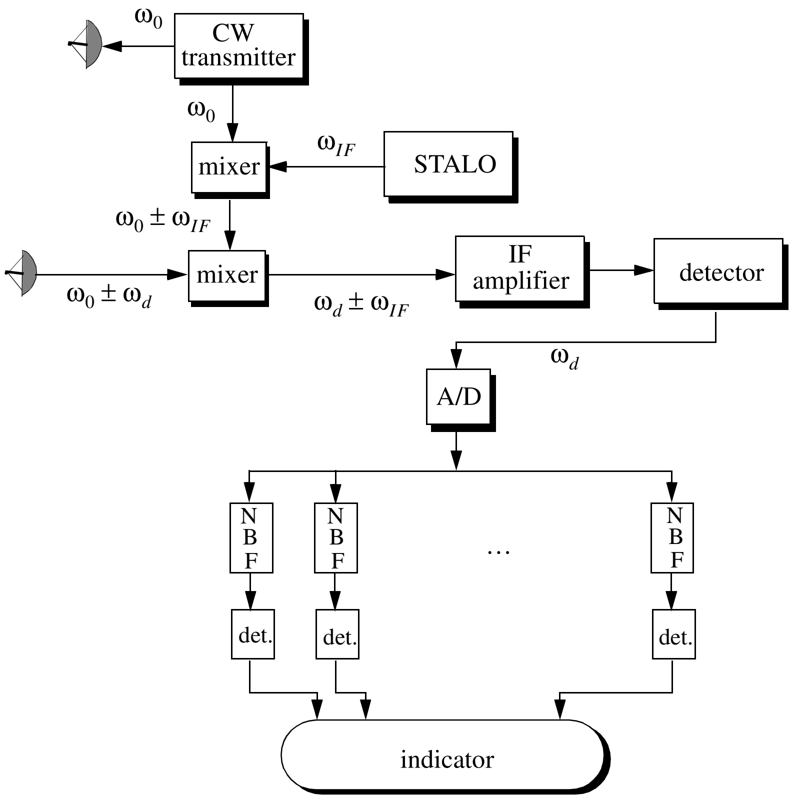


Figure 3.1. CW radar block diagram.

The NBF bank (Doppler filter bank) can be implemented using a Fast Fourier Transform (FFT). If the Doppler filter bank is implemented using an FFT of size N_{FFT}, and if the individual NBF bandwidth (FFT bin) is Δf, then the effective radar Doppler bandwidth is $N_{FFT}\Delta f/2$. The reason for the one-half factor is to account for both negative and positive Doppler shifts.

Since range is computed from the radar echoes by measuring a two-way time delay, then single frequency CW radars cannot measure target range. In order for CW radars to be able to measure target range, the transmit and receive waveforms must have some sort of timing marks. By comparing the timing marks at transmit and receive, CW radars can extract target range.

The timing mark can be implemented by modulating the transmit waveform, and one commonly used technique is Linear Frequency Modulation (LFM). Before we discuss LFM signals, we will first introduce the CW radar equation and briefly address the general Frequency Modulated (FM) waveforms using sinusoidal modulating signals.

3.2. CW Radar Equation

As indicated by Fig. 3.1, the CW radar receiver declares detection at the output of a particular Doppler bin if that output value passes the detection threshold within the detector box. Since the NBF bank is inplemented by an FFT, only finite length data sets can be processed at a time. The length of such blocks is normally referred to as the dwell time or dwell interval. The dwell interval determines the frequency resolution or the bandwidth of the individual NBFs. More precisely,

$$\Delta f = 1/T_{Dwell} \tag{3.1}$$

T_{Dwell} is the dwell interval. Therefore, once the maximum resolvable frequency by the NBF bank is chosen the size of the NBF bank is computed as

$$N_{FFT} = 2B/\Delta f \tag{3.2}$$

B is the maximum resolvable frequency by the FFT. The factor 2 is needed to account for both positive and negative Doppler shifts. It follows that

$$T_{Dwell} = N_{FFT}/2B \tag{3.3}$$

The CW radar equation can now be derived from the high PRF radar equation given in Eq. (1.69) and repeated here as Eq. (3.4)

$$SNR = \frac{P_{av}T_iG^2\lambda^2\sigma}{(4\pi)^3R^4kT_eFL} \tag{3.4}$$

In the case of CW radars, P_{av} is replaced by the CW average transmitted power over the dwell interval P_{CW}, and T_i must be replaced by T_{Dwell}. Thus, the CW radar equation can be written as

$$SNR = \frac{P_{CW}T_{Dwell}G_tG_r\lambda^2\sigma}{(4\pi)^3R^4kT_eFLL_{win}} \tag{3.5}$$

where G_t and G_r are the transmit and receive antenna gains, respectively. The factor L_{win} is a loss term associated with the type of window (weighting) used in computing the FFT. Other terms in Eq. (3.5) have been defined earlier.

3.3. *Frequency Modulation*

The discussion presented in this section will be restricted to sinusoidal modulating signals. In this case, the general formula for an FM waveform can be expressed by

$$s(t) = A\cos\left(2\pi f_0 t + k_f \int_0^t \cos 2\pi f_m u\,du\right) \tag{3.6}$$

f_0 is the radar operating frequency (carrier frequency), $\cos 2\pi f_m t$ is the modulating signal, A is a constant, and $k_f = 2\pi\Delta f_{peak}$, where Δf_{peak} is the peak frequency deviation. The phase is given by

$$\psi(t) = 2\pi f_0 t + 2\pi\Delta f_{peak}\int_0^t \cos 2\pi f_m u\,du = 2\pi f_0 t + \beta \sin 2\pi f_m t \tag{3.7}$$

where β is the FM modulation index given by

$$\beta = \frac{\Delta f_{peak}}{f_m} \tag{3.8}$$

Let $s_r(t)$ be the received radar signal from a target at range R. It follows that

$$s_r(t) = A_r \cos(2\pi f_0 (t - \Delta t) + \beta \sin 2\pi f_m (t - \Delta t)) \tag{3.9}$$

where the delay Δt is

$$\Delta t = \frac{2R}{c} \tag{3.10}$$

c is the speed of light. CW radar receivers utilize phase detectors in order to extract target range from the instantaneous frequency, as illustrated in Fig. 3.2. A good measurement of the phase detector output $o(t)$ implies a good measurement of Δt, and hence range.

