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*The Transforms and Applications Handbook: Second Edition.*  
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THE  

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TRANSFORMS  
AND  
APPLICATIONS  
HANDBOOK  

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SECOND EDITION

Editor-in-Chief

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# Preface

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The purpose of *The Transforms and Applications Handbook, Second Edition* is to include in a single volume the most important mathematical transforms frequently used by engineers and scientists. The book also was written with the advanced undergraduate and graduate students in mind. Each chapter covers one of the transforms, accompanied by a number of examples that are included to elucidate the use of the transform and its properties. Applications to different areas are included in each chapter as well. This inclusion gives readers of different backgrounds the opportunity to become familiar with the wide spectrum of applications of these transforms. We believe that having all of these useful transforms included in one book will be of great value to scientists, engineers, and students.

The information is now organized into 15 chapters, each covering one of the transforms, except for Chapter 1 which enhances some topics that are treated less extensively in the other chapters. Over the past 3 years, a number of communications have been received concerning different aspects of the Handbook. All of the comments regarding typographical errors have been incorporated in the second edition. The editor and the contributors wish to thank the readers for their contributions and encouragement which prompted this second edition.

In the second edition to the Handbook, we have added three new chapters: *Lapped Transforms*, *Discrete Time and Discrete Fourier Transforms*, and *Fractional Fourier Transforms*. In the original chapters, we have corrected typographical errors, replaced the table of Laplace transforms with another table containing a larger number of entries, the chapter on Mellin transforms was rewritten, the cosine and sine transforms were rewritten, and the Wavelet transforms were updated.

The Editor would be extremely grateful if the readers forwarded their opinion about the Handbook, any errors they may detect, suggestions for new material in new editions, and material that they feel may be neglected. The reader also may consult the following references:

1. Yu. A. Brychkov and A.P. Prudnikov, *Integral Transforms of Generalized Functions*, Gordon and Breach, 1989.
2. D.G. Duffy, *Transform Methods for Solving Partial Differential Equations*, CRC Press, Boca Raton, FL, 1994.
3. I.S. Gradshteyn and I.M. Ryzhik, *Tables of Integrals, Series and Products*, Academic Press, New York, 1980.
4. F. Oberhettinger, *Tables of Fourier Transforms and Fourier Transforms of Distributions*, Springer-Verlag, 1990.
5. A.P. Prudnikov, Y.A. Brychkov, and O.I. Marichev, *Integrals and Series, Direct Laplace Transforms*, Vol. 4; *Inverse Laplace Transforms*, Vol. 5, Gordon and Breach, 1992.

The Editor would like to thank Richard Dorf, the series editor, for his help. A special thanks goes to Nora Konopka, the acquiring editor of the Handbook, for her help and consistent and cheerful prodding to finish the project.

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# Editor-in-Chief

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**Alexander D. Poularikas**, as a Fulbright scholar, attended the University of Arkansas where he received his Ph.D. in Electrical Engineering in 1966. He joined the University of Rhode Island as an assistant professor of Electrical and Computer Engineering the same year and became full professor in 1974. Poularikas joined the University of Denver as chairman of the Department of Engineering in 1983, and 2 years later moved to the University of Alabama in Huntsville as chairman of its Department of Electrical and Computer Engineering.

Poularikas has visited and done scientific work at the Massachusetts Institute of Technology, the Underwater Systems Center at Newport, Goddard Spaceflight Center in Maryland, and at Stanford University. He is a senior member of IEEE, a charter member of the Arkansas Academy of Electrical Engineers, a member of Tau Beta Pi, Sigma Xi, Sigma Pi Sigma, and received the Outstanding Educator Award from the IEEE Huntsville Section in both 1990 and 1996. He also is a member of numerous professional societies.

He has published more than 60 papers in scientific and engineering magazines and published the following books:

*Electromagnetics*, Marcel Dekker, 1979

*Electrical Engineering: Introduction and Concepts*, Matrix Publishers, 1982

*Workbook*, Matrix Publishers, 1982

*Signals and Systems*, Brooks/Cole, 1985

*Elements of Signals and Systems*, PWS-Kent, 1988

*Signals and Systems, 2nd edition*, PWS-Kent, 1992

*The Transforms and Applications Handbook*, CRC Press, 1995

*The Handbook of Formulas and Tables for Signal Processing*, CRC Press, 1998

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