Permeability and Hysteresis Measurement

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Magnetic fields are typically conceptualized with so-called "flux lines" or "lines of force." When such flux lines encounter any sort of matter, an interaction takes place in which the number of flux lines is either increased or decreased. The original magnetic field therefore becomes amplified or diminished in the body of matter as a result of the interaction. This is true whether the matter is a typical "magnetic" material, such as iron or nickel, or a so-called "nonmagnetic" material, such as copper or air.

The *magnetic permeability* of a substance is a numerical description of the extent to which that substance interacts with an applied magnetic field. In other words, permeability refers to the degree to which a substance can be magnetized.

Different substances possess varying degrees of magnetization. The aforementioned examples of strongly magnetic materials have the ability to strengthen an applied magnetic field by a factor of several thousand. Such highly magnetizable materials are called *ferromagnetic*. Certain other substances, such as Al, only marginally increase an applied magnetic field. Such weakly magnetizable materials are called *paramagnetic*. Still other substances, such as Cu and the rare gases, slightly weaken an applied magnetic field. Such "negatively magnetizable" substances are called *diamagnetic*.

In common parlance, diamagnetic and paramagnetic substances are often called *nonmagnetic*. However, as detailed below, all substances are magnetic to some extent. Only empty space is truly nonmagnetic.

The term *hysteresis* has been used to describe many instances where an effect lags behind the cause. However, Ewing was apparently the first to use the term in science [1] when he applied it to the particular magnetic phenomenon displayed by ferromagnetic materials. Magnetic hysteresis occurs during the cyclical magnetization of a ferromagnet. The magnetization path created while increasing an externally applied field is not retraced on subsequent decrease (and even reversal) of the field. Some magnetization, known as *remanence*, remains in the ferromagnet after the external field has been removed. This remanence, if appreciable, allows for the permanent magnetization observed in common bar magnets.

TABLE 49.1   Conversion Factors Between the mks and cgs
Systems for Important Quantities in Magnetism

Quantity	mks	cgs
H, applied field	A/m	$= 4\pi \times 10^{-3}$ Oe
B, flux density	Wb/m²	$= 10^4$ G
M, magnetization	Wb/m²	$= 10^4/4\pi$ emu/cm³
κ, susceptibility	Wb/(A·m)	$= 16\pi^2 \times 10^{-7}$ emu/Oe·cm³

49.1   Definition of Permeability

Let an externally applied field be described by the vector quantity **H**. This field may be produced by a solenoid or an electromagnet. Regardless of its source, **H** has units of ampere turns per meter (A m^{-1}). On passing through a body of interest, **H** magnetizes the body to a degree, **M**, formally defined as the magnetic moment per unit volume. The units of **M** are usually webers per square meter. A secondary coil (and associated electronics) is typically used to measure the combined effects of the applied field and the body's magnetization. This sum total flux-per-unit-area (flux density) is known as the induction, **B**, which typically has units of Wb/m², commonly refered to as a Tesla (T). Because **H**, **M**, and **B** are usually parallel to one another, the vector notation can be dropped, so that:

$$B = \mu_0 H + M \tag{49.1}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Wb/A^{-1} m^{-1}).

The absolute permeability, μ, of a magnetized body is defined as the induction achieved for a given strength of applied field, or:

$$\mu = \frac{B}{H} \tag{49.2}$$

Often, the absolute permeability is normalized by μ_0 to result in the relative permeability, μ_r ($=\mu/\mu_0$). This relative permeability is numerically equal and physically equivalent to the cgs version of permeability. This, unfortunately, is still in common usage, and often expressed in units of gauss per oersted (G Oe^{-1}), although the cgs permeability is actually dimensionless. Conversion factors between the mks and cgs systems are listed in Table 49.1 for the important quantities encountered.

49.2   Types of Material Magnetization

All substances fall into one of three magnetic groups: diamagnetic, paramagnetic, or ferromagnetic. Two important subclasses, antiferromagnetic and ferrimagnetic, will not be included here. The interested reader can find numerous discussions of these subclasses; for example, see [1].