Figure 3.2. Extracting range from an FM signal return. K_1 is a constant.

Consider the FM waveform $s(t)$ given by

$$s(t) = A\cos(2\pi f_0 t + \beta \sin 2\pi f_m t) \tag{3.11}$$

which can be written as

$$s(t) = ARe\{e^{j2\pi f_0 t} \; e^{j\beta \sin 2\pi f_m t}\} \tag{3.12}$$

where $Re\{\ \cdot\ \}$ denotes the real part. Since the signal $\exp(j\beta \sin 2\pi f_m t)$ is periodic with period $T = 1/f_m$, it can be expressed using the complex exponential Fourier series as

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi f_m t} \tag{3.13}$$

where the Fourier series coefficients C_n are given by

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin 2\pi f_m t} \; e^{-jn2\pi f_m t} \; dt \tag{3.14}$$

Make the change of variable $u = 2\pi f_m t$, and recognize that the Bessel function of the first kind of order n is

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin u - nu)} \; du \tag{3.15}$$

Thus, the Fourier series coefficients are $C_n = J_n(\beta)$, and consequently Eq. (3.13) can now be written as

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t} \tag{3.16}$$

which is known as the Bessel-Jacobi equation. Fig. 3.3 shows a plot of Bessel functions of the first kind for $n = 0, 1, 2, 3$.

The total power in the signal $s(t)$ is

$$P = \frac{1}{2}A^2 \sum_{n=-\infty}^{\infty} |J_n(\beta)|^2 = \frac{1}{2}A^2 \tag{3.17}$$

Substituting Eq. (3.16) into Eq. (3.12) yields

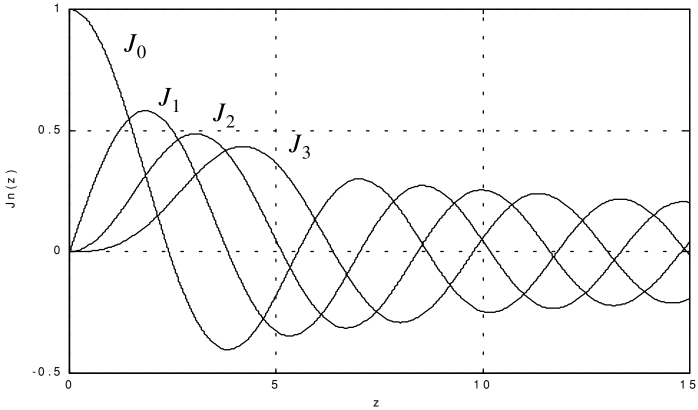


Figure 3.3. Plot of Bessel functions of order 0, 1, 2, and 3.

$$s(t) = ARe\left\{ e^{j2\pi f_0 t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t} \right\} \tag{3.18}$$

Expanding Eq. (3.18) yields

$$s(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi f_0 + n2\pi f_m)t \tag{3.19}$$

Finally, since $J_n(\beta) = J_{-n}(\beta)$ for n odd and $J_n(\beta) = -J_{-n}(\beta)$ for n even we can rewrite Eq. (3.19) as

$$\begin{aligned} s(t) = A\{&J_0(\beta)\cos 2\pi f_0 t + \\ &J_1(\beta)[\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t] \\ &+ J_2(\beta)[\cos(2\pi f_0 + 4\pi f_m)t + \cos(2\pi f_0 - 4\pi f_m)t] \\ &+ J_3(\beta)[\cos(2\pi f_0 + 6\pi f_m)t - \cos(2\pi f_0 - 6\pi f_m)t] \\ &+ J_4(\beta)[\cos((2\pi f_0 + 8\pi f_m)t + \cos(2\pi f_0 - 8\pi f_m)t)] + \ldots\} \end{aligned} \tag{3.20}$$

The spectrum of $s(t)$ is composed of pairs of spectral lines centered at f_0, as sketched in Fig. 3.4. The spacing between adjacent spectral lines is f_m. The central spectral line has an amplitude equal to $AJ_o(\beta)$, while the amplitude of the nth spectral line is $AJ_n(\beta)$.

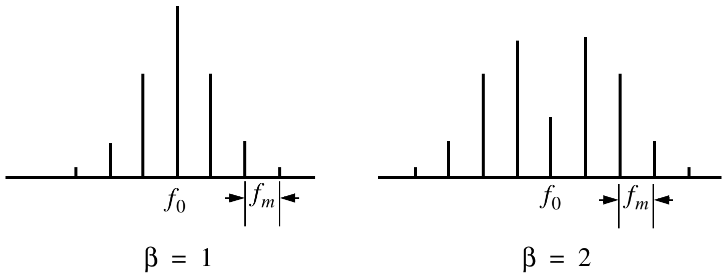


Figure 3.4. Amplitude line spectra sketch for FM signal.

As indicated by Eq. (3.20) the bandwidth of FM signals is infinite. However, the magnitudes of spectral lines of the higher orders are small, and thus the bandwidth can be approximated using Carson's rule,

$$B \approx 2(\beta + 1)f_m \tag{3.21}$$

When β is small, only $J_0(\beta)$ and $J_1(\beta)$ have significant values. Thus, we may approximate Eq. (3.20) by

$$\begin{aligned} s(t) \approx A\{&J_0(\beta)\cos 2\pi f_0 t + J_1(\beta) \\ &[\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t]\} \end{aligned} \tag{3.22}$$

Finally, for small β, the Bessel functions can be approximated by

$$J_0(\beta) \approx 1 \tag{3.23}$$

$$J_1(\beta) \approx \frac{1}{2}\beta \tag{3.24}$$

Thus, Eq. (3.22) may be approximated by

$$s(t) \approx A\left\{\cos 2\pi f_0 t + \frac{1}{2}\beta[\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t]\right\} \tag{3.25}$$

Example 3.1: If the modulation index is $\beta = 0.5$*, give an expression for the signal* $s(t)$.

