

Advanced  
Digital Signal  
Processing  
and Noise  
Reduction

**Second Edition**

# Advanced Digital Signal Processing and Noise Reduction

**Second Edition**

**SAEED V. VASEGHI**

*Professor of Communications and Signal Processing,  
Department of Electronics and Computer Engineering,  
Brunel University, UK*

**JOHN WILEY & SONS, LTD**

Chichester · New York · Weinheim · Brisbane · Singapore · Toronto

First Edition published in 1996 jointly by John Wiley & Sons, Ltd. and B. G. Teubner as Advanced Signal Processing and Digital Noise Reduction.

Copyright © 2000 by John Wiley & Sons, Ltd  
Baffins Lane, Chichester,  
West Sussex, PO19 1UD, England

National 01243 779777  
International (+44) 1243 779777

e-mail (for orders and customer service enquiries): [cs-books@wiley.co.uk](mailto:cs-books@wiley.co.uk)

Visit our Home Page on <http://www.wiley.co.uk> or <http://www.wiley.com>

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except under the terms of the Copyright Designs and Patents Act 1988 or under the terms of a licence issued by the Copyright Licensing Agency, 90 Tottenham Court Road, London, W1P 9HE, UK, without the permission in writing of the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the publication.

Neither the author(s) nor John Wiley & Sons Ltd accept any responsibility or liability for loss or damage occasioned to any person or property through using the material, instructions, methods or ideas contained herein, or acting or refraining from acting as a result of such use. The author(s) and Publisher expressly disclaim all implied warranties, including merchantability of fitness for any particular purpose.

Designations used by companies to distinguish their products are often claimed as trademarks. In all instances where John Wiley & Sons is aware of a claim, the product names appear in initial capital or capital letters. Readers, however, should contact the appropriate companies for more complete information regarding trademarks and registration.

### ***Other Wiley Editorial Offices***

John Wiley & Sons, Inc., 605 Third Avenue,  
New York, NY 10158-0012, USA

Weinheim • Brisbane • Singapore • Toronto

### ***Library of Congress Cataloging-in-Publication Data***

Vaseghi, Saeed V.

Advanced digital signal processing and noise reduction / Saeed V. Vaseghi.—2<sup>nd</sup> ed.  
p.cm.

Includes bibliographical references and index.

ISBN 0-471-62692-9 (alk.paper)

1. Signal processing. 2. Electronic noise. 3. Digital Filters (Mathematics) I. Title.

TK5102.9. V37 2000

621.382'2—dc21

00-032091

### ***British Library Cataloguing in Publication Data***

A catalogue record for this book is available from the British Library

ISBN 0 471 62692 9

Produced from PostScript files supplied by the author.

Printed and bound in Great Britain by Antony Rowe Ltd, Chippenham, Wiltshire.

This book is printed on acid-free paper responsibly manufactured from sustainable forestry, in which at least two trees are planted for each one used for paper production.

# **To my parents**

**With thanks to Peter Rayner, Ben Milner, Charles Ho and Aimin Chen**

# CONTENTS

<b>PREFACE .....</b>	<b>xvii</b>
<b>FREQUENTLY USED SYMBOLS AND ABBREVIATIONS.....</b>	<b>xxi</b>
<b>CHAPTER 1 INTRODUCTION.....</b>	<b>1</b>
1.1 Signals and Information .....	2
1.2 Signal Processing Methods .....	3
1.2.1 Non-parametric Signal Processing .....	3
1.2.2 Model-Based Signal Processing .....	4
1.2.3 Bayesian Statistical Signal Processing .....	4
1.2.4 Neural Networks.....	5
1.3 Applications of Digital Signal Processing .....	5
1.3.1 Adaptive Noise Cancellation and Noise Reduction .....	5
1.3.2 Blind Channel Equalisation.....	8
1.3.3 Signal Classification and Pattern Recognition .....	9
1.3.4 Linear Prediction Modelling of Speech.....	11
1.3.5 Digital Coding of Audio Signals .....	12
1.3.6 Detection of Signals in Noise.....	14
1.3.7 Directional Reception of Waves: Beam-forming .....	16
1.3.8 Dolby Noise Reduction .....	18
1.3.9 Radar Signal Processing: Doppler Frequency Shift .....	19
1.4 Sampling and Analog-to-Digital Conversion .....	21
1.4.1 Time-Domain Sampling and Reconstruction of Analog Signals .....	22
1.4.2 Quantisation.....	25
Bibliography.....	27
<b>CHAPTER 2 NOISE AND DISTORTION.....</b>	<b>29</b>
2.1 Introduction.....	30
2.2 White Noise .....	31
2.3 Coloured Noise .....	33
2.4 Impulsive Noise .....	34
2.5 Transient Noise Pulses.....	35
2.6 Thermal Noise.....	36

2.7 Shot Noise.....	38
2.8 Electromagnetic Noise .....	38
2.9 Channel Distortions .....	39
2.10 Modelling Noise .....	40
2.10.1 Additive White Gaussian Noise Model (AWGN).....	42
2.10.2 Hidden Markov Model for Noise .....	42
Bibliography.....	43
<b>CHAPTER 3 PROBABILITY MODELS .....</b>	<b>44</b>
3.1 Random Signals and Stochastic Processes .....	45
3.1.1 Stochastic Processes .....	47
3.1.2 The Space or Ensemble of a Random Process .....	47
3.2 Probabilistic Models .....	48
3.2.1 Probability Mass Function (pmf).....	49
3.2.2 Probability Density Function (pdf).....	50
3.3 Stationary and Non-Stationary Random Processes.....	53
3.3.1 Strict-Sense Stationary Processes.....	55
3.3.2 Wide-Sense Stationary Processes .....	56
3.3.3 Non-Stationary Processes .....	56
3.4 Expected Values of a Random Process.....	57
3.4.1 The Mean Value .....	58
3.4.2 Autocorrelation.....	58
3.4.3 Autocovariance.....	59
3.4.4 Power Spectral Density .....	60
3.4.5 Joint Statistical Averages of Two Random Processes.....	62
3.4.6 Cross-Correlation and Cross-Covariance .....	62
3.4.7 Cross-Power Spectral Density and Coherence .....	64
3.4.8 Ergodic Processes and Time-Averaged Statistics .....	64
3.4.9 Mean-Ergodic Processes .....	65
3.4.10 Correlation-Ergodic Processes .....	66
3.5 Some Useful Classes of Random Processes .....	68
3.5.1 Gaussian (Normal) Process .....	68
3.5.2 Multivariate Gaussian Process .....	69
3.5.3 Mixture Gaussian Process .....	71
3.5.4 A Binary-State Gaussian Process .....	72
3.5.5 Poisson Process .....	73
3.5.6 Shot Noise .....	75
3.5.7 Poisson–Gaussian Model for Clutters and Impulsive Noise.....	77
3.5.8 Markov Processes.....	77
3.5.9 Markov Chain Processes .....	79

