Problem 8.1

(a) Free space loss $= 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2$

$$= 20\log_{10}\left(\frac{4 \times \pi \times 150}{3 \times 10^8 / 4 \times 10^9}\right)\text{dB}$$

$$= 88 \text{ dB}$$

(b) The power gain of each antenna is

$$10\log_{10}G_r = 10\log_{10}G_t = 10\log_{10}\left(\frac{4 \times \pi \times A}{\lambda^2}\right)$$

$$= 10\log_{10}\left(\frac{4 \times \pi \times \pi \times 0.6}{(3/40)^2}\right)$$

$$= 36.24 \text{ dB}$$

(c) Received Power= Transmitted power $+G_r$ - Free space loss
= 1 + 36.24 - 88
= -50.76 dBW

Problem 8.2

The antenna gain and free-space loss at 12 GHz can be calculated by simply adding $20\log_{10}(12/4)$ for the values calculated in Problem 8.1 for downlink frequency 4 GHz. Specifically, we have:

(a) Free-space loss= $88 + 20\log_{10}(3)$
= 97.54 dB

(b) Power gain of each antenna
= $36.24 + 20\log_{10}(3)$
= 45.78 dB

(c) Received power = -50.76 dBW

The important points to note from the solutions to Problems 8.1 and 8.3 are:
1. Increasing the operating frequency produces a corresponding increase in free-space loss, and an equal increase in the power gain of each antenna.
2. The net result is that, in theory, the received power remains unchanged.

Problem 8.3

The Friis free-space equation is given by

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

(a) Using the relationship

$$A_r = \frac{\lambda^2}{4\pi} G_r, \quad \text{and} \quad A_t = \frac{\lambda^2}{4\pi} G_t \text{, we may write}$$

$$P_r = P_t \left[\frac{4\pi A_t}{\lambda^2}\right] \left[\frac{4\pi A_r}{\lambda^2}\right] \left[\frac{\lambda}{4\pi d}\right]^2$$

$$= \frac{P_t A_t A_r}{\lambda^2 d^2} \qquad (1)$$

(b) $$P_r = P_t \left[\frac{4\pi A_t}{\lambda^2}\right] G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

$$= \frac{P_t A_t G_r}{4\pi d^2} \qquad (2)$$

In both Eqs. (1) and (2) the dependent variable is the received signal power, but the independent variables are different.

(c) Equation (1) is the appropriate choice for calculating P_r performance when the dimensions of both the transmitting and receiving antennas are already fixed. Equation (1) states that for fixed size antennas, the received power increases as the wavelength is decreased.

Equation (2) is the appropriate choice when both A_t and G_r are fixed and the requirement is to determine the required value of the average transmitted power P_t in order to realize a specified P_r.

Problem 8.4

The free space loss is given by

$$L_{\text{free space}} = \left(\frac{4\pi d}{\lambda}\right)^2$$

According to the above formulation for free space loss, free space loss is frequency dependent. Path loss, as characterized in this formulation, is a definition based on the use of an isotropic receiving antenna ($G_r = 1$).

The power density, $\rho(d)$, is a function of distance and is equal to

$$\rho(d) = \frac{\text{EIRP}}{4\pi d^2}$$

The received power of an isotropic antenna is equal to

$$P_r = \rho(d) \times \frac{\lambda^2}{4\pi}$$

$$= \frac{\text{EIRP}}{4\pi d^2} \times \frac{\lambda^2}{4\pi}$$

$$= \frac{\text{EIRP}}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$$= \text{EIRP}/L_{\text{free-space}} \qquad (1)$$

Equation (1) states the power received by an isotropic antenna is equal to the effective transmitted power EIRP, reduced only by the path loss. However, when the receiving antenna is not isotropic, the received power is modified by the receiving antenna gain G_r, that is, Eq. (1) is multiplied by G_r

