Detection, Estimation, and Modulation Theory, Part I:
Detection, Estimation, and Linear Modulation Theory. Harry L. Van Trees

ISBNs: 0-471-09517-6 (Paperback); 0-471-22108-2 (Electronic)

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Part I. Detection, Estimation, and Linear Modulation Theory

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A Wiley-Interscience Publication

JOHN WILEY & SONS, INC.

New York • Chichester • Weinheim • Brisbane • Singapore • Toronto

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Library of Congress Cataloging in Publication Data is available.

ISBN 0-471-22108-2
This title is also available in print as **ISBN 0-471-09517-6**.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

To Diane

and Stephen, Mark, Kathleen, Patricia,
Eileen, Harry, and Julia

and the next generation—
Brittany, Erin, Thomas, Elizabeth, Emily,
Dillon, Bryan, Julia, Robert, Margaret,
Peter, Emma, Sarah, Harry, Rebecca,
and Molly

Preface for Paperback Edition

In 1968, Part I of *Detection, Estimation, and Modulation Theory* [VT68] was published. It turned out to be a reasonably successful book that has been widely used by several generations of engineers. There were thirty printings, but the last printing was in 1996. Volumes II and III ([VT71a], [VT71b]) were published in 1971 and focused on specific application areas such as analog modulation, Gaussian signals and noise, and the radar–sonar problem. Volume II had a short life span due to the shift from analog modulation to digital modulation. Volume III is still widely used as a reference and as a supplementary text. In a moment of youthful optimism, I indicated in the the Preface to Volume III and in Chapter III-14 that a short monograph on optimum array processing would be published in 1971. The bibliography lists it as a reference, *Optimum Array Processing,* Wiley, 1971, which has been subsequently cited by several authors. After a 30-year delay, *Optimum Array Processing,* Part IV of *Detection, Estimation, and Modulation Theory* will be published this year.

A few comments on my career may help explain the long delay. In 1972, MIT loaned me to the Defense Communication Agency in Washington, D.C. where I spent three years as the Chief Scientist and the Associate Director of Technology. At the end of the tour, I decided, for personal reasons, to stay in the Washington, D.C. area. I spent three years as an Assistant Vice-President at COMSAT where my group did the advanced planning for the INTELSAT satellites. In 1978, I became the Chief Scientist of the United States Air Force. In 1979, Dr. Gerald Dinneen, the former Director of Lincoln Laboratories, was serving as Assistant Secretary of Defense for C3I. He asked me to become his Principal Deputy and I spent two years in that position. In 1981, I joined M/A-COM Linkabit. Linkabit is the company that Irwin Jacobs and Andrew Viterbi had started in 1969 and sold to M/A-COM in 1979. I started an Eastern operation which grew to about 200 people in three years. After Irwin and Andy left M/A-COM and started Qualcomm, I was responsible for the government operations in San Diego as well as Washington, D.C. In 1988, M/A-COM sold the division. At that point I decided to return to the academic world.

I joined George Mason University in September of 1988. One of my priorities was to finish the book on optimum array processing. However, I found that I needed to build up a research center in order to attract young research-oriented faculty and doctoral students. The process took about six years. The Center for Excellence in Command, Control, Communications, and Intelligence has been very successful and has generated over $300 million in research funding during its existence. During this growth period, I spent some time on array processing but a concentrated effort was not possible. In 1995, I started a serious effort to write the Array Processing book.

Throughout the *Optimum Array Processing* text there are references to Parts I and III of *Detection, Estimation, and Modulation Theory*. The referenced material is available in several other books, but I am most familiar with my own work. Wiley agreed to publish Part I and III in paperback so the material will be readily available. In addition to providing background for Part IV, Part I is still useful as a text for a graduate course in Detection and Estimation Theory. Part III is suitable for a second level graduate course dealing with more specialized topics.

In the 30-year period, there has been a dramatic change in the signal processing area. Advances in computational capability have allowed the implementation of complex algorithms that were only of theoretical interest in the past. In many applications, algorithms can be implemented that reach the theoretical bounds.