- 3.6 Transformation of a Random Process.....81
  - 3.6.1 Monotonic Transformation of Random Processes .....81
  - 3.6.2 Many-to-One Mapping of Random Signals .....84
- 3.7 Summary .....86
- Bibliography.....87

**CHAPTER 4 BAYESIAN ESTIMATION.....89**

- 4.1 Bayesian Estimation Theory: Basic Definitions .....90
  - 4.1.1 Dynamic and Probability Models in Estimation.....91
  - 4.1.2 Parameter Space and Signal Space.....92
  - 4.1.3 Parameter Estimation and Signal Restoration .....93
  - 4.1.4 Performance Measures and Desirable Properties of Estimators .....94
  - 4.1.5 Prior and Posterior Spaces and Distributions .....96
- 4.2 Bayesian Estimation.....100
  - 4.2.1 Maximum A Posteriori Estimation .....101
  - 4.2.2 Maximum-Likelihood Estimation .....102
  - 4.2.3 Minimum Mean Square Error Estimation .....105
  - 4.2.4 Minimum Mean Absolute Value of Error Estimation.....107
  - 4.2.5 Equivalence of the MAP, ML, MMSE and MAVE for Gaussian Processes With Uniform Distributed Parameters .....108
  - 4.2.6 The Influence of the Prior on Estimation Bias and Variance.....109
  - 4.2.7 The Relative Importance of the Prior and the Observation.....113
- 4.3 The Estimate–Maximise (EM) Method .....117
  - 4.3.1 Convergence of the EM Algorithm .....118
- 4.4 Cramer–Rao Bound on the Minimum Estimator Variance.....120
  - 4.4.1 Cramer–Rao Bound for Random Parameters .....122
  - 4.4.2 Cramer–Rao Bound for a Vector Parameter.....123
- 4.5 Design of Mixture Gaussian Models .....124
  - 4.5.1 The EM Algorithm for Estimation of Mixture Gaussian Densities .....125
- 4.6 Bayesian Classification .....127
  - 4.6.1 Binary Classification .....129
  - 4.6.2 Classification Error.....131
  - 4.6.3 Bayesian Classification of Discrete-Valued Parameters .132
  - 4.6.4 Maximum A Posteriori Classification.....133
  - 4.6.5 Maximum-Likelihood (ML) Classification.....133
  - 4.6.6 Minimum Mean Square Error Classification .....134
  - 4.6.7 Bayesian Classification of Finite State Processes .....134

4.6.8 Bayesian Estimation of the Most Likely State Sequence.....	136
4.7 Modelling the Space of a Random Process.....	138
4.7.1 Vector Quantisation of a Random Process.....	138
4.7.2 Design of a Vector Quantiser: <i>K</i> -Means Clustering.....	138
4.8 Summary.....	140
Bibliography.....	141
<b>CHAPTER 5 HIDDEN MARKOV MODELS.....</b>	<b>143</b>
5.1 Statistical Models for Non-Stationary Processes.....	144
5.2 Hidden Markov Models.....	146
5.2.1 A Physical Interpretation of Hidden Markov Models.....	148
5.2.2 Hidden Markov Model as a Bayesian Model.....	149
5.2.3 Parameters of a Hidden Markov Model.....	150
5.2.4 State Observation Models.....	150
5.2.5 State Transition Probabilities.....	152
5.2.6 State–Time Trellis Diagram.....	153
5.3 Training Hidden Markov Models.....	154
5.3.1 Forward–Backward Probability Computation.....	155
5.3.2 Baum–Welch Model Re-Estimation.....	157
5.3.3 Training HMMs with Discrete Density Observation Models.....	159
5.3.4 HMMs with Continuous Density Observation Models...	160
5.3.5 HMMs with Mixture Gaussian pdfs.....	161
5.4 Decoding of Signals Using Hidden Markov Models.....	163
5.4.1 Viterbi Decoding Algorithm.....	165
5.5 HMM-Based Estimation of Signals in Noise.....	167
5.6 Signal and Noise Model Combination and Decomposition.....	170
5.6.1 Hidden Markov Model Combination.....	170
5.6.2 Decomposition of State Sequences of Signal and Noise.	171
5.7 HMM-Based Wiener Filters.....	172
5.7.1 Modelling Noise Characteristics.....	174
5.8 Summary.....	174
Bibliography.....	175
<b>CHAPTER 6 WIENER FILTERS.....</b>	<b>178</b>
6.1 Wiener Filters: Least Square Error Estimation.....	179
6.2 Block-Data Formulation of the Wiener Filter.....	184
6.2.1 QR Decomposition of the Least Square Error Equation .	185