Problem 8.5

In a satellite communication system, satellite power is limited by the permissible antenna size. Accordingly, a sensible design strategy is to have the path loss on the downlink smaller than the pass loss on the uplink. Recognizing the inverse dependence of path loss on the wavelength λ, it follows that we should have

$$\lambda_{\text{uplink}} < \lambda_{\text{downlink}}$$

or, equivalently,

$$f_{\text{uplink}} > f_{\text{downlink}}$$

Problem 8.6

Received power in dBW is defined by

$$Pr = \text{EIRP} + G_r - \text{Free-space loss} \tag{1}$$

For these three components, we have

(1) $\text{EIRP} = 10\log_{10}(P_t G_t)$

$$= 10\log_{10}P_t + 10\log_{10}(G_t)$$

$$= 10\log_{10}(0.1) + 10\log_{10}(G_t) \tag{2}$$

Transmit antenna gain (in dB):

$$10\log_{10}G_t = 10\log_{10}\left(\frac{4\times\pi\times 0.7\times\pi/4}{(3/40)^2}\right)$$

$$= 30.89 \text{ dB} \tag{3}$$

(2) Receive antenna gain:

$$10\log_{10}G_r = 10\log_{10}\left(\frac{4\times\pi\times 0.55\times\pi\times 5^2}{(3/40)^2}\right)$$

$$= 49.84 \text{ dB} \tag{4}$$

(3) Free-space loss:

$$L_p = 20\log_{10}\left(\frac{4\times\pi\times R}{\lambda}\right)$$

$$= 20\log_{10}\left(\frac{4\times\pi\times 4\times 10^7}{3/40}\right)$$

$$= 196.25 \text{ dB} \tag{5}$$

Hence, using Eqs. (1) to (5), we find that

$$P_r = 10\log_{10}(0.1) + 30.89 + 49.84 - 196.52$$

$$= -206.52 + 8.073$$

$$= -125 \text{ dBW}$$

Problem 8.7

(a) RMS value of thermal noise = $\sqrt{E[v^2]} = \sqrt{4kTR\Delta f}$ volts, where k is Boltzmann's constant equal to 1.38 x 10^{-23}, T is the absolute temperature in degrees Kelvin, and R is the resistance in ohmns. Hence,

$$\text{RMS value} = \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 75 \times 1 \times 10^6}$$

$$= \sqrt{4 \times 1.38 \times 290 \times 75 \times 10^{-17}}$$

$$= 1.096 \text{ x } 10^{-6} \text{ volts}$$

(b) The maximum available noise power delivered to a matched load is

$$kT\Delta f = 1.38 \times 10^{-23} \times 290 \times 10^6$$

$$= 4.0 \text{ x } 10^{-15} \text{ watts}$$

Problem 8.8

The wavegude loss is 1 dB; that is,

$$G_{\text{waveguide}} = 0.78$$

The noise temperature at the input to the LNA due to the combined presence of antenna and waveguide is

$$T_e = G_{\text{waveguide}} \times T_{\text{antenna}} + (1 - G_{\text{waveguide}})T_{\text{waveguide}}$$

$$= 0.78 \times 50 + 290(1 - 0.78)$$

$$= 102.8K$$

The overall noise temperature of the system is

$$T_{\text{system}} = T_e + 50 + \frac{500}{200} + \frac{1000}{200}$$

$$= 160.3K$$

The system noise temperature referred to the antenna terminal is

$$160.3/0.78 = 205.5K$$

Problem 8.9

In this problem, we are given the noise figures (F) and the available power gains (G) of the devices. By using the following relationship, we can estimate the equivalent noise temperature of each device:

$$F = \frac{T + T_e}{T}$$

$$T_e = T(F\text{-}1)$$

where T is room temperature (290K) and T_e is the equivalent noise temperature.