Diamagnetism

Diamagnetic and paramagnetic (see next section) substances are usually characterized by their magnetic susceptibility rather than permeability. Susceptibility is derived by combining Equations 49.1 and 49.2, viz.

$$\mu_r = 1 + \frac{M}{\mu_0 H} = 1 + \frac{\kappa}{\mu_0} \tag{49.3}$$
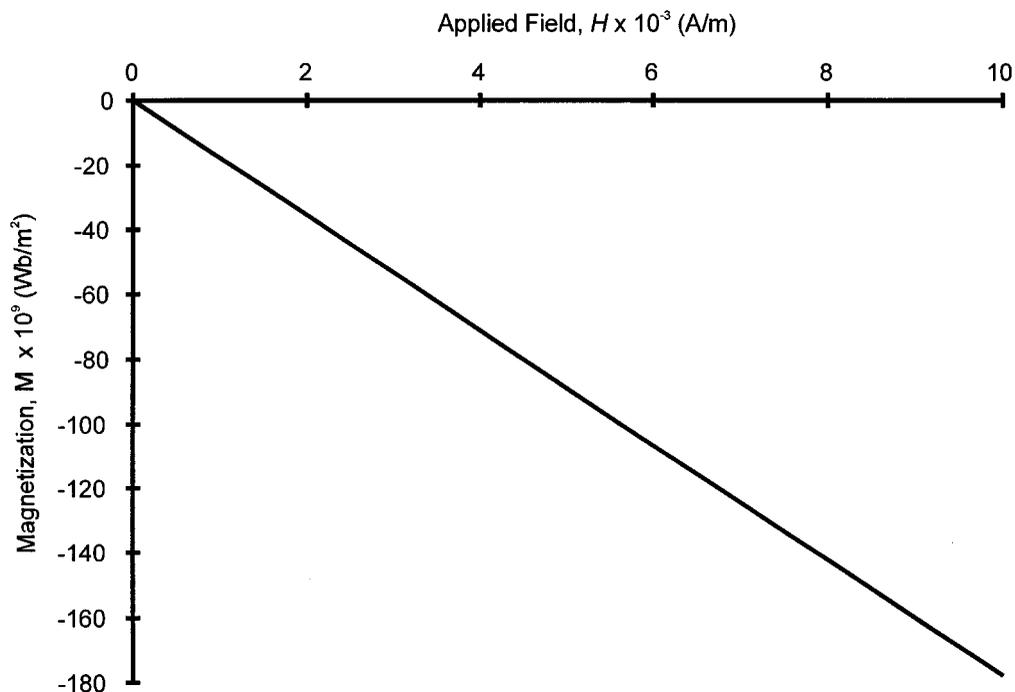
FIGURE 49.1 For diamagnetic substances, magnetization M is small and opposite the applied field H as in this schematic example for graphite ($\kappa = -1.78 \times 10^{-11}$ Wb A^{-1} m^{-1}).

where κ is the susceptibility with units of Wb A^{-1} m^{-1}. This so-called *volume susceptibility* is often converted to a mass susceptibility (χ) or a molar susceptibility (χ_M). Values for the latter are readily available for many pure substances and compounds [2]. Susceptibility is also often called "intrinsic permeability" [3].

In any atom, the orbiting and spinning electrons behave like tiny current loops. As with any charge in motion, a magnetic moment is associated with each electron. The strength of the moment is typically expressed in units of Bohr magnetons.

Diamagnetism represents the special case in which the moments contributed by all electrons cancel. The atom as a whole possesses a net zero magnetic moment. An applied field, however, can induce a moment in the diamagnetic material, and the induced moment opposes the applied field. The magnetization, **M**, in Equation 49.3 is therefore antiparallel to the applied field, **H**, and the susceptibility, κ, is negative. For diamagnetic materials, $\mu < 1$. Figure 49.1 shows a schematic M vs. H curve for graphite with $\kappa = -1.78 \times 10^{-11}$ Wb A^{-1} m^{-1}. Note that κ is a constant up to very high applied field values.