- 6.3 Interpretation of Wiener Filters as Projection in Vector Space ...187
- 6.4 Analysis of the Least Mean Square Error Signal .....189
- 6.5 Formulation of Wiener Filters in the Frequency Domain.....191
- 6.6 Some Applications of Wiener Filters.....192
  - 6.6.1 Wiener Filter for Additive Noise Reduction .....193
  - 6.6.2 Wiener Filter and the Separability of Signal and Noise ..195
  - 6.6.3 The Square-Root Wiener Filter .....196
  - 6.6.4 Wiener Channel Equaliser.....197
  - 6.6.5 Time-Alignment of Signals in Multichannel/Multisensor Systems.....198
  - 6.6.6 Implementation of Wiener Filters .....200
- 6.7 The Choice of Wiener Filter Order .....201
- 6.8 Summary .....202
- Bibliography.....202
  
- CHAPTER 7 ADAPTIVE FILTERS.....205**
  - 7.1 State-Space Kalman Filters.....206
  - 7.2 Sample-Adaptive Filters .....212
  - 7.3 Recursive Least Square (RLS) Adaptive Filters .....213
  - 7.4 The Steepest-Descent Method .....219
  - 7.5 The LMS Filter .....222
  - 7.6 Summary .....224
  - Bibliography.....225
  
- CHAPTER 8 LINEAR PREDICTION MODELS .....227**
  - 8.1 Linear Prediction Coding.....228
    - 8.1.1 Least Mean Square Error Predictor .....231
    - 8.1.2 The Inverse Filter: Spectral Whitening .....234
    - 8.1.3 The Prediction Error Signal.....236
  - 8.2 Forward, Backward and Lattice Predictors.....236
    - 8.2.1 Augmented Equations for Forward and Backward Predictors.....239
    - 8.2.2 Levinson–Durbin Recursive Solution .....239
    - 8.2.3 Lattice Predictors.....242
    - 8.2.4 Alternative Formulations of Least Square Error Prediction.....244
    - 8.2.5 Predictor Model Order Selection.....245
  - 8.3 Short-Term and Long-Term Predictors.....247

8.4 MAP Estimation of Predictor Coefficients .....	249
8.4.1 Probability Density Function of Predictor Output.....	249
8.4.2 Using the Prior pdf of the Predictor Coefficients .....	251
8.5 Sub-Band Linear Prediction Model .....	252
8.6 Signal Restoration Using Linear Prediction Models.....	254
8.6.1 Frequency-Domain Signal Restoration Using Prediction Models .....	257
8.6.2 Implementation of Sub-Band Linear Prediction Wiener Filters .....	259
8.7 Summary .....	261
Bibliography.....	261
<b>CHAPTER 9 POWER SPECTRUM AND CORRELATION .....</b>	<b>263</b>
9.1 Power Spectrum and Correlation .....	264
9.2 Fourier Series: Representation of Periodic Signals .....	265
9.3 Fourier Transform: Representation of Aperiodic Signals.....	267
9.3.1 Discrete Fourier Transform (DFT).....	269
9.3.2 Time/Frequency Resolutions, The Uncertainty Principle .....	269
9.3.3 Energy-Spectral Density and Power-Spectral Density ....	270
9.4 Non-Parametric Power Spectrum Estimation .....	272
9.4.1 The Mean and Variance of Periodograms .....	272
9.4.2 Averaging Periodograms (Bartlett Method).....	273
9.4.3 Welch Method: Averaging Periodograms from Overlapped and Windowed Segments.....	274
9.4.4 Blackman–Tukey Method .....	276
9.4.5 Power Spectrum Estimation from Autocorrelation of Overlapped Segments .....	277
9.5 Model-Based Power Spectrum Estimation .....	278
9.5.1 Maximum–Entropy Spectral Estimation .....	279
9.5.2 Autoregressive Power Spectrum Estimation .....	282
9.5.3 Moving-Average Power Spectrum Estimation.....	283
9.5.4 Autoregressive Moving-Average Power Spectrum Estimation.....	284
9.6 High-Resolution Spectral Estimation Based on Subspace Eigen- Analysis .....	284
9.6.1 Pisarenko Harmonic Decomposition.....	285
9.6.2 Multiple Signal Classification (MUSIC) Spectral Estimation.....	288
9.6.3 Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT).....	292

9.7 Summary .....294  
 Bibliography.....294

**CHAPTER 10 INTERPOLATION.....297**

10.1 Introduction.....298  
 10.1.1 Interpolation of a Sampled Signal .....298  
 10.1.2 Digital Interpolation by a Factor of  $I$ .....300  
 10.1.3 Interpolation of a Sequence of Lost Samples .....301  
 10.1.4 The Factors That Affect Interpolation Accuracy .....303  
 10.2 Polynomial Interpolation.....304  
 10.2.1 Lagrange Polynomial Interpolation .....305  
 10.2.2 Newton Polynomial Interpolation .....307  
 10.2.3 Hermite Polynomial Interpolation .....309  
 10.2.4 Cubic Spline Interpolation.....310  
 10.3 Model-Based Interpolation .....313  
 10.3.1 Maximum A Posteriori Interpolation .....315  
 10.3.2 Least Square Error Autoregressive Interpolation .....316  
 10.3.3 Interpolation Based on a Short-Term Prediction Model  
 .....317  
 10.3.4 Interpolation Based on Long-Term and Short-term  
 Correlations .....320  
 10.3.5 LSAR Interpolation Error.....323  
 10.3.6 Interpolation in Frequency–Time Domain .....326  
 10.3.7 Interpolation Using Adaptive Code Books.....328  
 10.3.8 Interpolation Through Signal Substitution .....329  
 10.4 Summary .....330  
 Bibliography.....331

**CHAPTER 11 SPECTRAL SUBTRACTION.....333**

11.1 Spectral Subtraction.....334  
 11.1.1 Power Spectrum Subtraction .....337  
 11.1.2 Magnitude Spectrum Subtraction .....338  
 11.1.3 Spectral Subtraction Filter: Relation to Wiener Filters .339  
 11.2 Processing Distortions .....340  
 11.2.1 Effect of Spectral Subtraction on Signal Distribution...342  
 11.2.2 Reducing the Noise Variance .....343  
 11.2.3 Filtering Out the Processing Distortions .....344  
 11.3 Non-Linear Spectral Subtraction .....345  
 11.4 Implementation of Spectral Subtraction .....348  
 11.4.1 Application to Speech Restoration and Recognition....351