(a) The equivalent noise temperatures of the given four components are

Waveguide

$$T_{\text{waveguide}} = 290(2-1)$$
$$= 290K$$

Mixer

$$T_{\text{mixer}} = 290(3-1)$$
$$= 580K$$

Low-noise RF amplifier

$$T_{RF} = 290(1.7\text{-}1)$$
$$= 203K$$

IF amplifier

$$T_{IF} = 290(5-1)$$
$$= 1160K$$

(b) The effective noise temperature at the input to the LNA due to the antenna and waveguide is

$$T_e = G_{\text{waveguide}} \times T_{\text{antenna}} + (1 - G_{\text{waveguide}}) \times T_{\text{waveguide}}$$

$$= 0.2 \times 50 + 290(1 - 0.2)$$
$$= 242K$$

The effective noise temperature of the system is

$$T_{\text{system}} = T_e + T_{\text{RF}} + \frac{T_{\text{mixer}}}{G_{\text{RF}}} + \frac{T_{\text{IF}}}{G_{\text{RF}} \times G_{\text{mixer}}}$$

$$= 242 + 203 + \frac{580}{10} + \frac{1160}{10 \times 5}$$

$$= 526.2K$$

Problem 8.10

(a) For the uplink power budget, the ratio $\frac{C}{N}$ is given by

$$\left.\frac{C}{N}\right|_{\text{uplink}} = \phi_s - G_I - BO_I + \frac{G}{T} - k - L_r$$

where

ϕ_s = Power density at saturation
G_I = Gain of 1m^2
BO_I = Power back-off
$\frac{G}{T}$ = Figure of Merit
k = Boltzmann constant in dBK
L_r = Losses due to rain

For the given satellite system, we have

$$\left.\frac{C}{N}\right|_{\text{uplink}} = -81 - 44.5 - 0.0 + 1.9 + 228.6 + 0.0$$

$$= 105.0 \text{ dB-Hz}$$

where we have used the following gain of 1m^2 antenna:

$$G_l = 10\log_{10}\left(\frac{4\times\pi\times 1}{(3\times 10^8/14\times 10^9)^2}\right)$$

$$= 44.5 \text{ dB}$$

Boltzmann constant k = -228.6 dB

(b) Given the data rate in the uplink = 33.9 Mb/s and link margin of 6 dB, the required $\frac{E_b}{N_0}$ is

$$\left(\frac{E_b}{N_0}\right)_{\text{required}} = \left(\frac{C}{N_0}\right)_{\text{uplink}} - (10\log_{10}M + 10\log_{10}R)$$

$$= 105 - 6 - 10\log_{10}(33.9 \times 10^6)$$

$$= 105 - 6 - 75.3$$

$$= 23.7 \text{ dB}$$

Equivalently, we have

$$\frac{E_b}{N_0} = 234$$

Given the use of 8-PSK, the symbol error rate is defined by

$$P_e = \text{erfc}\left(\sqrt{\frac{E}{N_0}}\sin(\pi/8)\right)$$

For 8-PSK

$$\frac{E}{N_0} = \frac{3E_b}{N_0} = 234\times 3 = 702$$

Hence,

$$P_e = \text{erfc}(\sqrt{702}\times\sin(\pi/8))$$
$$\approx 0$$

This result further confirms the statement we made in Example 8.2 in that the satellite communication system is essentially downlink-limited. Recognizing that we have more powerful resources available at an earth station then at a satellite, it would seem reasonable that the BER at the satellite can be made practically zero by transmitting enough signal power along the uplink.

Problem 8.11

For the downlink, the relationship between

$\left(\frac{C}{N_0}\right)$ and $\left(\frac{E_b}{N_0}\right)_{\text{req}}$, expressed in decibels, is described by

$$\left(\frac{C}{N_0}\right)_{\text{downlink}} = \left(\frac{E_b}{N_0}\right)_{\text{req}} + 10\log M + 10\log R \tag{1}$$

where M is the margin and R is the bit rate in bits/second.