Solution: From Bessel function tables we get $J_0(0.5) = 0.9385$ *and* $J_1(0.5) = 0.2423$*; then using Eq. (3.17) we get*

$$s(t) \approx A\{(0.9385)\cos 2\pi f_0 t + (0.2423) \\ [\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t]\} .$$

Example 3.2: Consider an FM transmitter with output signal $s(t) = 100\cos(2000\pi t + \varphi(t))$. *The frequency deviation is* $4Hz$, *and the modulating waveform is* $x(t) = 10\cos 16\pi t$. *Determine the FM signal bandwidth. How many spectral lines will pass through a band pass filter whose bandwidth is* $58Hz$ *centered at* $1000Hz$ *?*

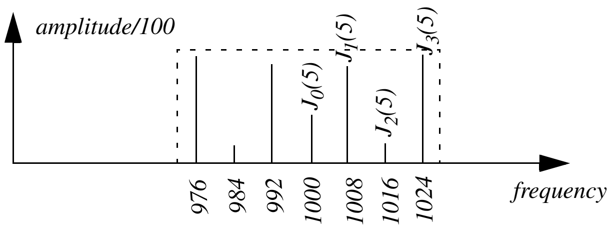
Solution: The peak frequency deviation is $\Delta f_{peak} = 4 \times 10 = 40Hz$. *It follows that*

$$\beta = \frac{\Delta f_{peak}}{f_m} = \frac{40}{8} = 5$$

Using Eq. (3.16) we get

$$B \approx 2(\beta + 1)f_m = 2 \times (5 + 1) \times 8 = 96Hz$$

However, only seven spectral lines pass through the band pass filter as illustrated in the figure shown below.



3.4. Linear FM (LFM) CW Radar

CW radars may use LFM waveforms so that both range and Doppler information can be measured. In practical CW radars, the LFM waveform cannot be continually changed in one direction, and thus periodicity in the modulation is normally utilized. Fig. 3.5 shows a sketch of a triangular LFM waveform. The modulation does not need to be triangular; it may be sinusoidal, saw-tooth, or some other form. The dashed line in Fig 3.5 represents the return waveform from a stationary target at range R. The beat frequency f_b is also sketched in Fig. 3.5. It is defined as the difference (due to heterodyning) between the transmitted and received signals. The time delay Δt is a measure of target range, as defined in Eq. (3.10).

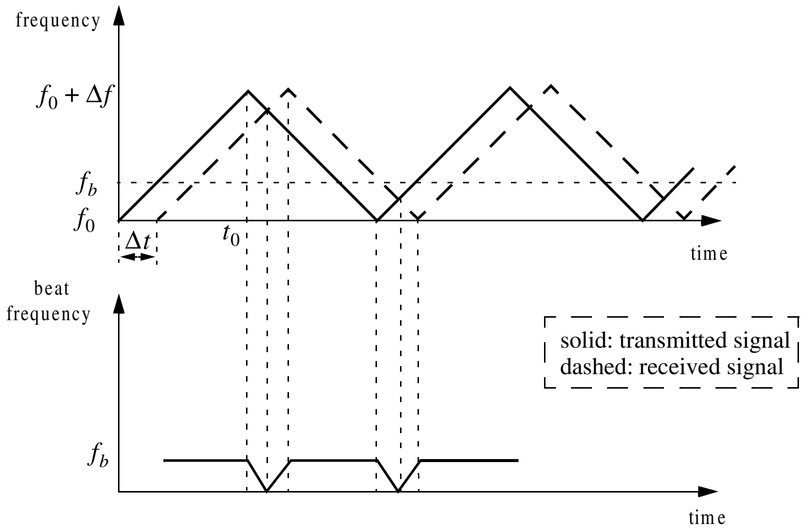


Figure 3.5. **Transmitted and received triangular LFM signals and beat frequency for stationary target.**

In practice, the modulating frequency f_m is selected such that

$$f_m = \frac{1}{2t_0} \tag{3.26}$$

The rate of frequency change, $\dot{f}$, is

$$\dot{f} = \frac{\Delta f}{t_0} = \frac{\Delta f}{(1/2f_m)} = 2f_m\Delta f \tag{3.27}$$

where Δf is the peak frequency deviation. The beat frequency f_b is given by

$$f_b = \Delta t\dot{f} = \frac{2R}{c}\dot{f} \tag{3.28}$$

Eq. (3.28) can be rewritten as

$$\dot{f} = \frac{c}{2R} f_b \tag{3.29}$$

Equating Eqs. (3.27) and (3.29) and solving for f_b yield

$$f_b = \frac{4Rf_m\Delta f}{c} \tag{3.30}$$

Now consider the case when Doppler is present (i.e., non-stationary target). The corresponding triangular LFM transmitted and received waveforms are sketched in Fig. 3.6, along with the corresponding beat frequency. As before the beat frequency is defined as

$$f_b = f_{received} - f_{transmitted} \tag{3.31}$$

When the target is not stationary the received signal will contain a Doppler shift term in addition to the frequency shift due to the time delay Δt. In this case, the Doppler shift term subtracts from the beat frequency during the positive portion of the slope. Alternatively, the two terms add up during the negative portion of the slope. Denote the beat frequency during the positive (up) and negative (down) portions of the slope, respectively, as f_{bu} and f_{bd}.

It follows that

$$f_{bu} = \frac{2R}{c}\dot{f} - \frac{2\dot{R}}{\lambda} \tag{3.32}$$

where $\dot{R}$ is the range rate or the target radial velocity as seen by the radar. The first term of the right-hand side of Eq. (3.32) is due to the range delay defined by Eq. (3.28), while the second term is due to the target Doppler. Similarly,