11.5 Summary .....	352
Bibliography.....	352
<b>CHAPTER 12 IMPULSIVE NOISE .....</b>	<b>355</b>
12.1 Impulsive Noise .....	356
12.1.1 Autocorrelation and Power Spectrum of Impulsive Noise .....	359
12.2 Statistical Models for Impulsive Noise.....	360
12.2.1 Bernoulli–Gaussian Model of Impulsive Noise .....	360
12.2.2 Poisson–Gaussian Model of Impulsive Noise.....	362
12.2.3 A Binary-State Model of Impulsive Noise .....	362
12.2.4 Signal to Impulsive Noise Ratio.....	364
12.3 Median Filters .....	365
12.4 Impulsive Noise Removal Using Linear Prediction Models .....	366
12.4.1 Impulsive Noise Detection .....	367
12.4.2 Analysis of Improvement in Noise Detectability .....	369
12.4.3 Two-Sided Predictor for Impulsive Noise Detection .....	372
12.4.4 Interpolation of Discarded Samples .....	372
12.5 Robust Parameter Estimation.....	373
12.6 Restoration of Archived Gramophone Records .....	375
12.7 Summary .....	376
Bibliography.....	377
<b>CHAPTER 13 TRANSIENT NOISE PULSES.....</b>	<b>378</b>
13.1 Transient Noise Waveforms .....	379
13.2 Transient Noise Pulse Models .....	381
13.2.1 Noise Pulse Templates .....	382
13.2.2 Autoregressive Model of Transient Noise Pulses .....	383
13.2.3 Hidden Markov Model of a Noise Pulse Process.....	384
13.3 Detection of Noise Pulses .....	385
13.3.1 Matched Filter for Noise Pulse Detection .....	386
13.3.2 Noise Detection Based on Inverse Filtering .....	388
13.3.3 Noise Detection Based on HMM .....	388
13.4 Removal of Noise Pulse Distortions.....	389
13.4.1 Adaptive Subtraction of Noise Pulses .....	389
13.4.2 AR-based Restoration of Signals Distorted by Noise Pulses .....	392
13.5 Summary .....	395

Bibliography.....395

**CHAPTER 14 ECHO CANCELLATION.....396**

14.1 Introduction: Acoustic and Hybrid Echoes .....397
14.2 Telephone Line Hybrid Echo .....398
14.3 Hybrid Echo Suppression .....400
14.4 Adaptive Echo Cancellation .....401
14.4.1 Echo Canceller Adaptation Methods.....403
14.4.2 Convergence of Line Echo Canceller.....404
14.4.3 Echo Cancellation for Digital Data Transmission.....405
14.5 Acoustic Echo .....406
14.6 Sub-Band Acoustic Echo Cancellation.....411
14.7 Summary .....413
Bibliography.....413

**CHAPTER 15 CHANNEL EQUALIZATION AND BLIND DECONVOLUTION.....416**

15.1 Introduction.....417
15.1.1 The Ideal Inverse Channel Filter .....418
15.1.2 Equalization Error, Convolutional Noise .....419
15.1.3 Blind Equalization.....420
15.1.4 Minimum- and Maximum-Phase Channels.....423
15.1.5 Wiener Equalizer .....425
15.2 Blind Equalization Using Channel Input Power Spectrum.....427
15.2.1 Homomorphic Equalization .....428
15.2.2 Homomorphic Equalization Using a Bank of High-Pass Filters .....430
15.3 Equalization Based on Linear Prediction Models.....431
15.3.1 Blind Equalization Through Model Factorisation.....433
15.4 Bayesian Blind Deconvolution and Equalization .....435
15.4.1 Conditional Mean Channel Estimation .....436
15.4.2 Maximum-Likelihood Channel Estimation.....436
15.4.3 Maximum A Posteriori Channel Estimation .....437
15.4.4 Channel Equalization Based on Hidden Markov Models.....438
15.4.5 MAP Channel Estimate Based on HMMs.....441
15.4.6 Implementations of HMM-Based Deconvolution.....442
15.5 Blind Equalization for Digital Communication Channels.....446

- 15.5.1 LMS Blind Equalization.....448
- 15.5.2 Equalization of a Binary Digital Channel.....451
- 15.6 Equalization Based on Higher-Order Statistics .....453
  - 15.6.1 Higher-Order Moments, Cumulants and Spectra .....454
  - 15.6.2 Higher-Order Spectra of Linear Time-Invariant Systems .....457
  - 15.6.3 Blind Equalization Based on Higher-Order Cepstra ....458
- 15.7 Summary .....464
- Bibliography.....465
- INDEX .....467**

# PREFACE

Signal processing theory plays an increasingly central role in the development of modern telecommunication and information processing systems, and has a wide range of applications in multimedia technology, audio-visual signal processing, cellular mobile communication, adaptive network management, radar systems, pattern analysis, medical signal processing, financial data forecasting, decision making systems, etc. The theory and application of signal processing is concerned with the identification, modelling and utilisation of patterns and structures in a signal process. The observation signals are often distorted, incomplete and noisy. Hence, noise reduction and the removal of channel distortion is an important part of a signal processing system. The aim of this book is to provide a coherent and structured presentation of the theory and applications of statistical signal processing and noise reduction methods.

This book is organised in 15 chapters.

Chapter 1 begins with an introduction to signal processing, and provides a brief review of signal processing methodologies and applications. The basic operations of sampling and quantisation are reviewed in this chapter.

Chapter 2 provides an introduction to noise and distortion. Several different types of noise, including thermal noise, shot noise, acoustic noise, electromagnetic noise and channel distortions, are considered. The chapter concludes with an introduction to the modelling of noise processes.