The advances in computational capability have also changed how the material is taught. In Parts I and III, there is an emphasis on compact analytical solutions to problems. In Part IV, there is a much greater emphasis on efficient iterative solutions and simulations. All of the material in parts I and III is still relevant. The books use continuous time processes but the transition to discrete time processes is straightforward. Integrals that were difficult to do analytically can be done easily in Matlab®. The various detection and estimation algorithms can be simulated and their performance compared to the theoretical bounds. We still use most of the problems in the text but supplement them with problems that require Matlab® solutions.

We hope that a new generation of students and readers find these reprinted editions to be useful.

HARRY L. VAN TREES

Fairfax, Virginia
June 2001

Preface

The area of detection and estimation theory that we shall study in this book represents a combination of the classical techniques of statistical inference and the random process characterization of communication, radar, sonar, and other modern data processing systems. The two major areas of statistical inference are decision theory and estimation theory. In the first case we observe an output that has a random character and decide which of two possible causes produced it. This type of problem was studied in the middle of the eighteenth century by Thomas Bayes [1]. In the estimation theory case the output is related to the value of some parameter of interest, and we try to estimate the value of this parameter. Work in this area was published by Legendre [2] and Gauss [3] in the early nineteenth century. Significant contributions to the classical theory that we use as background were developed by Fisher [4] and Neyman and Pearson [5] more than 30 years ago. In 1941 and 1942 Kolmogoroff [6] and Wiener [7] applied statistical techniques to the solution of the optimum linear filtering problem. Since that time the application of statistical techniques to the synthesis and analysis of all types of systems has grown rapidly. The application of these techniques and the resulting implications are the subject of this book.

This book and the subsequent volume, Detection, Estimation, and Modulation Theory, Part II, are based on notes prepared for a course entitled "Detection, Estimation, and Modulation Theory," which is taught as a second-level graduate course at M.I.T. My original interest in the material grew out of my research activities in the area of analog modulation theory. A preliminary version of the material that deals with modulation theory was used as a text for a summer course presented at M.I.T. in 1964. It turned out that our viewpoint on modulation theory could best be understood by an audience with a clear understanding of modern detection and estimation theory. At that time there was no suitable text available to cover the material of interest and emphasize the points that I felt were important, so I started writing notes. It was clear that in order to present the material to graduate students in a reasonable amount of time it would be necessary to develop a unified presentation of the three topics: detection, estimation, and modulation theory, and exploit the fundamental ideas that connected them. As the development proceeded, it grew in size until the material that was originally intended to be background for modulation theory occupies the entire contents of this book. The original material on modulation theory starts at the beginning of the second book. Collectively, the two books provide a unified coverage of the three topics and their application to many important physical problems.

For the last three years I have presented successively revised versions of the material in my course. The audience consists typically of 40 to 50 students who have completed a graduate course in random processes which covered most of the material in Davenport and Root [8]. In general, they have a good understanding of random process theory and a fair amount of practice with the routine manipulation required to solve problems. In addition, many of them are interested in doing research in this general area or closely related areas. This interest provides a great deal of motivation which I exploit by requiring them to develop many of the important ideas as problems. It is for this audience that the book is primarily intended. The appendix contains a detailed outline of the course.

On the other hand, many practicing engineers deal with systems that have been or should have been designed and analyzed with the techniques developed in this book. I have attempted to make the book useful to them. An earlier version was used successfully as a text for an in-plant course for graduate engineers.

From the standpoint of specific background little advanced material is required. A knowledge of elementary probability theory and second moment characterization of random processes is assumed. Some familiarity with matrix theory and linear algebra is helpful but certainly not necessary. The level of mathematical rigor is low, although in most sections the results could be rigorously proved by simply being more careful in our derivations. We have adopted this approach in order not to obscure the important ideas with a lot of detail and to make the material readable for the kind of engineering audience that will find it useful. Fortunately, in almost all cases we can verify that our answers are intuitively logical. It is worthwhile to observe that this ability to check our answers intuitively would be necessary even if our derivations were rigorous, because our ultimate objective is to obtain an answer that corresponds to some physical system of interest. It is easy to find physical problems in which a plausible mathematical model and correct mathematics lead to an unrealistic answer for the original problem.