$$f_{bd} = \frac{2R}{c}\dot{f} + \frac{2\dot{R}}{\lambda} \tag{3.33}$$

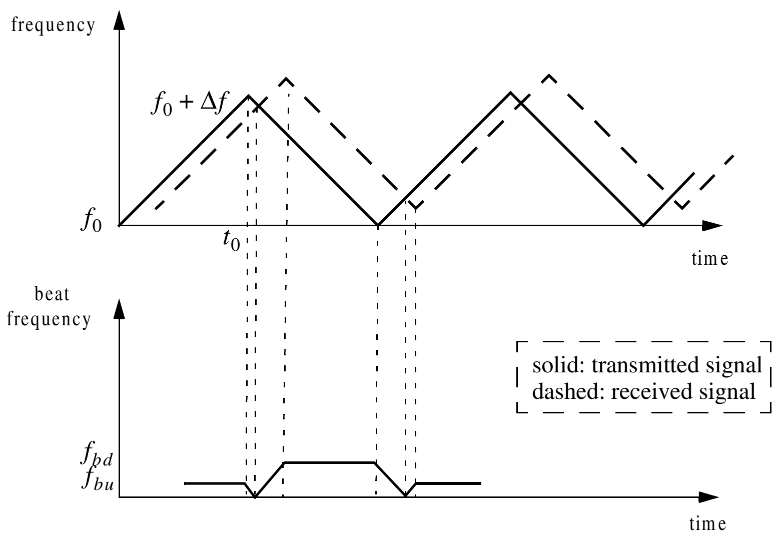


Figure 3.6. Transmited and received LFM signals and beat frequency, for a moving target.

Range is computed by adding Eq. (3.32) and Eq. (3.33). More precisely,

$$R = \frac{c}{4\dot{f}}(f_{bu} + f_{bd}) \tag{3.34}$$

The range rate is computed by subtracting Eq. (3.33) from Eq. (3.32),

$$\dot{R} = \frac{\lambda}{4}(f_{bd} - f_{bu}) \tag{3.35}$$

As indicated by Eq. (3.34) and Eq. (3.35), CW radars utilizing triangular LFM can extract both range and range rate information. In practice, the maximum time delay Δt_{max} is normally selected as

$$\Delta t_{max} = 0.1 t_0 \tag{3.36}$$

Thus, the maximum range is given by

$$R_{max} = \frac{0.1 c t_0}{2} = \frac{0.1 c}{4 f_m} \tag{3.37}$$

and the maximum unambiguous range will correspond to a shift equal to $2t_0$.

3.5. *Multiple Frequency CW Radar*

CW radars do not have to use LFM waveforms in order to obtain good range measurements. Multiple frequency schemes allow CW radars to compute very adequate range measurements, without using frequency modulation. In order to illustrate this concept, first consider a CW radar with the following waveform:

$$s(t) = A \sin 2\pi f_0 t \tag{3.38}$$

The received signal from a target at range R is

$$s_r(t) = A_r \sin(2\pi f_0 t - \varphi) \tag{3.39}$$

where the phase φ is equal to

$$\varphi = 2\pi f_0 \frac{2R}{c} \tag{3.40}$$

Solving for R we obtain

$$R = \frac{c\varphi}{4\pi f_0} = \frac{\lambda}{4\pi}\varphi \tag{3.41}$$

Clearly, the maximum unambiguous range occurs when φ is maximum, i.e., $\varphi = 2\pi$. Therefore, even for relatively large radar wavelengths, R is limited to impractical small values.

Next, consider a radar with two CW signals, denoted by $s_1(t)$ and $s_2(t)$. More precisely,

$$s_1(t) = A_1 \sin 2\pi f_1 t \tag{3.42}$$

$$s_2(t) = A_2 \sin 2\pi f_2 t \tag{3.43}$$

The received signals from a moving target are

$$s_{1r}(t) = A_{r1} \sin(2\pi f_1 t - \varphi_1) \tag{3.44}$$

and

$$s_{2r}(t) = A_{r2} \sin(2\pi f_2 t - \varphi_2) \tag{3.45}$$

where $\varphi_1 = (4\pi f_1 R)/c$ and $\varphi_2 = (4\pi f_2 R)/c$. After heterodyning (mixing) with the carrier frequency, the phase difference between the two received signals is

$$\varphi_2 - \varphi_1 = \Delta\varphi = \frac{4\pi R}{c}(f_2 - f_1) = \frac{4\pi R}{c}\Delta f \tag{3.46}$$

Again R is maximum when $\Delta\varphi = 2\pi$; it follows that the maximum unambiguous range is now

$$R = \frac{c}{2\Delta f} \tag{3.47}$$

and since $\Delta f \ll c$, the range computed by Eq. (3.47) is much greater than that computed by Eq. (3.41).

3.6. Pulsed Radar

Pulsed radars transmit and receive a train of modulated pulses. Range is extracted from the two-way time delay between a transmitted and received pulse. Doppler measurements can be made in two ways. If accurate range measurements are available between consecutive pulses, then Doppler frequency can be extracted from the range rate $\dot{R} = \Delta R / \Delta t$. This approach works fine as long as the range is not changing drastically over the interval Δt. Otherwise, pulsed radars utilize a Doppler filter bank.

Pulsed radar waveforms can be completely defined by the following: (1) carrier frequency which may vary depending on the design requirements and radar mission; (2) pulse width, which is closely related to the bandwidth and defines the range resolution; (3) modulation; and finally (4) the pulse repetition frequency. Different modulation techniques are usually utilized to enhance the radar performance, or to add more capabilities to the radar that otherwise would not have been possible. The PRF must be chosen to avoid Doppler and range ambiguities as well as maximize the average transmitted power.

Radar systems employ low, medium, and high PRF schemes. Low PRF waveforms can provide accurate, long, unambiguous range measurements, but exert severe Doppler ambiguities. Medium PRF waveforms must resolve both range and Doppler ambiguities; however, they provide adequate average transmitted power as compared to low PRFs. High PRF waveforms can provide superior average transmitted power and excellent clutter rejection capabilities. Alternatively, high PRF waveforms are extremely ambiguous in range. Radar systems utilizing high PRFs are often called Pulsed Doppler Radars (PDR). Range and Doppler ambiguities for different PRFs are summarized in Table 3.1.

Distinction of a certain PRF as low, medium, or high PRF is almost arbitrary and depends on the radar mode of operations. For example, a $3KHz$ PRF is considered low if the maximum detection range is less than $30Km$. However, the same PRF would be considered medium if the maximum detection range is well beyond $30Km$.

Radars can utilize constant and varying (agile) PRFs. For example, Moving Target Indicator (MTI) radars use PRF agility to avoid blind speeds. This kind of agility is known as PRF staggering. PRF agility is also used to avoid range and Doppler ambiguities, as will be explained in the next three sections. Additionally, PRF agility is also used to prevent jammers from locking onto the radar's PRF. These two latter forms of PRF agility are sometimes referred to as PRF jitter.