Chapter 3 provides an introduction to the theory and applications of probability models and stochastic signal processing. The chapter begins with an introduction to random signals, stochastic processes, probabilistic models and statistical measures. The concepts of stationary, non-stationary and ergodic processes are introduced in this chapter, and some important classes of random processes, such as Gaussian, mixture Gaussian, Markov chains and Poisson processes, are considered. The effects of transformation of a signal on its statistical distribution are considered.

Chapter 4 is on Bayesian estimation and classification. In this chapter the estimation problem is formulated within the general framework of Bayesian inference. The chapter includes Bayesian theory, classical estimators, the estimate-maximise method, the Cramér-Rao bound on the minimum-variance estimate, Bayesian classification, and the modelling of the space of a random signal. This chapter provides a number of examples on Bayesian estimation of signals observed in noise.

Chapter 5 considers hidden Markov models (HMMs) for non-stationary signals. The chapter begins with an introduction to the modelling of non-stationary signals and then concentrates on the theory and applications of hidden Markov models. The hidden Markov model is introduced as a Bayesian model, and methods of training HMMs and using them for decoding and classification are considered. The chapter also includes the application of HMMs in noise reduction.

Chapter 6 considers Wiener Filters. The least square error filter is formulated first through minimisation of the expectation of the squared error function over the space of the error signal. Then a block-signal formulation of Wiener filters and a vector space interpretation of Wiener filters are considered. The frequency response of the Wiener filter is derived through minimisation of mean square error in the frequency domain. Some applications of the Wiener filter are considered, and a case study of the Wiener filter for removal of additive noise provides useful insight into the operation of the filter.

Chapter 7 considers adaptive filters. The chapter begins with the state-space equation for Kalman filters. The optimal filter coefficients are derived using the principle of orthogonality of the innovation signal. The recursive least squared (RLS) filter, which is an exact sample-adaptive implementation of the Wiener filter, is derived in this chapter. Then the steepest-descent search method for the optimal filter is introduced. The chapter concludes with a study of the LMS adaptive filters.

Chapter 8 considers linear prediction and sub-band linear prediction models. Forward prediction, backward prediction and lattice predictors are studied. This chapter introduces a modified predictor for the modelling of the short-term and the pitch period correlation structures. A maximum a posteriori (MAP) estimate of a predictor model that includes the prior probability density function of the predictor is introduced. This chapter concludes with the application of linear prediction in signal restoration.

Chapter 9 considers frequency analysis and power spectrum estimation. The chapter begins with an introduction to the Fourier transform, and the role of the power spectrum in identification of patterns and structures in a signal process. The chapter considers non-parametric spectral estimation, model-based spectral estimation, the maximum entropy method, and high-resolution spectral estimation based on eigenanalysis.

Chapter 10 considers interpolation of a sequence of unknown samples. This chapter begins with a study of the ideal interpolation of a band-limited signal, a simple model for the effects of a number of missing samples, and the factors that affect interpolation. Interpolators are divided into two



categories: polynomial and statistical interpolators. A general form of polynomial interpolation as well as its special forms (Lagrange, Newton, Hermite and cubic spline interpolators) are considered. Statistical interpolators in this chapter include maximum a posteriori interpolation, least squared error interpolation based on an autoregressive model, time–frequency interpolation, and interpolation through search of an adaptive codebook for the best signal.

Chapter 11 considers spectral subtraction. A general form of spectral subtraction is formulated and the processing distortions that result from spectral subtraction are considered. The effects of processing-distortions on the distribution of a signal are illustrated. The chapter considers methods for removal of the distortions and also non-linear methods of spectral subtraction. This chapter concludes with an implementation of spectral subtraction for signal restoration.

Chapters 12 and 13 cover the modelling, detection and removal of impulsive noise and transient noise pulses. In Chapter 12, impulsive noise is modelled as a binary–state non-stationary process and several stochastic models for impulsive noise are considered. For removal of impulsive noise, median filters and a method based on a linear prediction model of the signal process are considered. The materials in Chapter 13 closely follow Chapter 12. In Chapter 13, a template-based method, an HMM-based method and an AR model-based method for removal of transient noise are considered.

Chapter 14 covers echo cancellation. The chapter begins with an introduction to telephone line echoes, and considers line echo suppression and adaptive line echo cancellation. Then the problem of acoustic echoes and acoustic coupling between loudspeaker and microphone systems are considered. The chapter concludes with a study of a sub-band echo cancellation system

Chapter 15 is on blind deconvolution and channel equalisation. This chapter begins with an introduction to channel distortion models and the ideal channel equaliser. Then the Wiener equaliser, blind equalisation using the channel input power spectrum, blind deconvolution based on linear predictive models, Bayesian channel equalisation, and blind equalisation for digital communication channels are considered. The chapter concludes with equalisation of maximum phase channels using higher-order statistics.

