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Angle Measurement

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An *angle* is defined as the figure formed by two lines or planes that intersect one another. Such lines or planes may be real, such as the edges and surfaces of a object, or they may be defined, such as lines from an observer to two distant stars.

The units of measurement of angle are degrees (°) (1° is defined as 1/360 of a circle), and radians (rad) (1 rad is defined as 1/(2 π) of a circle). One radian is equal to 57.29578°, and small angles may be expressed in the unit of milliradians (1×10^{-3} rad). A degree of angle is further divided into 60′ (minutes), and 1′ of angle is divided into 60″ (seconds). One second of angle, being 1/1,296,000 of a circle, is a very small unit of measurement when related to manufactured parts, but is a significant unit when related to much larger dimensions such as the Earth (1″ of angle equals approximately 30 m of a great circle), or in space travel (an included angle of 1″ represents about 9 km on the surface of the moon when it is observed from the Earth during its closest approach to Earth.)

Many terms are used to describe angles in many different fields of expertise. Table 14.1 lists some of these terms, along with very basic definitions.

Many devices and instruments are used to measure or set angles. The following paragraphs describe the variety of equipment involved. See Table 14.2 for a partial list of manufacturers and suppliers of this equipment. See Table 14.3 for a partial list of specific models and approximate prices of angle measurement devices and systems.

14.1 Angle Gage Blocks

Angle gage blocks are right triangle-shaped, hardened and ground steel, flat, about 7 mm thick and 60 mm long. They are supplied in sets that include blocks with one of the acute angles equal to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, or 30°. These blocks can be used in combination to set work pieces or measure objects in 1° increments. Other sets include blocks with as small as 1″ steps. Special angle gage blocks can be made to any acute angle with the aid of a sine bar and thickness gage blocks.

Angle gage blocks provide a durable, simple, and inexpensive method for measuring and setting angles; for example, positioning work pieces in a vice or fixture prior to a machining operation such as milling, drilling, or grinding.

Because they are not adjustable and made of hardened steel, their accuracy can be assumed by simple observation of their physical condition. Look for indications of wear, nicks, or dents before using.

TABLE 14.1 Defining Terms Relating to Angles

Term	Definition
Angle	A figure formed by two lines or planes that intersect one another.
Acute angle	An angle less than 90°.
Azimuth	The horizontal angle measured along the Earth's horizon, between a fixed reference (usually due south) and an object.
Bank	A lateral inclination.
Circle	A closed plane curve where all of its points are the same distance from its center point.
Declination = declivity	A negative slope.
Degree	Equal to 1/360 of a circle.
Goniometer	An instrument for measuring angles (from the Greek word *gonio*).
Incline = Slope = Bias = Slant = Gradient = Grade	The deviation, plus or minus, from horizontal as defined by gravity.
Latitude	An angle measured north or south from the equator on a meridian to a point on the earth.
Lean = List = Tilt	The deviation from vertical as defined by gravity.
Longitude	The angle between the prime meridian (through Greenwich, England) and the meridian of a given place on Earth. This angle is defined as positive moving west.
Milliradian	An angle equal to 1/1000 rad.
Minute	An angle equal to 1/60°.
Oblique angle	An obtuse or acute angle.
Obtuse angle	An angle greater than 90°.
Quadrant	One quarter of a circle (90°).
Radian	The angle subtended by an arc of a circle equal to the radius of that circle. One radian is equal to 57.29578°.
Rake	Equals the deviation in degrees from being perpendicular (90°) to a line or a plane.
Right angle	An angle of 90°.
Rise	A positive incline.
Second	An angle equal to 1/60′ (1/3600°).
Straight	An angle equal to 180°.
Taper	The change in diameter or thickness per unit length of axis.
Twist	The angle of turn per unit length of axis, as in a gun barrel or a screw thread.

14.2 Clinometers

A *clinometer* is an electronic device that measures vertical angle with respect to gravitational level. It is rectangular, with each side being a 90° to its adjacent sides. With a range of readings of at least ±45°, this shape allows measurements up to a full 360°. Floating zero can be set anywhere and resolutions of ±0.01° are obtainable. Some models will convert readings to inches per foot, % of grade, and millimeters per meter ($mm\ m^{-1}$).

A clinometer can be used anywhere the angle of a surface with respect to gravity or another surface needs to be measured. High accuracy and resolution are obtainable, but calibration should be checked periodically with respect to a known level surface and a known angle. Surfaces that are remote to one another or have an intervening obstruction pose no problem for a clinometer.