TABLE 3.1. PRF ambiguities.

PRF	Range Ambiguous	Doppler Ambiguous
Low PRF	No	Yes
Medium PRF	Yes	Yes
High PRF	Yes	No

Fig. 3.7 shows a simplified pulsed radar block diagram. The range gates can be implemented as filters that open and close at time intervals that correspond to the detection range. The width of such an interval corresponds to the desired range resolution. The radar receiver is often implemented as a series of contiguous (in time) range gates, where the width of each gate is matched to the radar pulse width. The NBF bank is normally implemented using an FFT, where

bandwidth of the individual filters corresponds to the FFT frequency resolution.

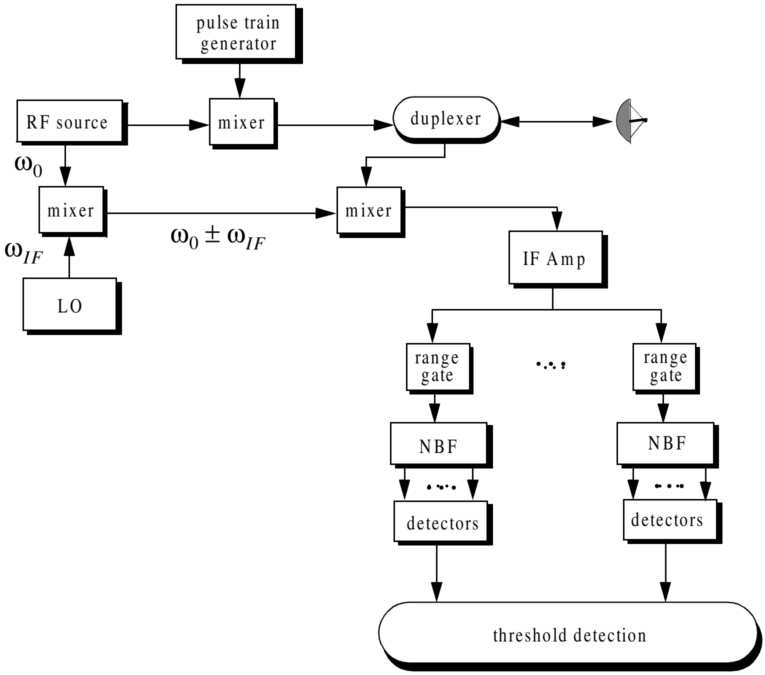


Figure 3.7. Pulsed radar block diagram.

3.7. Range and Doppler Ambiguities

As explained earlier, a pulsed radar can be range ambiguous if a second pulse is transmitted prior to the return of the first pulse. In general, the radar PRF is chosen such that the unambiguous range is large enough to meet the radar's operational requirements. Therefore, long-range search (surveillance) radars would require relatively low PRFs.

The line spectrum of a train of pulses has $\sin x/x$ envelope, and the line spectra are separated by the PRF, f_r, as illustrated in Fig. 3.8. The Doppler filter bank is capable of resolving target Doppler as long as the anticipated Doppler shift is less than one half the bandwidth of the individual filters (i.e., one half the width of an FFT bin). Thus, pulsed radars are designed such that

$$f_r = 2f_{dmax} = \frac{2v_{rmax}}{\lambda} \tag{3.48}$$

where f_{dmax} is the maximum anticipated target Doppler frequency, v_{rmax} is the maximum anticipated target radial velocity, and λ is the radar wavelength.

If the Doppler frequency of the target is high enough to make an adjacent spectral line move inside the Doppler band of interest, the radar can be Doppler ambiguous. Therefore, in order to avoid Doppler ambiguities, radar systems require high PRF rates when detecting high speed targets. When a long-range radar is required to detect a high speed target, it may not be possible to be both range and Doppler unambiguous. This problem can be resolved by using multiple PRFs. Multiple PRF schemes can be incorporated sequentially within each dwell interval (scan or integration frame) or the radar can use a single PRF in one scan and resolve ambiguity in the next. The latter technique, however, may have problems due to changing target dynamics from one scan to the next.

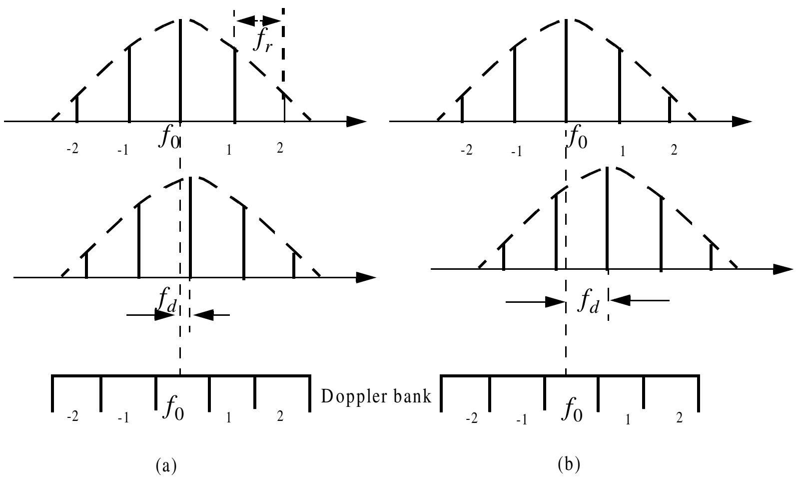


Figure 3.8. Spectra of transmitted and received waveforms, and Doppler bank. (a) Doppler is resolved. (b) Spectral lines have moved into the next Doppler filter. This results in an ambiguous Doppler measurement.