Saeed Vaseghi  
June 2000

## FREQUENTLY USED SYMBOLS AND ABBREVIATIONS

AWGN	Additive white Gaussian noise
ARMA	Autoregressive moving average process
AR	Autoregressive process
$A$	Matrix of predictor coefficients
$a_k$	Linear predictor coefficients
$\mathbf{a}$	Linear predictor coefficients vector
$a_{ij}$	Probability of transition from state $i$ to state $j$ in a Markov model
$\alpha_i(t)$	Forward probability in an HMM
bps	Bits per second
$b(m)$	Backward prediction error
$b(m)$	Binary state signal
$\beta_i(t)$	Backward probability in an HMM
$c_{xx}(m)$	Covariance of signal $x(m)$
$c_{XX}(k_1, k_2, \dots, k_N)$	$k^{\text{th}}$ order cumulant of $x(m)$
$C_{XX}(\omega_1, \omega_2, \dots, \omega_{k-1})$	$k^{\text{th}}$ order cumulant spectra of $x(m)$
$D$	Diagonal matrix
$e(m)$	Estimation error
$\mathcal{E}[x]$	Expectation of $x$
$f$	Frequency variable
$f_X(\mathbf{x})$	Probability density function for process $X$
$f_{X,Y}(\mathbf{x}, \mathbf{y})$	Joint probability density function of $X$ and $Y$
$f_{X Y}(\mathbf{x} \mathbf{y})$	Probability density function of $X$ conditioned on $Y$
$f_{X;\boldsymbol{\theta}}(\mathbf{x};\boldsymbol{\theta})$	Probability density function of $X$ with $\boldsymbol{\theta}$ as a parameter
$f_{X s,\mathcal{M}}(\mathbf{x} s, \mathcal{M})$	Probability density function of $X$ given a state sequence $s$ of an HMM $\mathcal{M}$ of the process $X$
$\Phi(m, m-1)$	State transition matrix in Kalman filter
$\mathbf{h}$	Filter coefficient vector, Channel response
$\mathbf{h}_{\max}$	Maximum-phase channel response
$\mathbf{h}_{\min}$	Minimum-phase channel response
$\mathbf{h}^{\text{inv}}$	Inverse channel response
$H(f)$	Channel frequency response

$H^{\text{inv}}(f)$	Inverse channel frequency response
$H$	Observation matrix, Distortion matrix
$I$	Identity matrix
$J$	Fisher's information matrix
$ J $	Jacobian of a transformation
$K(m)$	Kalman gain matrix
LSE	Least square error
LSAR	Least square AR interpolation
$\lambda$	Eigenvalue
$\Lambda$	Diagonal matrix of eigenvalues
MAP	Maximum a posterior estimate
MA	Moving average process
ML	Maximum likelihood estimate
MMSE	Minimum mean squared error estimate
$m$	Discrete time index
$m_k$	$k^{\text{th}}$ order moment
$\mathcal{M}$	A model, e.g. an HMM
$\mu$	Adaptation convergence factor
$\mu_x$	Expected mean of vector $x$
$n(m)$	Noise
$\mathbf{n}(m)$	A noise vector of $N$ samples
$n_i(m)$	Impulsive noise
$N(f)$	Noise spectrum
$N^*(f)$	Complex conjugate of $N(f)$
$\overline{N(f)}$	Time-averaged noise spectrum
$\mathcal{N}(x, \mu_{xx}, \Sigma_{xx})$	A Gaussian pdf with mean vector $\mu_{xx}$ and covariance matrix $\Sigma_{xx}$
$O(\cdot)$	In the order of $(\cdot)$
$P$	Filter order (length)
pdf	Probability density function
pmf	Probability mass function
$P_x(x_i)$	Probability mass function of $x_i$
$P_{x,y}(x_i, y_j)$	Joint probability mass function of $x_i$ and $y_j$
$P_{x y}(x_i y_j)$	Conditional probability mass function of $x_i$ given $y_j$
$P_{NN}(f)$	Power spectrum of noise $n(m)$
$P_{XX}(f)$	Power spectrum of the signal $x(m)$

$P_{XY}(f)$	Cross–power spectrum of signals $x(m)$ and $y(m)$
$\theta$	Parameter vector
$\hat{\theta}$	Estimate of the parameter vector $\theta$
$r_k$	Reflection coefficients
$r_{xx}(m)$	Autocorrelation function
$\mathbf{r}_{xx}(m)$	Autocorrelation vector
$\mathbf{R}_{xx}$	Autocorrelation matrix of signal $\mathbf{x}(m)$
$\mathbf{R}_{xy}$	Cross–correlation matrix
$s$	State sequence
$s^{ML}$	Maximum–likelihood state sequence
SNR	Signal-to-noise ratio
SINR	Signal-to-impulsive noise ratio
$\sigma_n^2$	Variance of noise $n(m)$
$\Sigma_{nn}$	Covariance matrix of noise $\mathbf{n}(m)$
$\Sigma_{xx}$	Covariance matrix of signal $\mathbf{x}(m)$
$\sigma_x^2$	Variance of signal $x(m)$
$\sigma_n^2$	Variance of noise $n(m)$
$x(m)$	Clean signal
$\hat{x}(m)$	Estimate of clean signal
$\mathbf{x}(m)$	Clean signal vector
$X(f)$	Frequency spectrum of signal $x(m)$
$X^*(f)$	Complex conjugate of $X(f)$
$\overline{X(f)}$	Time-averaged frequency spectrum of $x(m)$
$X(f,t)$	Time-frequency spectrum of $x(m)$
$\mathbf{X}$	Clean signal matrix
$\mathbf{X}^H$	Hermitian transpose of $\mathbf{X}$
$y(m)$	Noisy signal
$\mathbf{y}(m)$	Noisy signal vector
$\hat{y}(m m-i)$	Prediction of $y(m)$ based on observations up to time $m-i$
$\mathbf{Y}$	Noisy signal matrix
$\mathbf{Y}^H$	Hermitian transpose of $\mathbf{Y}$
Var	Variance
$w_k$	Wiener filter coefficients
$\mathbf{w}(m)$	Wiener filter coefficients vector
$W(f)$	Wiener filter frequency response
$z$	$z$ -transform variable

# INDEX

## A

Absolute value of error, 374  
Acoustic feedbacks, 407  
Acoustic noise, 30  
Adaptation formula, 212  
Adaptation step size, 220, 404  
Adaptive filter, 205, 212, 448  
Adaptive noise cancellation, 6  
Additive white Gaussian noise, 42  
Algorithm, 165  
Aliasing, 23  
All-pole digital filter, 231  
Analog signals, 22  
Autocorrelation, 58, 271, 359  
Autocorrelation of impulsive noise, 62  
Autocorrelation of the output of a linear time-invariant system, 59  
Autocorrelation of white noise, 61  
Autocovariance, 59  
Autoregressive, 115, 278  
Autoregressive (AR) model, 46, 78, 144, 316, 383  
Auto regressive-moving-average model, 278  
AWGN, 109