14.3 Optical Comparator

An *optical comparator* measures angles, along with other dimensions or profiles, by referencing a visual image (usually magnified) of an object to a reticule that is calibrated in the measurement units desired. A hand-held optical comparator is placed directly over the object to be measured and the operator's eye is moved to the proper distance above the comparator for good focus of the magnified image. Some models contain a battery- or line-powered light source. Reticules for these hand-held devices are generally graduated in 1° increments.

TABLE 14.2 A Partial List of Manufacturers and Supplies of Angle Measurement Equipment

Company	Address
Flexbar Machine Corporation (Representative for Erich Preissr & Co., West Germany)	250 Gibbs Road Islandia, NY 11722-2697 Tel: (800) 879-7575
Fred V. Fowler Co., Inc.	66 Rowe Street P.O. Box 299 Newton, MA 02166 Tel: (617) 332-7004
L. S. Starrett Company	121 Crescent Street Athol, MA 01331 Tel: See local distributor
Brown & Sharpe Mfg. Co.	931 Oakton Street Elk Grove Village, IL 60007 Tel: (312) 593-5950
Swiss Precision Instruments, Inc. SPI	2206 Lively Blvd. Elk Grove Village, IL 60007 Tel: (708) 981-1300
Edmund Scientific Co.	101 East Gloucester Pike Barrington, NJ 08007-1380 Tel: (609) 573-6250

Projection-type optical comparators are available as bench or floor models and are made for either horizontal or vertical beam viewing. They use a high-intensity light source and magnifying optics to display an image of an object onto a rear-projection, frosted glass screen that is inscribed with angular as well as linear markings. The image displayed is the result of light being projected past the object, referred to as a shadow graph, or of light being reflected off the surface of the object. The method used is determined by the shape of the viewed object and its surface quality.

Magnification of the optical system in these devices can range from 10× by 100×, with screen diameters ranging from 0.3 m to 1 m.

These instruments are useful for measuring profiles of parts after final matching for the purpose of quality control or duplication.

As the name implies, the image that is projected can be superimposed on a mask or outline drawing placed directly on the view screen so that any deviations from the required shape can easily be determined. Optical comparators are heavy, nonportable devices that require a fairly high amount of maintenance and are best used in a fairly dark room.

14.4 Protractor

A *protractor* is an instrument used for measuring and constructing angles. A direct-reading protractor usually is graduated in 1° increments and can be semicircular or circular in shape. The simplest models are of one-piece construction and made from metal or plastic. Other models include a blade or pointer pivoted in the center of the graduated circle.

More precise protractors are equipped with a vernier scale that allows an angle to be indicated to 5′ of arc. See Figure 14.1 for an explanation of how to read such a vernier scale.