Solving Eq. (1) for the the link margin in dB and evaluating it for the problem at hand, we get

$$\begin{aligned} 10\log_{10}M &= 85 - 10 - 10\log_{10}(10^6) \\ &= 5\text{dB} \end{aligned}$$

For the downlink budget, the equation for $\left(\frac{C}{N_0}\right)$, expressed in decibels, is as follows:

$$\left(\frac{C}{N_0}\right)_{\text{downlink}} = \text{EIRP} + \left(\frac{G_r}{T}\right)_{\text{dB}} - L_{\text{freespace}} - 10\log_{10}k$$

where k is Boltzmann's constant.

For a satisfactory reception at any situation, we consider additional losses due to rain etc. up to the calculated link margin of 5 dB. Hence, we may write

$$\left(\frac{C}{N_0}\right)_{\text{downlink}} = \text{EIRP} + \left(\frac{G_r}{T}\right)_{\text{dB}} - L_{\text{freespace}} - 10\log_{10}k - 10\log_{10}M(\text{dB}) \tag{2}$$

where

$$\begin{aligned} \text{EIRP} &= 57 \text{ dBW} \\ L_{\text{freespace}} &= \text{free-space loss} \end{aligned}$$

$$= 92.4 + 20\log_{10}(12.5) + 20\log_{10}(40,000)$$

$$= 206 \text{ dB}$$

$$10\log_{10}k = 228.6 \text{ dBK}$$

$$10\log_{10}M = 5 \text{ dB}$$

Using these values in Eq. (2) and solving for G_r/T, we get

$$\left(\frac{G_r}{T}\right)_{\text{dB}} = 85 - 57 + 206 - 228.6 + 5$$

$$= 10.4 \text{ dB}$$

With $T = 310\text{K}$, we thus find

$$G_r = 10.4 + 10\log_{10}(310)$$

$$= 35.31 \text{ dB}$$

The receiving antenna gain in is given by

$$10\log_{10}G_r = 10\log_{10}\left(\frac{4\pi A\eta}{\lambda^2}\right)$$

For a dish antenna (circular) with diameter D, the area A equals $\pi D^2/4$. Thus,

$$10\log_{10}G_r = 20\log_{10}D + 20\log_{10}f + 10\log_{10}(\eta) + 20.4(\text{dB})$$

where D is measured in meters and f is measured in GHz. Solving for the antenna diameter for the given system, we finally get

$D_{\min} = 0.6$ meters

Problem 8.12

(a) Similarities between satellite and wireless communications:

- They are both bandwidth-limited.
- They both rely on multiple-access techniques for their operation.
- They both have uplink and downlink data transmissions.
- The performance of both systems is influenced by intersymbol interference and external interference signals.

(b) Major differences between satellite and wireless communications:

- Multipath fading and user mobility are characteristic features of wireless communications, which have no counterparts in satellite communications.
- The carrier frequency for satellite communications is in the gigahertz range (Ku-band), whereas in satellite communications it is in the megahertz range.
- Satellite communication systems provide broad area coverage, whereas wireless communications provide local coverage with provision for mobility in a cellular type of layout.

Problem 8.13

In a wireless communication system, transmit power is limited at the mobile unit, whereas no such limitation exists at the base station. A sensible design strategy is to make the path loss (i.e., free-space loss) on the downlink as small as possible, which, in turn, suggests that we make

$(\text{Path loss})_{\text{uplink}} < (\text{Path loss})_{\text{downlink}}$

Recognizing that path loss is inversely proportional to wavelength, it follows that