Paramagnetism

In a paramagnetic substance, the individual electronic moments do not cancel and the atom possesses a net nonzero moment. In an applied field, the weak diamagnetic response is dominated by the atom's tendency to align its moment parallel with the applied field's direction. Paramagnetic materials have relatively small positive values for κ, and $\mu > 1$.

Thermal energy retards a paramagnet's ability to align with an applied field. Over a considerable range of applied field and temperature, the paramagnetic susceptibility is constant. However, with very high applied fields and low temperatures, a paramagnetic material can be made to approach saturation — the condition of complete alignment with the field. This is illustrated in Figure 49.2 for potassium chromium alum, a paramagnetic salt. Even at a temperature as low as 1.30 K, an applied field in excess of about 3.8×10^6 A m^{-1} is necessary to approach saturation. [Note in Figure 49.2, that 1 Bohr magneton = 9.27×10^{-24} J T^{-1}.]
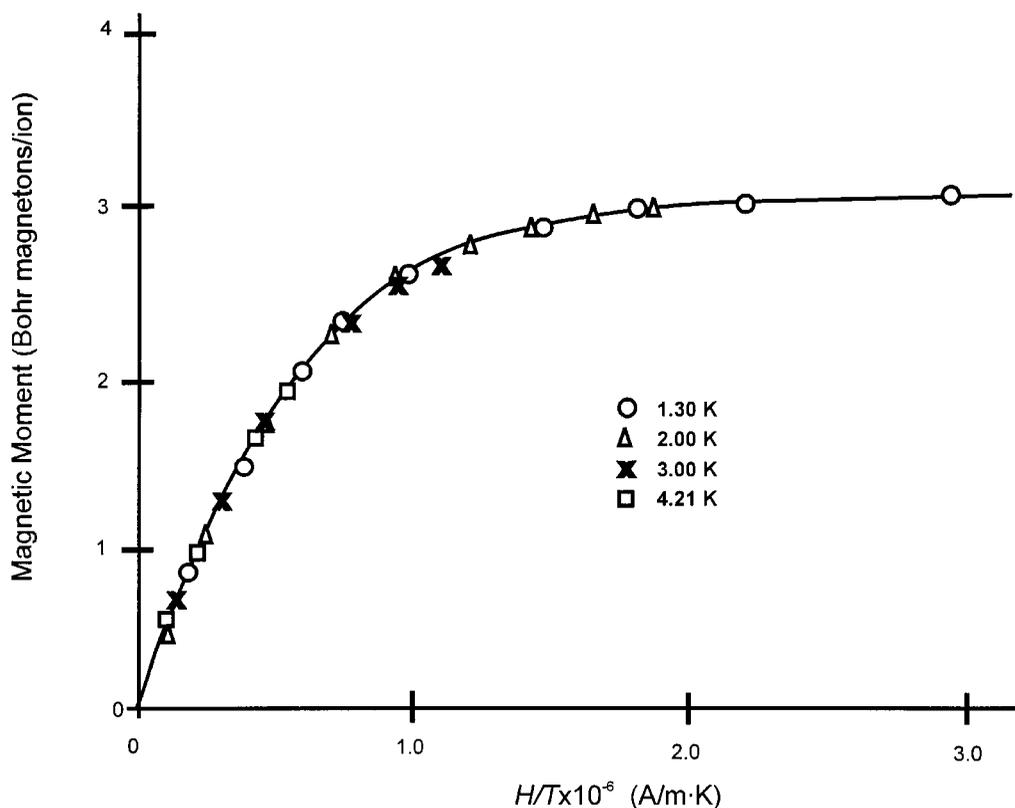
FIGURE 49.2 For paramagnetic substances, the susceptibility is constant over a wide range of applied field and temperature. However, at very high H and low T, saturation can be approached, as in this example for potassium chromium alum. (After W. E. Henry, *Phys. Rev.*, 88, 559-562, 1952.)