TABLE 14.3 Instruments and Devices Used to Measure or Indicate Angles

Type	Manufacturer	Model	Description	Approx. price
Sine bar	Flexbar	16292	5 in. × 15/16 in. wide	$130.00
		16293	10 in. × 1 in. wide	
		16294	5 in. × 2 in. wide	
		12202	5 in. × 1 in. wide, economy	$30.00
	Fowler	52-455-010	5 in. center to center, 15/16 in. wide	
		52-455-015	10 in. C. to C., 1 in. wide	
		52-455-030	2.5 in. C. to C., 1 in. wide	
	SPI	30-712-4	10 in. C. to C., universal bench center	$3048.00
		98-379-1	5 in. C. to C., 1 in. wide, accuracy between rolls = 0.0003 in.	$31.00
		30-091-3	10 in. C. to C., 1 in. wide, accuracy between rolls = 0.0001 in.	$203.00
	Brown & Sharpe	598-291-121-1	5 in. C. to C., 1 in. wide	
		598-293-121-1	10 in. C. to C., 1 1/8 in. wide	
Sine plate	Flexbar	14612	5 in. C. to C., 6 in. × 3 in. × 2 in.	$320.00
		14615	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	$1000.00
	Fowler	57-374-001	5 in. C. to C., 6 in. × 3 in. × 2 in.	
		57-374-004	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	
	SPI	77-026-3	10 in. C. to C., 12 in. × 6 in. × 2 5/8 in.	$872.00
	Brown & Sharpe	599-925-10	10 in. C. to C., 12 in. × 6 in. × 2 3/8 in.	
Compound sine plate	Flexbar	14616	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	$1100.00
	Fowler	57-375-001	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	
	SPI	7-072-7	5 in. C. to C., 6 in. × 6 in. × 3 1/8 in.	$926.00
	Brown & Sharpe	599-926-5	5 in. C. to C., 6 in. × 6 in. × 3 1/2 in.	
Angle Computer	Flexbar	19860	3-axis with vernier protractors	$3750.00
Protractor-Direct	Flexbar	16337	Rectangular Head, 0–180°	$25.00
	Starrett	RP1224W	Head only, To fit 12 in., 18 in. & 24 in. blades	
		C183	Rectangular head, 0–180° 6 in. Blade	
	SPI	30-393-3	Rectangular head, 0–180°	$23.00
		31-804-8	Head only. To fit 12 in., 18 in. & 24 in. Blades	$39.00
Protractor-Vernier	Flexbar	16339	360° range, 1′ reading with magnifier, 12 in. & 6 in. blades incl.	$400.00
		16338	360° range, 5′ reading	$75.00
	Starrett	C364DZ	12 in. Blade, 0–90° range thru 360°, 5′ graduations	
	SPI	30-395-8	6 in. Blade, 0–90° range thru 360°, 5′ graduations	$65.00
		30-390-9	6 in. & 12 in. Blades, 0–90° range thru 360°, 1′ graduations with magnifier	$540.00
	Brown & Sharpe	599-490-8	8 in. Blade, 0–90° range thru 360°, Magnifier optional	
Protractor, Digital (Inclinometer)	Flexbar	17557	±45° range, ±0.1° resolution	$260.00
		17556	±60°, ±0.01° resolution, SPC output	$450.00
	Fowler	54-635-600	±45° range, ±0.01° resolution, RS232 output available	
	SPI	31-040-9	±45° range, resolution: ±0.01° (0 to ±10°), 0.1° (10° to 90°)	$329.00
Protractor, Dial Bevel		30-150-7	8 in. Blade, 1 3/8 in. diameter dial, geared to direct read to 5′	$527.30
	Brown & Sharpe	599-4977-8	8 in. Blade, dial read degrees and 5′	
Square-Reference	SPI	30-392-5	90° fixed angle	$55.00
Optical Comparator (Projector)	Fowler	53-912-000	12 in. screen diameter, 10×, 20×, 25× lens available, horizontal beam, with separate light source for surface illumination	
	Starrett	HB350	14 in. screen diameter, 10×, 20×, 25×, 31.25×, 50×, 100× lens available, horizontal beam	

TABLE 14.3 (continued) Instruments and Devices Used to Measure or Indicate Angles

Type	Manufacturer	Model	Description	Approx. price
		VB300	12 in. screen diameter, 10× through 100× lens available, Vertical beam	
		HS1000	40 in. screen diameter, 10× thru 100× lens available, Horizontal beam	
	SPI	40-350-1	14 in. screen diameter, 10×, 20×, 50× lens available, Horizontal beam	$2995.00
Optical Comparator (hand-held)	SPI	40-145-3	10× Magnification, Pocket style	$57.50
			Additional Reticles	$11.00
		40-140-6	7× Magnification, pocket style with illuminator	$62.50
			Additional Reticles	$10.50
	Edmund Scientific	A2046	6× Magnification, pocket style, 360° protractor reticle, 1° increments	$58.75
Angle Plate	Fowler	52-456-000	Set of 2, 9/32 in. thick, steel, 30 × 60 × 90°, 45 × 45 × 90°	
	SPI	98-463	Set of 2, 5/16 in. thick, steel, 30 × 60 × 90°, 45 × 45 × 90°	$32.00
Angle Positioning Block	SPI	70-997-2	0 to 60°, 10′ vernier (for setting workpiece in a vice)	$122.00
Angle Gage	Fowler	52-470-180	18 leaves, spring steel, 1 thru 10, 14, 14.5, 15, 20, 25, 30, 35, and 45°	
	SPI	31-375-9	18 gage set, 5° thru 90° in 5° steps, 5′ accuracy	$49.60
Angle Gage Blocks	Starrett	Ag18.TR	18 block set, use in combination for steps of 1″, 1″ accuracy	
	Starrett AG16.LM		16 block set, use in combination for steps of 1″, 1/4″ accuracy	
	SPI	30-140-8	10 block set, 1, 2, 3, 4, 5, 10, 15, 20, 25, 30°, Accuracy = ±0.0001″ per inch.	$170.00
			1/4° and 1/2° blocks optional (each)	$18.00

14.5 Sine Bar

A *sine bar* is a device used to accurately measure angles or to position work pieces prior to grinding and other machining procedures. It is constructed from a precisely hardened and ground steel rectangular bar to which are attached two hardened and ground steel cylindrical rods of the same diameter. The axis of each rod is very accurately positioned parallel to the other and to the top surface of the bar.