## B

Backward predictor, 237  
Backward probability, 156  
Band-limited white noise, 31, 32  
Bartlett periodogram, 273

Baum–Welch model re-  
Estimation, 157  
Bayes' rule, 50, 167, 249  
Bayesian estimation, 89, 100  
Bayesian inference, 4  
Bayesian MMSE, 105  
Bayesian risk function, 100  
Beam-forming, 16  
Bernoulli-Gaussian model, 360  
Bias, 94  
Bi-cepstrum, 459  
Binary-state classifier, 9  
Binary-state Gaussian Process, 72  
Bi-spectrum, 457  
Bivariate pdf, 51  
Block least square (BLS) error estimation, 185  
Boltzmann constant, 37  
Brown noise, 33  
Brownian motion, 47  
Burg's method, 244

## C

Car noise, 41  
Central limit theorem, 65, 68, 449  
Channel distortions, 30, 39, 416  
Channel equalisation, 8,416  
Channel impulse response, 34, 358  
Channel response, 417  
Characteristic function, 454  
Classification, 127  
Clutters, 77  
Coherence, 64  
Coloured Noise, 33

Complete data, 117  
 Conditional multivariate Gaussian probability, 70  
 Conditional probability density, 52  
 Consistent estimator, 95  
 Continuous density HMM, 151, 160  
 Continuously variable state process, 144  
 Continuous-valued random variables, 51  
 Convergence rate, 222  
 Convolutional noise, 449  
 Correlation subtraction, 255  
 Correlation-ergodic, 67, 183  
 Correlator, 14  
 Cost function, 374  
 Cost of error function, 100, 374  
 Cramer-Rao lower bound, 120  
 Cross-correlation, 62, 390  
 Cross-covariance, 63  
 Cross-power spectral density, 64  
 Cumulants, 455  
 Cumulative distribution function, 51

## D

Decision-directed equalisation, 444  
 Decoding of signals, 163  
 Deconvolution, 417  
 Decorrelation filter, 235  
 Detection, 367  
 Detection of signals in noise, 14  
 Deterministic signals, 45  
 DFT, 349  
 Digital coding of audio, 12  
 Digital signal, 21

Discrete Density Observation Models, 159  
 Discrete Fourier transform, 13, 269  
 Discrete state observation HMM, 151  
 Discrete-time stochastic process, 47  
 Discrete-valued random variable, 50  
 Distortion, 29  
 Distortion matrix, 206  
 Distribution function, 69  
 Divided differences, 308  
 Dolby, 18  
 Doppler, 20  
 Durbins algorithm, 242

## E

Echo Cancellation, 396  
 Echo canceller, 401  
 Echo suppresser, 400  
 Echo synthesiser, 411  
 Efficient estimator, 95  
 Eigenvalue, 221  
 Eigenvalue spread, 222  
 Eigen analysis, 284  
 Electromagnetic noise, 30, 38  
 Electrostatic noise, 30  
 EM Algorithm, 118  
 Energy-spectral density, 270  
 Ensemble, 47  
 Entropy, 279  
 Equalisation, 417  
 Ergodic HMM, 147  
 Ergodic processes, 47,64  
 ESPIRIT algorithm, 292  
 Estimate–Maximise (EM), 117  
 Estimation, 90

Estimation of the Mean and  
Variance of a Gaussian  
Process, 102  
Expected values, 57

## F

Factorisation of linear prediction  
models, 433  
Finite state process, 144  
Fisher's information matrix, 123  
Forgetting factor, 215  
Forward predictor model, 236  
Forward probability, 155  
Fourier series, 265  
Fourier transform, 267  
Frequency resolution, 270

## G

Gaussian pdf, 151  
Gaussian process, 68  
Gaussian-AR process, 115  
Gauss–Markov process, 79

## H

Hard non-linearity, 452  
Hermite polynomials, 309  
Hermitian transpose, 289  
Hidden Markov model, 73, 143,  
363, 438  
Hidden Markov model for Noise,  
42  
High resolution spectral  
estimation, 284  
Higher-Order Spectra, 456  
Homogeneous Poisson process,  
74  
Homogenous Markov chain, 80  
Homomorphic equalisation, 428  
Howling, 407

Huber's function, 374  
Hybrid echo, 398  
Hypothesised-input HMM  
equalisation, 443

## I

Ideal equaliser, 418  
Ideal interpolation, 298  
Impulsive noise, 31, 34, 355  
Incomplete data, 117  
Influence function, 374  
Information, 2  
Inhomogeneous Markov chains,  
80  
Innovation signal, 206, 230, 255  
Inversion lemma, 216  
Interpolation, 297  
Interpolation error, 323  
Interpolation through signal  
substitution, 329  
Inter-symbol-interference, 446  
Inverse discrete Fourier transform,  
269  
Inverse filter, 234  
Inverse linear predictor, 234  
Inverse-channel filter, 418

## J

Jacobian, 85  
Joint characteristic function, 454

## K

Kalman filter, 206  
Kalman filtering algorithm, 210  
Kalman gain, 208  
K-means algorithm, 138  
Kronecker delta function, 359

**L**

Lagrange interpolation, 305  
 Leaky LMS algorithm, 224  
 Least squared AR (LSAR)  
   interpolation, 320  
 Left–right HMM, 148  
 Levinson–Durbin algorithm, 238,  
   239  
 Linear array, 16  
 Linear least square error filters,  
   178  
 Linear prediction, 228  
 Linear prediction models, 11, 227,  
   431  
 Linear time invariant channel, 197  
 Linear transformation, 86  
 Linear transformation of a  
   Gaussian process, 86  
 Line interpolator, 306  
 LMS adaptation algorithm, 405  
 LMS Filter, 222  
 Log-normal Process, 83