$\lambda_{\text{uplink}} > \lambda_{\text{downlink}}$

or, equivalently,

$f_{\text{uplink}} < f_{\text{downlink}}$

Problem 8.14

The phase difference between the direct and reflected waves can be expressed as

$$\phi = \frac{2\pi d}{\lambda}\left[\sqrt{\left(\frac{h_b + h_m}{d}\right)^2 + 1} - \sqrt{\left(\frac{h_b - h_m}{d}\right)^2 + 1}\right] \qquad (1)$$

where λ is the wave length. For large d, Eq. (1) may be approximated as

$$\phi \approx \frac{4\pi(h_b h_m)}{\lambda d} \text{ radians}$$

With perfect reflection (i.e., reflected coefficient of the ground is -1) and assuming small ϕ (i.e., large d), the received power P_r is defined by

$$P_r = P_o\left|1 - e^{j\phi}\right|^2 \approx P_o \sin^2\left(\frac{4\pi(h_b h_m)}{\lambda d}\right)$$

$$\approx P_o\left(\frac{4\pi(h_b h_m)}{\lambda d}\right)^2 \tag{2}$$

where $P_o = P_t G_b G_m \left(\frac{\lambda}{4\pi d}\right)^2$ (3)

Using Eq. (3) in (2):

$$P_r = P_t G_b G_m \left(\frac{4\pi(h_b h_m)}{\lambda d}\right)^2 \left(\frac{\lambda}{4\pi d}\right)^2$$

$$= P_t G_b G_m \left(\frac{h_b^2 h_m^2}{d^4}\right)$$

which shows that the received power is inversely proportional to the fourth power of distance d between the two antennas.

Problem 8.15

The complex (baseband) impulse response of a wireless channel may be described by

$$\tilde{h}(t) = a_1 e^{-j\phi_1}\delta(t-\tau) + a_2 e^{-j\phi_2}\delta(t-\tau) \tag{1}$$

where the amplitudes a_1 and a_2 are Rayleigh distributed, and the phase angles ϕ_1 and ϕ_2 are uniformly distributed. This model assumes (2) the presence of two different clusters with each one consisting of a large number of scatterers, and (2) the absence of line-of-sight paths in the wireless environment. Define

$$h(t) = \tilde{h}(t)e^{j\phi_1}$$

$$\theta = \phi_2 - \phi_1$$

We may then rewrite Eq. (1) in the form

$$h(t) = a_1\delta(t-\tau) + a_2 e^{-j\theta}\delta(t-\tau)$$

as stated in the problem.

(a) (i) The transfer function of the model is

$$H(f) = F[h(t)]$$

$$= a_1 e^{-j2\pi f\tau_1} + a_2 e^{-j(2\pi f\tau_2 + \theta)}$$

(ii) The power-delay profile of the model is

$$P_h = E[|h(t)|^2]$$

$$= E[a_1\delta(t-\tau_1) + a_2 e^{-j\theta}\delta(t-\tau_2)(a_1\delta(t-\tau)) + a_2 e^{j\theta}\delta(t-\tau_2)]$$

$$= E[a_1^2\delta^2(t-\tau_1) + a_2^2\delta^2(t-\tau_2) + a_1 a_2\cos\theta\delta(t-\tau_1)\delta(t-\tau_2)]$$

$$= E[a_1^2]\delta^2(t-\tau_1) + E[a_2^2]\delta^2(t-\tau_2) \qquad (1)$$

(b) The magnitude response of the model is

$$|H(f)| = \left|a_1 e^{-j2\pi f\tau_1} + a_2 e^{-j2\pi f(\tau_2 + \theta)}\right|$$

$$= \sqrt{a_1^2 + a_2^2 + 2a_1 a_2\cos(2\pi f(\tau_2 - \tau_1) + \theta)}$$

which exhibits frequency selectivity due to two factors: (1) variations in the coefficients a_1 and a_2, and (2) variations inthe delay difference τ_2-τ_1.

Problem 8.16

The multipath influence on a communication system is usually described in terms of two effects - selective fading and intersymbol interference. In a Rake receiver, selective fading is mitigated by detecting the echo signals individually, using a correlation method, and adding them algebraically (with the same sign) rather than vectorially, and intersymbol interference is dealt with by reinserting different delays into the various detected echoes so that they fall into step again.