A sine bar is used in conjunction with precision gage blocks that are placed under one of the two cylindrical rods to raise that rod above the other rod a distance H (see Figure 14.2) equal to the sine of the angle desired, times the distance D between the two rods. The standard distance between the rods is 250 mm (5 in.) or 500 mm (10 in.). The governing equation in using a sine bar is $\sin A = H/D$.

A work piece positioned using a sine bar is usually secured with the use of a precision vice. The vice may clamp directly to the work piece or, when using a sine bar that has tapped holes on its top surface, to the sine bar sides with the work piece bolted to the top of the sine bar.

14.6 Sine Plate

A variation of the sine bar is the *sine plate*. A sine plate consists of the three elements of a sine bar plus a bottom plate and side straps used to lock the plate in the desired position. In addition, one of the ground steel rods is arranged to form a hinge between the top and bottom plates. When using a sine plate, a work piece is secured to the top plate using bolts or clamps and the bottom plate is secured to a machine tool table using clamps or a magnetic chuck.

Compound sine plates are bidirectional, allowing angles to be measured and set in each of two orthogonal planes (true compound angles).

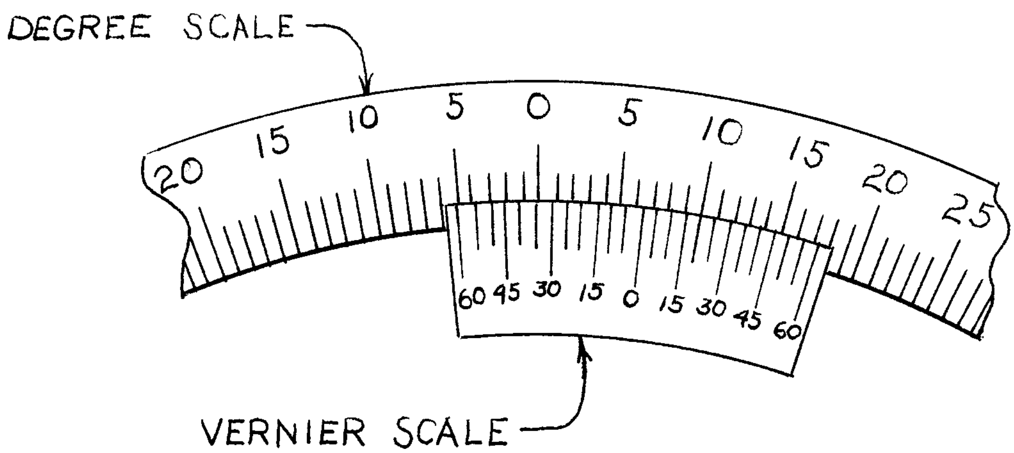

FIGURE 14.1 Vernier protractor. If the zero mark on the vernier scale is to the right of the zero mark on the main scale, as shown in this drawing, then the right side of the vernier scale must be used. Look for the mark on the vernier scale that best aligns with one of the marks of the protractor degree scale. Count the number of marks on the vernier scale from the zero mark to this mark. Each mark thus counted is, in this example, equal to 5′ of arc and, therefore, the number of minutes to be added to the number of degrees indicated is 5 times the vernier marks counted. (In this example, the fourth mark aligns the best with the main scale indicating 20′). The number of degree indicated is the degree mark just to the left of the zero mark on the vernier scale. The left side of the vernier is similarly used when the indicated angle is to the left of the zero mark on the degree scale.