Ferromagnetism

Ferromagnetic substances are actually a subclass of paramagnetic substances. In both cases, the individual electronic moments do not cancel, and the atom has a net nonzero magnetic moment that tends to align itself parallel to an applied field. However, a ferromagnet is much less affected by the randomizing action of thermal energy compared to a paramagnet. This is because the individual atomic moments of a ferromagnet are coupled in rigid parallelism, even in the absence of an applied field.

With no applied field, a demagnetized ferromagnet is comprised of several magnetic domains. Within each domain, the individual atomic moments are parallel to one another, or coupled, and the domain has a net nonzero magnetization. However, the direction of this magnetization is generally opposed by a neighboring domain. The vector sum of all magnetizations among the domains is zero. This condition is called the *state of spontaneous magnetization*.

With an increasing applied field, domains with favorable magnetization directions, relative to the applied field direction, grow at the expense of the less favorably oriented domains. This process is schematically illustrated in Figure 49.3. The exchange forces responsible for the ferromagnetic coupling are explained by Heisenberg's quantum mechanical model [4]. Above a critical temperature known as the Curie point, the exchange forces disappear and the formerly ferromagnetic material behaves exactly like a paramagnet.

During magnetization, ferromagnets show very different characteristics from diamagnets and paramagnets. Figure 49.4 is a so-called *B–H* curve for a typical soft ferromagnet. Note that B is no longer
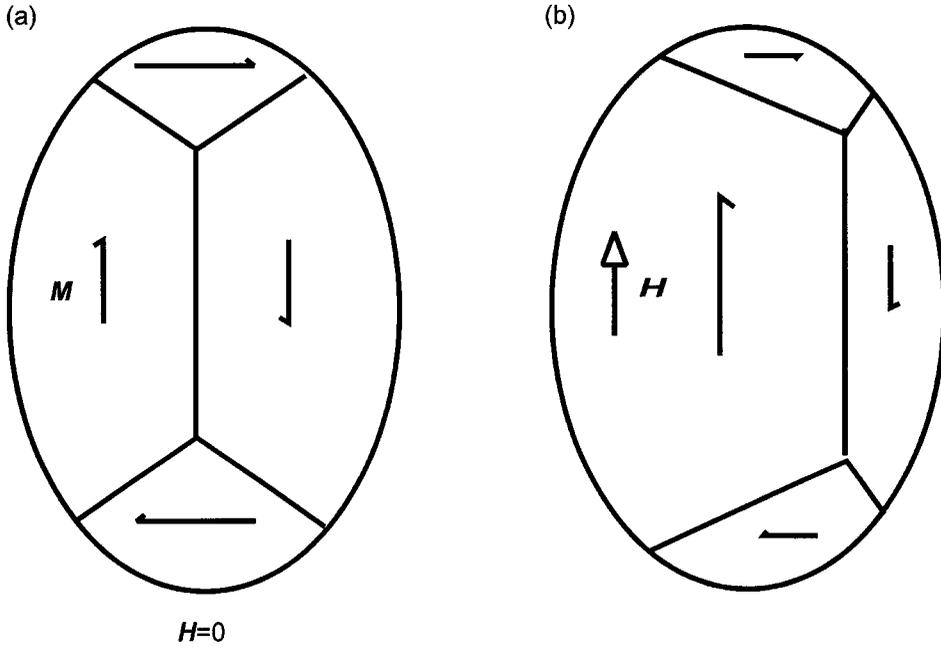
FIGURE 49.3 With no applied field (a) a ferromagnet assumes spontaneous magnetization. With an applied field (b) domains favorably oriented with **H** grow at the expense of other domains.
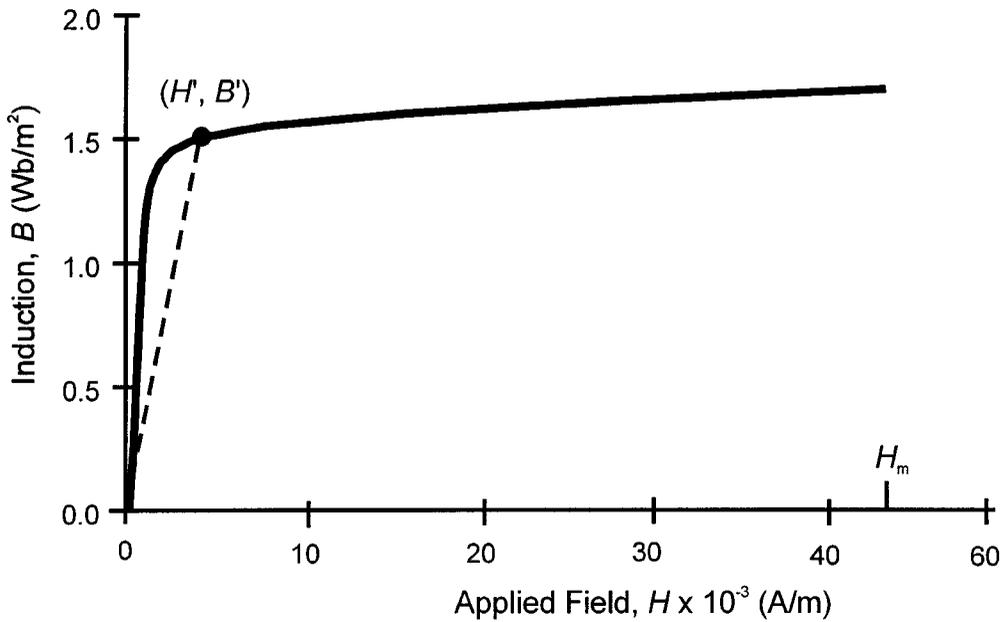
FIGURE 49.4 Magnetization (B–H) curve for a typical soft ferromagnet. Permeability at point (H', B') is the slope of the dashed line.