Making each correlator perform at its assigned value of delay can be done by inserting the right amount of delay in either the reference (called the *delayed-reference*) or received signals (called the *delayed-signal*). Independent of the form of the reference signals employed, the output SNR from the integrating filters is substantially the same for both configurations, under the assumption that the length of the delay T_d is significantly smaller than the symbol duration T. Each integrating filter responds to signals only within about $\pm 1/T$ of the frequency f. Therefore, the noises adding shorter than T, regardless of the form of reference signal. The only difference between the tap circuit contributions of the delayed-signal scheme and those of the delayed-reference scheme is that the latter are staggered in time by various fractions of T_d, and since such staggering is small compared to the significant fluctuation period of the contributions, we conclude that the noise outputs of the two configurations are equivalent.

However, there are three practical advantages of the delayed-signal scheme over the delayed-reference scheme. First, one delay line instead of two is required. Second, in the latter configuration, corresponding taps in the mark (symbol 0) and space (symbol 1) lines would have to be adjusted and be kept in phase coincidence. Third, coherent intersymbol interference (eliminated in the delayed-signal scheme) is still present in the latter scheme, (Price and Green 1958), see the Bibliography.

Problem 8.17

(a) The output of the linear combiner is given by

$$x(t) = \sum_{j=1}^{N} \alpha_j x_j(t)$$

$$= \sum_{j=1}^{N} \alpha_j (z_j m(t) + n_j(t))$$

$$= \underbrace{\sum_{j=1}^{N} \alpha_j z_j m(t)}_{\text{signal}} + \underbrace{\sum_{j=1}^{N} \alpha_j n_j(t)}_{\text{noise}}$$

The output signal-to-noise ratio is therefore

$$(\text{SNR})_0 = \frac{\text{Average signal power}}{\text{Average noise power}}$$

$$= \frac{E\left[\sum_{j=1}^{N} \alpha_j (z_j m(t))\right]^2}{E\left[\sum_{j=1}^{N} \alpha_j n_j(t)\right]^2}$$

$$= \frac{E\left[\sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_j\alpha_k z_j z_k m^2(t)\right]}{E\left[\sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_j\alpha_k n_j(t) n_k(t)\right]}$$

$$= \frac{\sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_j\alpha_k z_j z_k E[m^2(t)]}{\sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_j\alpha_k \; E[n_j(t) n_k(t)]} \qquad (1)$$

Using the following expectations

$E[m^2(t)] = 1$ for all t (i.e., unit message power)

$$E[n_j(t)n_k(t)] = \begin{cases} \sigma_j^2 & \text{for } k = j \\ 0 & \text{for } k \neq j \end{cases}$$

we find that Eq. (1) simplifies to

$$(\text{SNR})_0 = \frac{\sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_j\alpha_k z_j z_k}{\sum_{j=1}^{N}\alpha_j^2\sigma_j^2}$$

$$= \frac{\left(\sum_{j=1}^{N}\alpha_j z_j\right)^2}{\sum_{j=1}^{N}\alpha_j^2\sigma_j^2} \qquad (2)$$

(b) Equation (2) can be rewritten in the equivalent form

$$(\text{SNR})_0 = \frac{\left[\sum_{j=1}^{N} \alpha_j \sigma_j (z_j/\sigma_j)\right]^2}{\sum_{j=1}^{N} \alpha_j^2 \sigma_j^2}$$

$$= \frac{\left[\sum_{j=1}^{N} u_j (\text{SNR})_j^{1/2}\right]^2}{\sum_{j=1}^{N} u_j^2} \qquad (3)$$

where $u_j = \alpha_j \sigma_j$ and $(\text{SNR})_j = z_j^2/\sigma_j^2$. We now invoke the Schwarz inequality, which, in discrete form for the problem at hand, is stated as follows

$$\left(\sum_{j=1}^{N} u_j (\text{SNR})_j^{1/2}\right)^2 \le \left(\sum_{j=1}^{N} u_j^2\right)\left(\sum_{j=1}^{N} ((\text{SNR})_j^{1/2})^2\right) \qquad (4)$$

Hence, inserting this inequality into the right-hand side of Eq. (3), we may write

$$(\text{SNR})_0 \le \sum_{j=1}^{N} (\text{SNR})_j$$

which proves the formula under subpart (i).