**M**

Magnitude spectral subtraction,  
   335  
 Many-to-one Mapping, 84  
 MAP Estimation, 114  
 Marginal density, 78  
 Marginal probabilities, 73  
 Marginal probability mass  
   functions, 50  
 Markov chain, 79  
 Markov process, 77  
 Markovian prior, 439  
 Markovian state transition prior,  
   149  
 $M$ -ary pulse amplitude  
   modulation, 446, 450  
 Matched filter, 14, 386

Matrix inversion lemma, 216  
 Maximum a posteriori (MAP)  
   estimate, 101, 251  
 Maximum entropy correlation,  
   280  
 Maximum-phase channel, 423,  
   458  
 Maximum-phase information, 461  
 Mean value of a process, 58  
 Mean-ergodic, 65  
 Median, 107  
 Median Filters, 365  
 Minimisation of Backward and  
   Forward Prediction Error,  
   245  
 Minimum mean absolute value of  
   error, 107  
 Minimum mean squared error,  
   181  
 Minimum-phase channel, 423  
 Minimum-phase information, 461  
 Mixture Gaussian densities, 72  
 Mixture Gaussian density, 151  
 Mixture pdf, 450  
 Model order selection, 245  
 Model-based signal processing, 4  
 Modelling noise, 40, 174  
 Monotonic transformation, 81  
 Moving-average, 278  
 Multivariate Gaussian pdf, 69  
 Multi-variate probability mass  
   functions, 52  
 MUSIC algorithm, 288  
 Musical noise, 341, 344  
 $M$ -variate pdf, 52

**N**

Narrowband noise, 31  
 Neural networks, 5  
 Newton polynomials, 307



Noise, 29  
Noise reduction, 193  
Non-linear spectral subtraction,  
345  
Nonstationary process, 53, 56, 144  
Normal process, 68  
Normalised least mean square  
error, 404  
Nyquist sampling theorem, 23,  
298

## O

Observation equation, 206  
Orthogonality, 265  
Outlier, 365  
Over-subtraction, 345

## P

Parameter estimation, 93  
Parameter Space, 92  
Parseval's theorem, 191  
Partial correlation, 241  
Partial correlation (PARCOR)  
coefficients, 244  
Pattern recognition, 9  
Performance Measures, 94  
Periodogram, 272  
Pink noise, 33  
Poisson process, 73  
Poisson–Gaussian model, 362  
Poles and zeros, 433  
Posterior pdf, 90, 97  
Power, 55  
Power spectral density, 60, 271  
Power spectral subtraction, 335  
Power spectrum, 192, 264, 272,  
359, 428  
Power Spectrum Estimation, 263  
Power spectrum of a white noise,  
61

Power Spectrum of impulsive  
Noise., 61  
Power spectrum subtraction, 337  
Prediction error filter, 235  
Prediction error signal, 236  
Predictive model, 91  
Principal eigenvectors., 290  
Prior pdf, 97  
Prior pdf of predictor coefficients,  
251  
Prior space of a signal, 96  
Probability density function, 51  
Probability mass function, 50  
Probability models, 48  
Processing distortions, 341, 344  
Processing noise, 30

## Q

QR Decomposition, 185  
Quantisation, 22  
Quantisation noise, 25

## R

Radar, 19  
Random signals, 45  
Random variable, 48  
Rayner, 433  
Rearrangement matrices, 314  
Recursive least square error (RLS)  
filter, 213  
Reflection coefficient, 240, 242  
RLS adaptation algorithm, 218  
Robust estimator, 373  
Rotation matrix, 292

## S

Sample and hold, 22, 24  
Sampling, 22, 23

Scalar Gaussian random variable, 68  
 Second order statistics, 60  
 Short time Fourier transform (STFT), 326  
 Shot noise, 38, 76  
 Signal, 2  
 Signal classification, 9  
 Signal restoration, 93  
 Signal to impulsive noise ratio, 364  
 Signal to noise ratio, 195  
 Signal to quantisation noise ratio, 25  
 Signum non-linearity, 449  
 SINR, 364  
 Sinusoidal signal, 45  
 Soft non-linearity, 453  
 Source-filter model, 228  
 Spectral coherence, 64  
 Spectral subtraction, 335  
 Spectral whitening, 234, 235  
 Spectral-time representation, 326  
 Speech processing, 11  
 Speech recognition, 10  
 State observation models, 150  
 State transition probability, 158  
 State transition-probability matrix, 150  
 State-dependent Wiener filters, 173  
 State-equation model, 206  
 State-time diagram, 153  
 Statistical models, 44, 91  
 Stochastic processes, 47  
 Strict-sense stationary process, 55  
 Subspace eigen-analysis, 284

## T

Thermal noise, 36

Time-delay estimation, 63  
 Time delay of arrival, 198  
 Time/Frequency Resolutions, 269  
 Time-Alignment, 198  
 Time-averaged correlations, 183  
 Time-varying processes, 56  
 Toeplitz matrix, 183, 233  
 Transformation of a random process, 81  
 Transform-based coder, 13  
 Transient noise pulses, 31, 379  
 Transient Noise Pulses, 35  
 Trellis, 153  
 Tri-cepstrum, 461  
 Tri-spectrum, 457  
 Tukeys bi-weight function, 374

## U

Unbiased estimator, 94  
 Uncertainty principle, 269  
 Uniform cost function, 101  
 Uni-variate pdf, 51

## V

Vandermonde matrix, 305  
 Vector quantisation, 138  
 Vector space, 188  
 Viterbi decoding, 143

## W

Welch power spectrum, 275  
 White noise, 61  
 White Noise, 31  
 Wide-sense stationary processes, 56  
 Wiener equalisation, 425  
 Wiener filter, 7, 172, 178, 179, 339

Wiener filter in frequency  
domain, 191

Wiener-Kinchin, 60

**Z**

Zero-forcing filter, 447

Zero-inserted signal, 300