linear with H except in the very low- and very high-field regions. Because of this, the permeability μ for a ferromagnet must always be specified at a certain applied field, H, or, more commonly, a certain achieved induction, B. Note that μ is the slope of the line connecting a point of interest on the B–H curve to the origin. It is not the slope of the curve itself, although this value, dB/dH, is called the *differential permeability* [3].

Another ferromagnetic characteristic evident in Figure 49.4 is *saturation*. Once the applied field has exceeded a certain (and relatively low) value, the slope of the magnetization curve assumes a constant value of unity. At this point, M in Equation 49.1 has reached a maximum value, and the ferromagnet is said to be saturated. For all practical purposes, all magnetic moments in the ferromagnet are aligned with the applied field at saturation. This maximum magnetization is often called the saturation induction, B_s [5]. Note that B_s is an intrinsic property — it does not include the applied field in its value.

49.3   Definition of Hysteresis

If H is decreased from H_M in Figure 49.4, B does not follow the original magnetization path in reverse. Even if H is repeatedly cycled from H_M to $-H_M$, B will follow a path on increasing H that is different from decreasing H. The cyclical B–H relationship for a typical soft ferromagnet is displayed by the hysteresis loops in Figure 49.5. Two loops are included: a minor loop inside a major loop generated by independent measurements. The two differ in the value of maximum applied field: $H_{M'}$ for the minor loop was below saturation while $H_{M''}$ for the major loop was near saturation. Both loops are symmetrical about the origin as a point of inversion since in each case $H_M = \left| -H_M \right|$.

Notice for the minor loop that when the applied field is reduced from $H_{M'}$ to 0, the induction does not also decrease to zero. Instead, the induction assumes the value B_r, known as the *residual induction*. If the peak applied field exceeds the point of saturation, as for the major loop in Figure 49.5, B_r assumes a maximum value known as the *retentivity*, B_{rs}.