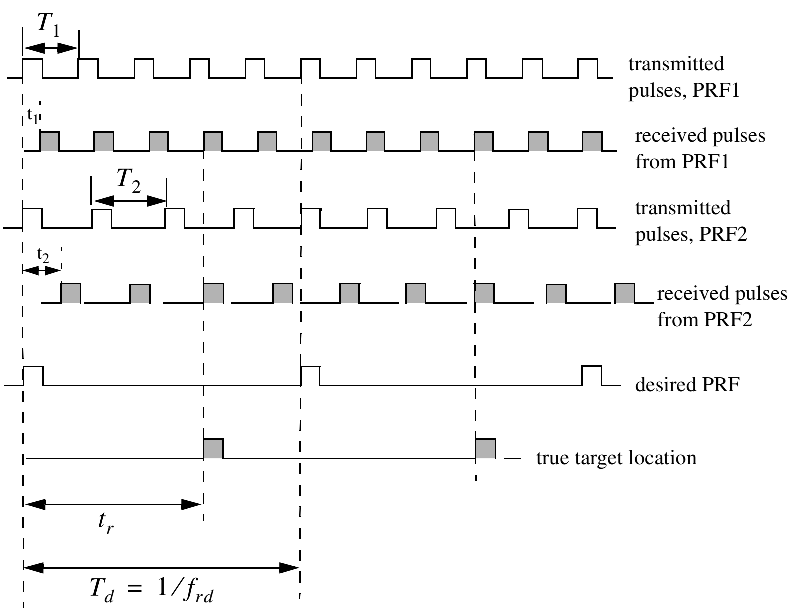
3.8. Resolving Range Ambiguity

Consider a radar that uses two PRFs, f_{r1} and f_{r2}, on transmit to resolve range ambiguity, as shown in Fig. 3.9. Denote R_{u1} and R_{u2} as the unambiguous ranges for the two PRFs, respectively. Normally, these unambiguous ranges are relatively small and are short of the desired radar unambiguous range R_u (where $R_u \gg R_{u1}, R_{u2}$). Denote the radar desired PRF that corresponds to R_u as f_{rd}.

We choose f_{r1} and f_{r2} such that they are relatively prime with respect to one another. One choice is to select $f_{r1} = Nf_{rd}$ and $f_{r2} = (N+1)f_{rd}$ for some integer N. Within one period of the desired PRI ($T_d = 1/f_{rd}$) the two PRFs f_{r1} and f_{r2} coincide only at one location, which is the true unambiguous target position. The time delay T_d establishes the desired unambiguous range. The time delays t_1 and t_2 correspond to the time between the transmit of a pulse on each PRF and receipt of a target return due to the same pulse.

Let M_1 be the number of PRF1 intervals between transmit of a pulse and receipt of the true target return. The quantity M_2 is similar to M_1 except it is for PRF2. It follows that, over the interval 0 to T_d, the only possible results are $M_1 = M_2 = M$ or $M_1 + 1 = M_2$. The radar needs only to measure t_1 and t_2. First, consider the case when $t_1 < t_2$. In this case,

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M}{f_{r2}} \tag{3.49}$$



Fgure 3.9. Resolving range ambiguity.

for which we get

$$M = \frac{t_2 - t_1}{T_1 - T_2} \tag{3.50}$$

where $T_1 = 1/f_{r1}$ and $T_2 = 1/f_{r2}$. It follows that the round trip time to the true target location is

$$t_r = MT_1 + t_1$$
$$t_r = MT_2 + t_2 \tag{3.51}$$

and the true target range is

$$R = ct_r/2 \tag{3.52}$$

Now if $t_1 > t_2$, then

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M+1}{f_{r2}} \tag{3.53}$$

Solving for M we get

$$M = \frac{(t_2 - t_1) + T_2}{T_1 - T_2} \tag{3.54}$$

and the round-trip time to the true target location is

$$t_{r1} = MT_1 + t_1 \tag{3.55}$$

and in this case, the true target range is

$$R = \frac{ct_{r1}}{2} \tag{3.56}$$

Finally, if $t_1 = t_2$, then the target is in the first ambiguity. It follows that

$$t_{r2} = t_1 = t_2 \tag{3.57}$$

and

$$R = ct_{r2}/2 \tag{3.58}$$

Since a pulse cannot be received while the following pulse is being transmitted, these times correspond to blind ranges. This problem can be resolved by using a third PRF. In this case, once an integer N is selected, then in order to guarantee that the three PRFs are relatively prime with respect to one another, we may choose $f_{r1} = N(N+1)f_{rd}$, $f_{r2} = N(N+2)f_{rd}$, and $f_{r3} = (N+1)(N+2)f_{rd}$.

3.9. Resolving Doppler Ambiguity

The Doppler ambiguity problem is analogous to that of range ambiguity. Therefore, the same methodology can be used to resolve Doppler ambiguity. In this case, we measure the Doppler frequencies f_{d1} and f_{d2} instead of t_1 and t_2.

If $f_{d1} > f_{d2}$, then we have

$$M = \frac{(f_{d2} - f_{d1}) + f_{r2}}{f_{r1} - f_{r2}} \tag{3.59}$$

And if $f_{d1} < f_{d2}$,

$$M = \frac{f_{d2} - f_{d1}}{f_{r1} - f_{r2}} \tag{3.60}$$

and the true Doppler is

$$\begin{aligned} f_d &= M f_{r1} + f_{d1} \\ f_d &= M f_{r2} + f_{d2} \end{aligned} \tag{3.61}$$

Finally, if $f_{d1} = f_{d2}$, then

$$f_d = f_{d1} = f_{d2} \tag{3.62}$$

Again, blind Dopplers can occur, which can be resolved using a third PRF.

Example 3.3: A certain radar uses two PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 100Km$. Choose $N = 59$. Compute f_{r1}, f_{r2}, R_{u1}, and R_{u2}.