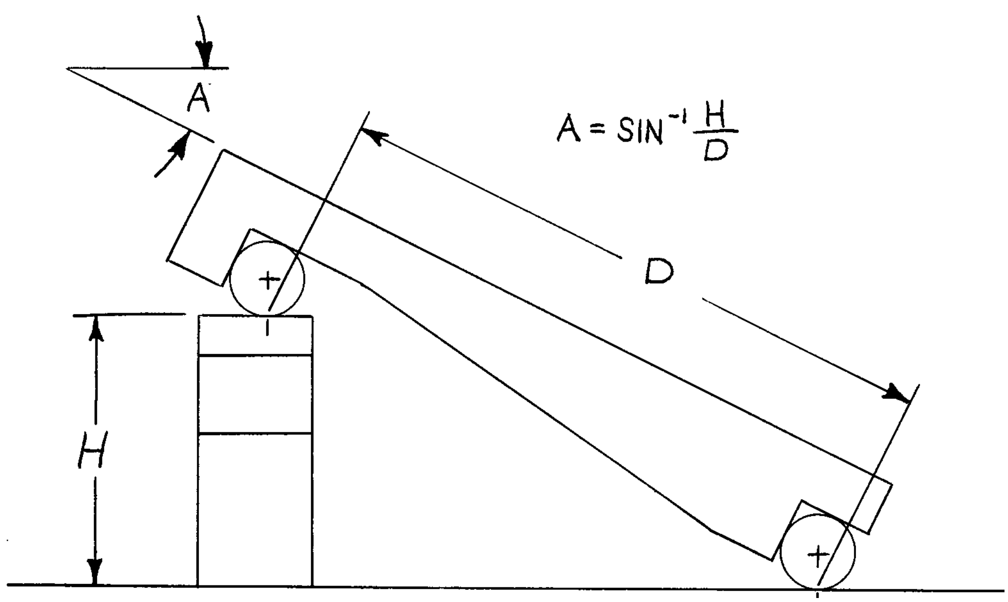

FIGURE 14.2 Sine bar. A sine bar is used in conjunction with precision gage blocks that are placed under one of the cylindrical rods, raising that end a distance, *H*, equal to the sine of the desired angle times the distance, *D*, between the two rods.

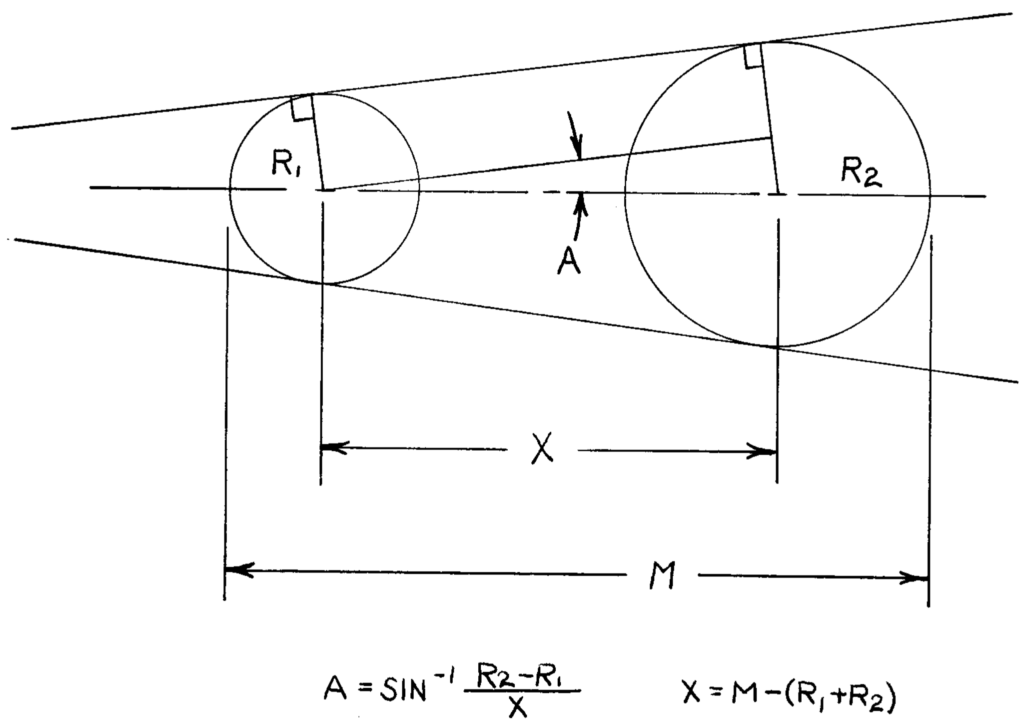

FIGURE 14.3 Measuring tapers. Two balls (for holes) or gage pins (for slots) should be selected to fit the hole size to be measured. The distance between the balls or pins should be as large as possible to allow for the best accuracy of measurement. The position of the balls can be determined and the equations shown in this figure applied to determined the angle of the taper.

14.7 Taper

An accurate method that can be used to measure a tapered hole is described in Figure 14.3. In this method, one uses a pair of steel balls of proper diameters to match the hole size and angle of taper. This method can also be used to measure the angle between two adjacent planes, using either balls or gage pins.

Further Information

Further information on this subject can be found in company catalogs in both print and Internet formats. An additional reference is *Machinery's Handbook*, 25th ed., Industrial Press Inc., 200 Madison Avenue, New York, NY 10016-4078, 1996.