Note that B_r and B_{rs} are short-lived quantities observable only during cyclical magnetization conditions. When the applied field is removed, B_r rapidly decays to a value B_d, known as the *remanent induction*. B_d is a measure of the permanent magnetization of the ferromagnet. If the maximum applied field was in excess of saturation, B_{rs} rapidly decays to a maximum value of permanent magnetization, or *remanence*, B_{dm}.

The minor loop in Figure 49.5 shows that in order to reduce the induction B to zero, a reverse applied field, H_c, is needed. This is known as the *coercive force*. If the maximum applied field was in excess of saturation, the coercive force assumes a maximum value, H_{cs}, known as the *coercivity*. Note that H_c and H_{cs} are usually expressed as positive quantities, although they are negative fields relative to $H_{M'}$ and $H_{M''}$.

The hysteresis loops in Figure 49.5 are known as *dc loops*. Typical sweep times for such loops range from 10 s to 120 s. At faster sweep times, the coercivity will show a frequency dependence, as shown experimentally in Figure 49.6. For soft magnetic materials, this dependence can be influenced by the metallurgical condition of the ferromagnet [6].

49.4   Core Loss

During ac magnetization, some of the input energy is converted to heat in ferromagnetic materials. This heat energy is called *core loss* and is classically comprised of three parts. The first, *hysteresis loss*, P_h, is proportional to the ac frequency, f, and the area of the (slow-sweep) dc hysteresis loop:

$$P_h = kf \int B \, dH \tag{49.4}$$

The second part is the *loss due to eddy current formation*, P_e. In magnetic testing of flat-rolled strips (e.g., the Epstein test; see next section), this core loss component is classically expressed as
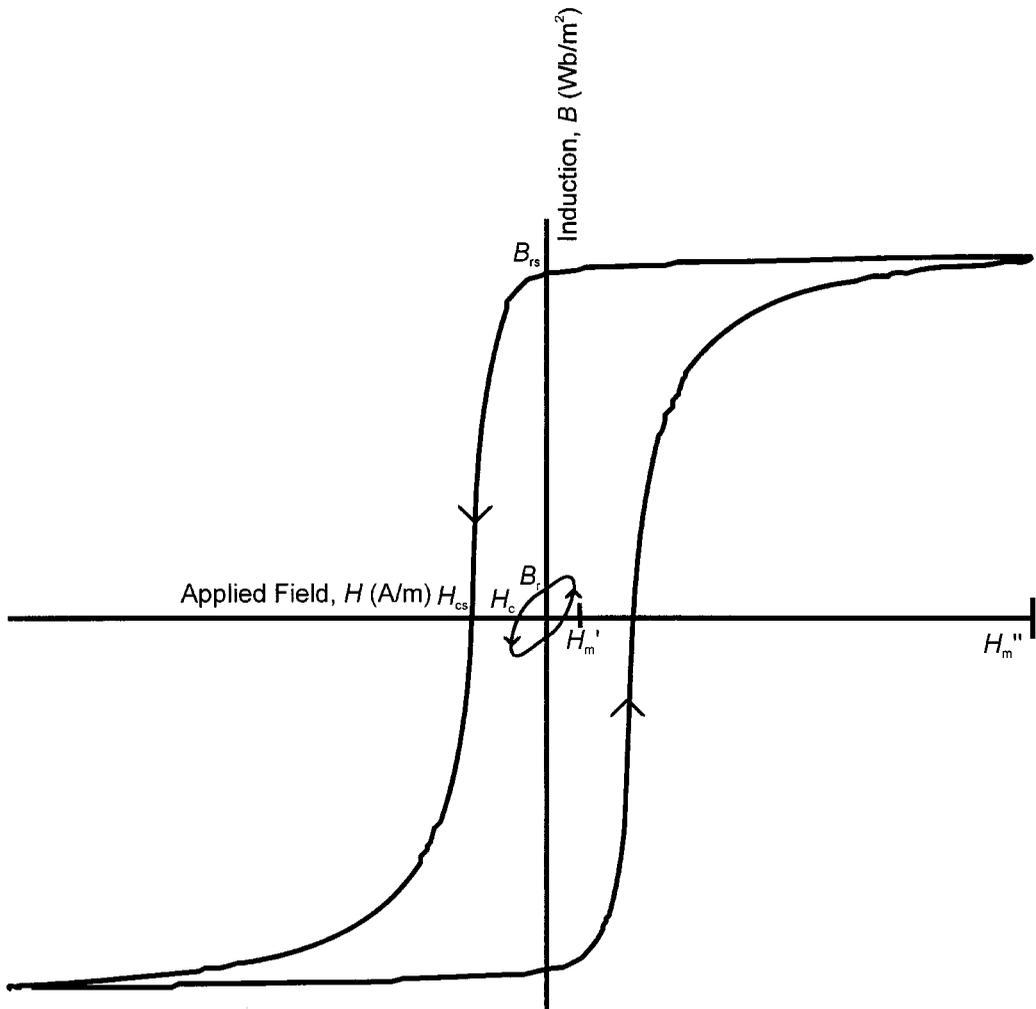
FIGURE 49.5 Major and minor dc hysteresis loops for a typical soft ferromagnet. Labeled points of interest are described in the text.