Solution: First let us compute the desired PRF, f_{rd}

$$f_{rd} = \frac{c}{2R_u} = \frac{3 \times 10^8}{200 \times 10^3} = 1.5KHz$$

It follows that

$$f_{r1} = N f_{rd} = (59)(1500) = 88.5KHz$$

$$f_{r2} = (N+1) f_{rd} = (59+1)(1500) = 90KHz$$

$$R_{u1} = \frac{c}{2f_{r1}} = \frac{3 \times 10^8}{2 \times 88.5 \times 10^3} = 1.695Km$$

$$R_{u2} = \frac{c}{2f_{r2}} = \frac{3 \times 10^8}{2 \times 90 \times 10^3} = 1.667 Km .$$

Example 3.4: Consider a radar with three PRFs; $f_{r1} = 15KHz$, $f_{r2} = 18KHz$, *and* $f_{r3} = 21KHz$. *Assume* $f_0 = 9GHz$. *Calculate the frequency position of each PRF for a target whose velocity is* $550m/s$. *Calculate* f_d *(Doppler frequency) for another target appearing at* $8KHz$, $2KHz$, *and* $17KHz$ *for each PRF.*

Solution: The Doppler frequency is

$$f_d = 2\frac{vf_0}{c} = \frac{2 \times 550 \times 9 \times 10^9}{3 \times 10^8} = 33KHz$$

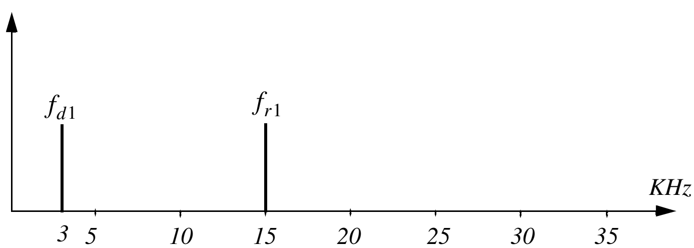
Then by using Eq. (3.61) $n_i f_{ri} + f_{di} = f_d$ *where* $i = 1, 2, 3$, *we can write*

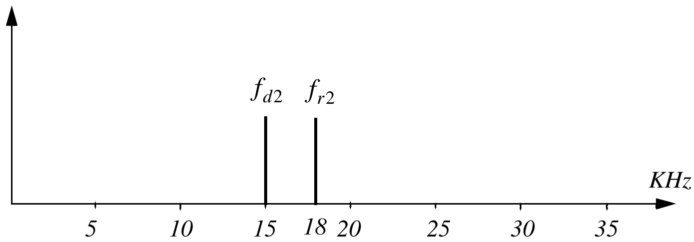
$$n_1 f_{r1} + f_{d1} = 15n_1 + f_{d1} = 33$$

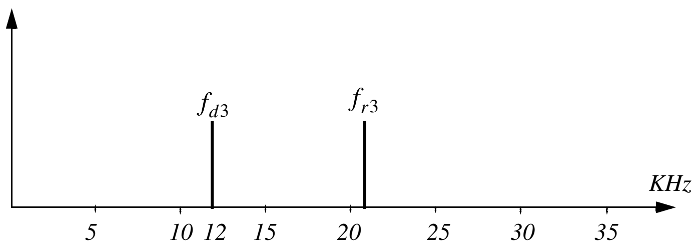
$$n_2 f_{r2} + f_{d2} = 18n_2 + f_{d2} = 33$$

$$n_3 f_{r3} + f_{d3} = 21n_3 + f_{d3} = 33$$

We will show here how to compute n_1, *and leave the computations of* n_2 *and* n_3 *to the reader. First, if we choose* $n_1 = 0$, *that means* $f_{d1} = 33KHz$, *which cannot be true since* f_{d1} *cannot be greater than* f_{r1}. *Choosing* $n_1 = 1$ *is also invalid since* $f_{d1} = 18KHz$ *cannot be true either. Finally, if we choose* $n_1 = 2$ *we get* $f_{d1} = 3KHz$, *which is an acceptable value. It follows that the minimum* n_1, n_2, n_3 *that may satisfy the above three relations are* $n_1 = 2$, $n_2 = 1$, *and* $n_3 = 1$. *Thus, the apparent Doppler frequencies are* $f_{d1} = 2KHz$, $f_{d2} = 15KHz$, *and* $f_{d3} = 12KHz$.







Now for the second part of the problem. Again by using Eq. (3.61) we have

$$n_1 f_{r1} + f_{d1} = f_d = 15n_1 + 8$$

$$n_2 f_{r2} + f_{d2} = f_d = 18n_2 + 2$$

$$n_3 f_{r3} + f_{d3} = f_d = 21n_3 + 17$$

We can now solve for the smallest integers n_1, n_2, n_3 *that satisfy the above three relations. See the table below.*

n	**0**	**1**	**2**	**3**	**4**
f_d from f_{r1}	8	23	<u>38</u>	53	68
f_d from f_{r2}	2	20	<u>38</u>	56	
f_d from f_{r3}	17	<u>38</u>	39		

Thus, $n_1 = 2 = n_2$, *and* $n_3 = 1$, *and the true target Doppler is* $f_d = 38KHz$. *It follows that*

$$v_r = 38000 \times \frac{0.0333}{2} = 632.7 \frac{m}{\text{sec}}$$

3.10. MATLAB Program "range_calc.m"

The program *"range_calc.m"* solves the radar range equation of the form

$$R = \left(\frac{P_t \tau f_r T_i G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_e F L (SNR)_o} \right)^{\frac{1}{4}} \tag{3.63}$$

where P_t is peak transmitted power, τ is pulse width, f_r is PRF, G_t is transmitting antenna gain, G_r receiving antenna gain, λ is wavelength, σ is target cross section, k is Boltzman's constant, T_e effective noise temperature, F is system noise figure, L is total system losses, and $(SNR)_o$ is the minimum SNR required for detection. This equation applies for both CW and pulsed radars. In the case of CW radars, the terms $P_t \tau f_r$ must be replaced by the average CW power P_{CW}. Additionally, the term T_i refers to the dwell interval; alternatively, in the case of pulse radars T_i denotes the time on target. MATLAB-based GUI is utilized in inputting and editing all input parameters. The outputs include the maximum detection range versus minimum SNR plots. This program can be executed by typing *"range_calc_driver"* which is included in this book's companion software. This software can be downloaded from CRC Press Web site "*www.crcpress.com*". The related MATLAB GUI workspace associated with this program is illustrated in Fig. 3.10.

Problems

3.1. Prove that

$$\sum_{n=-\infty}^{\infty} J_n(z) = 1 .$$

3.2. Show that $J_{-n}(z) = (-1)^n J_n(z)$. Hint: You may utilize the relation

$$J_n(z) = \frac{1}{\pi} \int_0^{\pi} \cos(z \sin y - ny) dy .$$

3.3. In a multiple frequency CW radar, the transmitted waveform consists of two continuous sinewaves of frequencies $f_1 = 105KHz$ and $f_2 = 115KHz$. Compute the maximum unambiguous detection range.