We have several idiosyncrasies that it might be appropriate to mention. In general, we look at a problem in a fair amount of detail. Many times we look at the same problem in several different ways in order to gain a better understanding of the meaning of the result. Teaching students a number of ways of doing things helps them to be more flexible in their approach to new problems. A second feature is the necessity for the reader to solve problems to understand the material fully. Throughout the course and the book we emphasize the development of an ability to work problems. At the end of each chapter are problems that range from routine manipulations to significant extensions of the material in the text. In many cases they are equivalent to journal articles currently being published. Only by working a fair number of them is it possible to appreciate the significance and generality of the results. Solutions for an individual problem will be supplied on request, and a book containing solutions to about one third of the problems is available to faculty members teaching the course. We are continually generating new problems in conjunction with the course and will send them to anyone who is using the book as a course text. A third issue is the abundance of block diagrams, outlines, and pictures. The diagrams are included because most engineers (including myself) are more at home with these items than with the corresponding equations.

One problem always encountered is the amount of notation needed to cover the large range of subjects. We have tried to choose the notation in a logical manner and to make it mnemonic. All the notation is summarized in the glossary at the end of the book. We have tried to make our list of references as complete as possible and to acknowledge any ideas due to other people.

A number of people have contributed in many ways and it is a pleasure to acknowledge them. Professors W. B. Davenport and W. M. Siebert have provided continual encouragement and technical comments on the various chapters. Professors Estil Hoversten and Donald Snyder of the M.I.T. faculty and Lewis Collins, Arthur Baggeroer, and Michael Austin, three of my doctoral students, have carefully read and criticized the various chapters. Their suggestions have improved the manuscript appreciably. In addition, Baggeroer and Collins contributed a number of the problems in the various chapters and Baggeroer did the programming necessary for many of the graphical results. Lt. David Wright read and criticized Chapter 2. L. A. Frasco and H. D. Goldfein, two of my teaching assistants, worked all of the problems in the book. Dr. Howard Yudkin of Lincoln Laboratory read the entire manuscript and offered a number of important criticisms. In addition, various graduate students taking the course have made suggestions which have been incorporated. Most of the final draft was typed by Miss Aina Sils. Her patience with the innumerable changes is sincerely appreciated. Several other secretaries, including Mrs. Jarmila Hrbek, Mrs. Joan Bauer, and Miss Camille Tortorici, typed sections of the various drafts.

As pointed out earlier, the books are an outgrowth of my research interests. This research is a continuing effort, and I shall be glad to send our current work to people working in this area on a regular reciprocal basis. My early work in modulation theory was supported by Lincoln Laboratory as a summer employee and consultant in groups directed by Dr. Herbert Sherman and Dr. Barney Reiffen. My research at M.I.T. was partly supported by the Joint Services and the National Aeronautics and Space Administration under the auspices of the Research Laboratory of Electronics. This support is gratefully acknowledged.

Harry L. Van Trees

Cambridge, Massachusetts
October, 1967.

REFERENCES

[1] Thomas Bayes, "An Essay Towards Solving a Problem in the Doctrine of Chances," *Phil. Trans*, **53**, 370–418 (1764).
[2] A. M. Legendre, *Nouvelles Méthodes pour La Détermination ces Orbites des Comètes*, Paris, 1806.
[3] K. F. Gauss, *Theory of Motion of the Heavenly Bodies Moving About the Sun in Conic Sections*, reprinted by Dover, New York, 1963.
[4] R. A. Fisher, "Theory of Statistical Estimation," *Proc. Cambridge Philos. Soc.*, **22**, 700 (1925).
[5] J. Neyman and E. S. Pearson, "On the Problem of the Most Efficient Tests of Statistical Hypotheses," *Phil. Trans. Roy. Soc. London*, **A 231**, 289, (1933).
[6] A. Kolmogoroff, "Interpolation and Extrapolation von Stationären Zufälligen Folgen," *Bull. Acad. Sci. USSR, Ser. Math.* **5**, 1941.
[7] N. Wiener, *Extrapolation, Interpolation, and Smoothing of Stationary Time Series*, Tech. Press of M.I.T. and Wiley, New York, 1949 (originally published as a classified report in 1942).
[8] W. B. Davenport and W. L. Root, *Random Signals and Noise*, McGraw-Hill, New York, 1958.

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