$$P_e = \frac{\left(\pi B f t\right)^2}{6 d \rho}$$

(49.5)

where B = Peak induction
$\quad\;\; t$ = Strip thickness
$\quad\;\; d$ = Material density
$\quad\;\; \rho$ = Material resistivity

The sum total $P_h + P_e$ almost never equals the observed total core loss, P_t. The discrepancy chiefly originates from the assumptions made in the derivation of Equation 49.5. To account for the additional observed loss, an anomalous loss term, P_a, has often been included, so that

$$P_t = P_h + P_e + P_a$$

(49.6)
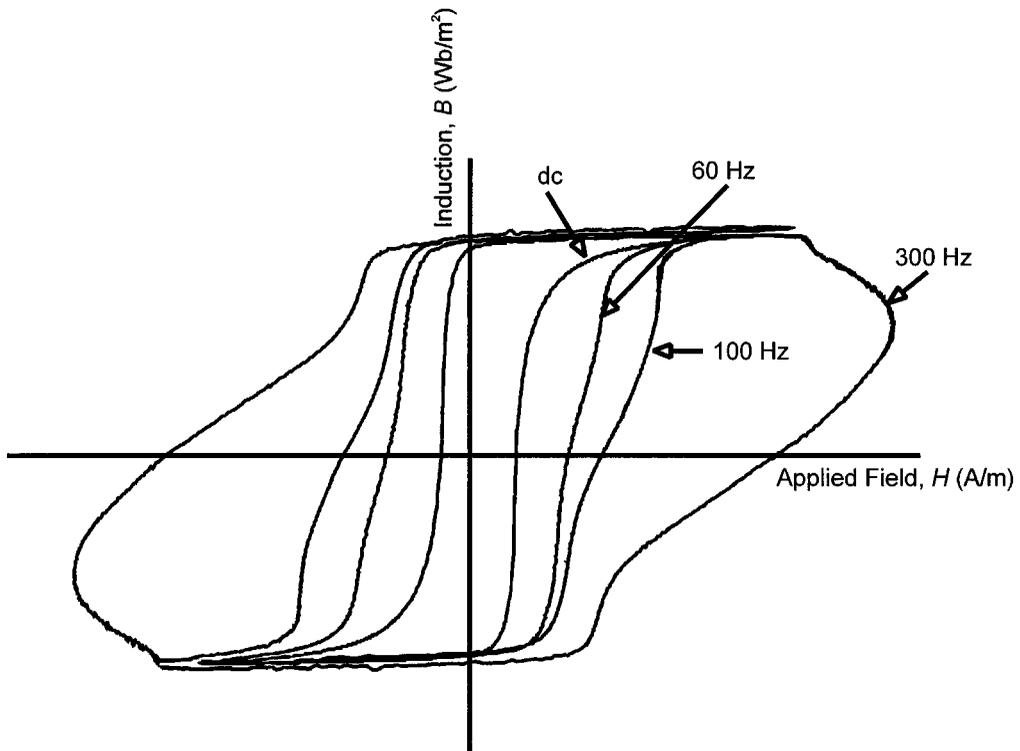
FIGURE 49.6 With increasing test frequency, coercivity for a soft ferromagnet also increases.