3.4. Consider a radar system using linear frequency modulation. Compute the range that corresponds to $\dot{f} = 20, 10MHz$. Assume a beat frequency $f_b = 1200Hz$.

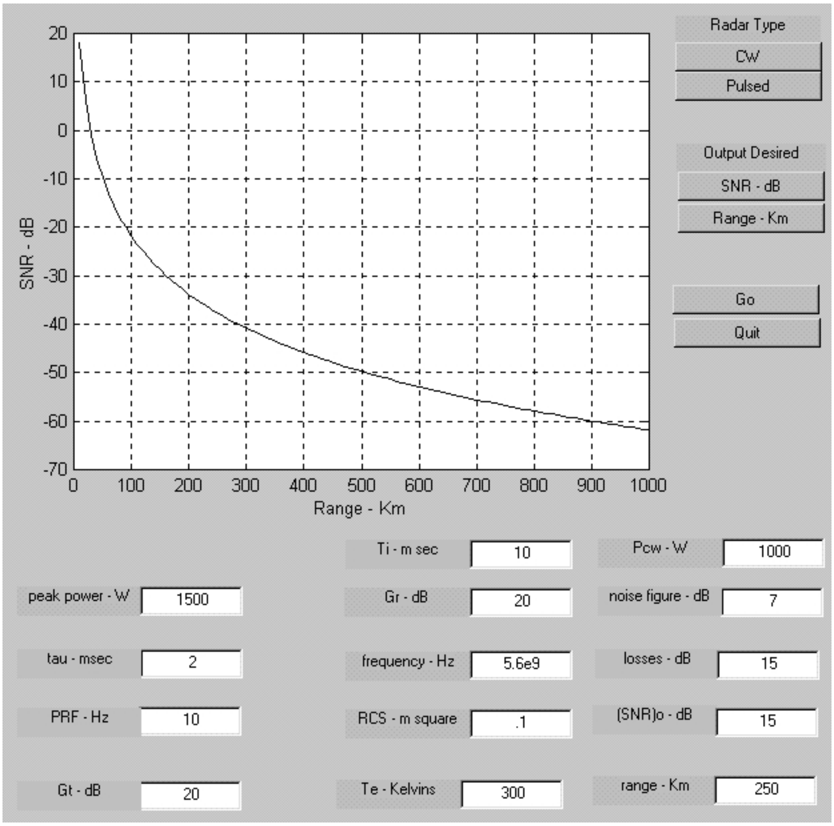
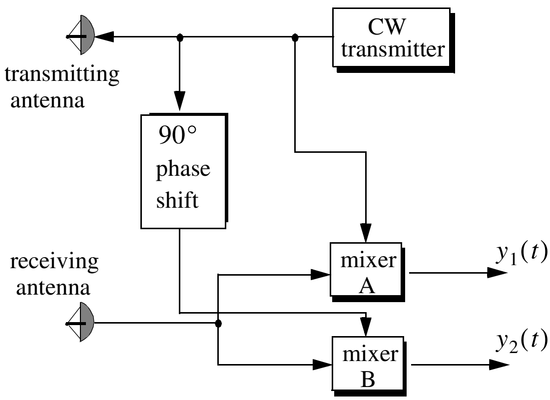


Figure 3.10. GUI work space associated with the program *"range_calc.m"*.

3.5. A certain radar using linear frequency modulation has a modulation frequency $f_m = 300Hz$, and frequency sweep $\Delta f = 50MHz$. Calculate the average beat frequency differences that correspond to range increments of 10 and 15 meters.

3.6. A CW radar uses linear frequency modulation to determine both range and range rate. The radar wavelength is $\lambda = 3cm$, and the frequency sweep is $\Delta f = 200KHz$. Let $t_0 = 20ms$. (a) Calculate the mean Doppler shift; (b) compute f_{bu} and f_{bd} corresponding to a target at range $R = 350Km$, which is approaching the radar with radial velocity of $250m/s$.

3.7. In Chapter 1 we developed an expression for the Doppler shift associated with a CW radar (i.e., $f_d = \pm 2v/\lambda$, where the plus sign is used for closing targets and the negative sign is used for receding targets). CW radars can use the system shown below to determine whether the target is closing or receding. Assuming that the emitted signal is $A\cos\omega_0 t$ and the received signal is $kA\cos((\omega_0 \pm \omega_d)t + \varphi)$, show that the direction of the target can be determined by checking the phase shift difference in the outputs $y_1(t)$ and $y_2(t)$.



3.8. Consider a medium PRF radar on board an aircraft moving at a speed of $350\ m/s$ with PRFs $f_{r1} = 10KHz$, $f_{r2} = 15KHz$, and $f_{r3} = 20KHz$; the radar operating frequency is $9.5GHz$. Calculate the frequency position of a nose-on target with a speed of $300\ m/s$. Also calculate the closing rate of a target appearing at 6, 5, and $18KHz$ away from the center line of PRF 10, 15, and $20KHz$, respectively.

3.9. Repeat Problem 3.8 when the target is $15°$ off the radar line of sight.

3.10. A certain radar operates at two PRFs, f_{r1} and f_{r2}, where $T_{r1} = (1/f_{r1}) = T/5$ and $T_{r2} = (1/f_{r2}) = T/6$. Show that this multiple PRF scheme will give the same range ambiguity as that of a single PRF with PRI T.

3.11. Consider an X-band radar with wavelength $\lambda = 3cm$ and bandwidth $B = 10MHz$. The radar uses two PRFs, $f_{r1} = 50KHz$ and $f_{r2} = 55.55KHz$. A target is detected at range bin 46 for f_{r1} and at bin 12 for f_{r2}. Determine the actual target range.

3.12. A certain radar uses two PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 150Km$. Select a reasonable value for N. Compute the corresponding f_{r1}, f_{r2}, R_{u1}, and R_{u2}.

3.13. A certain radar uses three PRFs to resolve range ambiguities. The desired unambiguous range is $R_u = 250Km$. Select $N = 43$. Compute the corresponding f_{r1}, f_{r2}, f_{r3}, R_{u1}, R_{u2}, and R_{u3}.