TABLE 49.2 Commercially Available Instruments for Measurement of Permeability and Hysteresis

Manufacturer	Model	Power	Material type[a]	Ferromagnetic type	Hysteresis loop?	Core loss?	Cost ($U.S.)
LDJ	3500/5600	ac/dc	F	Soft & hard	Y	Y	30–90k
Troy, MI	5500	dc	F	Soft & hard	Y	N	30–90k
(810) 528-2202	VSM	dc	D, P, F	Soft & hard	Y	N	50–110k
Lakeshore Cryotronics	VSM	dc	D, P, F	Soft & hard	Y	N	45–120k
Westerville, OH	Susceptometer	ac	F	Soft & hard	N	N	50–110k
(614) 891-2243	Magnetometer	dc	F	Soft & hard	N	N	50–110k
Donart Electronics	3401	dc	F	Soft	Y	N	20k+
Pittsburgh, PA	MS-2	ac	F	Soft	N	Y	20k+
(412) 796-5941							
Soken/Magnetech	DAC-BHW-2	ac	F	Soft	Y	Y	38k+
Racine, WI							
(501) 922-6899							

[a] D — diamagnetic, P — paramagnetic, F — ferromagnetic.

49.5   Measurement Methods

Reference [3] is a good source for the various accepted test methods for permeability and hysteresis in diamagnetic, paramagnetic, and ferromagnetic materials. Unfortunately, only a few of the instruments described there are available commercially. Examples of these are listed in Table 49.2.

The instruments in Table 49.2 include hysteresigraphs (LDJ models 3500, 5600, and 5500) and vibrating sample magnetometers (LDJ and Lakeshore Cryotronics VSM models). Also included are two Donart models of Epstein testers. The Epstein test is commonly used to characterize flat-rolled soft ferromagnets such as silicon electrical steels in sheet form. A recent alternative to the Epstein test is the single-sheet test method. The Soken instrument in Table 49.2 is an example of such a tester. This method requires much less sample volume than the Epstein test. It can also accommodate irregular sample geometries. However, the Soken instrument is not yet accepted by the American Society for Testing and Materials (ASTM) for reasons explained in the next section.

Note that all instruments in Table 49.2 can measure permeability (or susceptibility), but not all can provide hysteresis loop measurements. Diamagnetic and paramagnetic materials generally require VSM instruments unless one is prepared to construct their own specialty apparatus [3]. All instruments in Table 49.2 can measure ferromagnetic materials, although only a few can accommodate hard (i.e., permanently magnetizable) ferromagnets.

The price ranges in Table 49.2 account for such options as temperature controls, specialized test software, high-frequency capabilities, etc.

49.6 Validity of Measurements

For a ferromagnet under sinusoidal ac magnetization, the induction will show a waveform distortion (i.e., B is nonsinusoidal) once H_m exceeds the knee of the B–H curve in Figure 49.4. Brailsford [7] has discussed such waveform distortion in detail. With one exception, all ac instruments in Table 49.2 determine H from its sinusoidal waveform and B from its distorted waveform.

The single exception is the Soken instrument. Here, feedback amplification is employed to deliberately distort the H waveform in a way necessary to force a sinusoidal B waveform. In general, this will result in a smaller measured value for permeability compared to the case where feedback amplification is not used. Some suggest this to be the more precise method for permeability measurement, but the use of feedback amplification has prevented instruments such as the Soken from achieving ASTM acceptance to date.

Defining Terms

Permeability: The extent to which a material can be magnetized
Hysteresis: A ferromagnetic phenomenon in which the magnetic induction B is out of phase with the magnetic driving force H.

References

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