To prove subpart (ii), we recall that the Schwarz inequality of Eq. (4) is satisfied with the equality sign if (except for a scaling factor)

$$(\text{SNR})_j^{1/2} = u_j$$

or, equivalently,

$$\frac{z_j}{\sigma_j} = \alpha_j \sigma_j$$

That is,

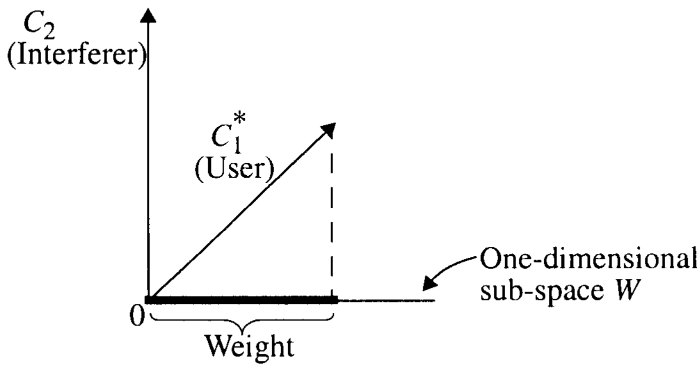
$$\alpha_j = \frac{z_j}{\sigma_j^2}$$

Problem 8.18

For the problem at hand, we have $M = N = 2$. Therefore,

$$M - N + 1 = 1$$

and so the weight subspace W is one-dimensional. We thus have the following representation for the action of the antenna array:



Problem 8.19

(a) The cost function is

$$J = \frac{1}{2}|e[n]|^2 = \frac{1}{2}e[n]e^*[n]$$

where the error signal is

$$e[n] = d[n] - \sum_{k=1}^{M} w_k^*[n]x_k[n]$$

Let

$$w_k[n] = a_k[n] + jb_k[n]$$

Hence

$$\frac{\partial J}{\partial a_k} = \frac{1}{2}e[n]\frac{\partial e^*[n]}{\partial a_k} + \frac{1}{2}e^*[n]\frac{\partial e[n]}{\partial a_k}$$

$$= -\frac{1}{2}e[n]x_k^*[n] - \frac{1}{2}e^*[n]x_k[n]$$

$$= -\frac{1}{2}\text{Re}\{x_k[n]e^*[n]\} \tag{1}$$

$$\frac{\partial J}{\partial b_k} = \frac{1}{2}e[n]\frac{\partial e^*[n]}{\partial b_k} + \frac{1}{2}e^*[n]\frac{\partial e[n]}{\partial b_k}$$

$$= -\frac{j}{2}e[n]x_k^*[n] + \frac{j}{2}e^*[n]x_k[n]$$

$$= -\text{Im}\{x_k[n](e^*[n])\} \tag{2}$$

The adjustment applied to the kth weight is therefore

$$\Delta w_k[n] = \Delta a_k[n] + j\Delta b_k[n]$$

$$= -\mu\frac{\partial J}{\partial a_k} - j\mu\frac{\partial J}{\partial b_k} \tag{3}$$

where μ is the step-size parameter. Substituting Eqs. (1) and (2) into (3),

$$\Delta w_k[n] = \mu\text{Re}\{x_k[n]e^*[n]\} + \mu\text{Im}\{x_k[n]e^*[n]\}$$

$$= \mu x_k[n]e^*[n]$$

(b) The complex LMS algorithm is described by the following pair of relations:

$$w_k[n+1] = w_k[n] + \Delta w_k[n]$$

$$= w_k[n] + \mu x_k[n]e^*[n], \qquad k = 1, 2, \cdots, M$$

$$e(n) = d[n] - \sum_{k=1}^{M} w_k^* x_k[n]$$

Problem 8.20

(a) We are told that the speed of response of the weights in the LMS algorithm is proportional to the average signal power at the antenna array input. Conversely, we may say that the average signal power at the array input is proportional to the speed of response of the weights in the LMS algorithm. Moreover, the maximum speed of response of the LMS weights is proportional to R_b/f_{max}, where R_b is the bit rate and f_{max} is the maximum fade rate in Hz. It follows therefore that the dynamic range of the average signal power at the antenna array input is proportional to R_b/f_{max}, as shown by

$$P_{max} = \alpha R_b / f_{max} \text{ watts} \tag{1}$$

where α is the proportionality constant.

(b) For $\alpha = 0.2$, $R_b = 32 \times 10^3$ b/s, and $f_{max} = 70$ Hz, the use of Eq. (1) yields

$$P_{max} = 0.2 \times 32 \times 10^3 / 70$$

$$= 640/7$$

$$= 91 \text{ watts}$$

which is somewhat limited in value.

Problem 8.21

(a) According to the Wiener filter, derived for the case of complex data, the optimum weight vector is defined by

$$\mathbf{R}_x \mathbf{w}_o = \mathbf{r}_{xd} \tag{1}$$

where

$\mathbf{R}_x$ = correlation matrix of the input signal vector $\mathbf{x}[n]$

$$= E[\mathbf{x}[n]\mathbf{x}^H[n]] \tag{2}$$

$\mathbf{r}_{xd}$ = cross-correlation vector between $\mathbf{x}[n]$ and desired response $d[n]$

$$= E[\mathbf{x}[n]\mathbf{d}^*[n]] \tag{3}$$

$\mathbf{w}_o$ = optimum weight vector.

Note that the formulation of Eq. (1) is based on the premise that the array output is defined as the inner product $\mathbf{w}^H\mathbf{x}[n]$. The Wiener filter for real data is a special case of Eq. (1), where the Hermitian transpose H in Eq. (2) is replaced by ordinary transposition and the complex conjugation in Eq. (3) is omitted. Assuming that the input $\mathbf{x}[n]$ and desired response $d[n]$ are jointly ergodic, we may use the following estimates for $\mathbf{R}_x$ and $\mathbf{r}_{xd}$:

$$\hat{\mathbf{R}}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}[n]\mathbf{x}^H[n] \tag{4}$$

$$\hat{\mathbf{r}}_{\mathbf{x}d} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}[n]d^*[n] \tag{5}$$

where K is the total number of snapshots used to train the antenna array. Correspondingly, the estimate of the optimum weight vector $\mathbf{w}_o$ is computed as

$$\hat{\mathbf{w}} = \hat{\mathbf{R}}_x^{-1}\hat{\mathbf{r}}_{\mathbf{x}d} \tag{6}$$

where $\mathbf{R}_x^{-1}$ is the inverse of $\mathbf{R}_x$.

(b) The DMI algorithm for computing the estimate $\hat{\mathbf{w}}$ may now proceed as follows:

1. Collect K snapshots of data denoted by

   $$\{\mathbf{x}[k], d[k]\}_{k=1}^{K}$$

   where K is sufficiently large for $\hat{\mathbf{w}}$ to approach $\mathbf{w}_o$ and yet small enough to ensure stationarity of the data.

2. Use Eqs. (4) and (5) to compute the correlation estimates $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{r}}_{\mathbf{x}d}$.

3. Invert the correlation matrix $\hat{\mathbf{R}}_x$ and then use Eq. (6) to compute the weight estimate $\hat{\mathbf{w}}$.

For an antenna array consisting of M elements, the matrix $\hat{\mathbf{R}}_x$ is an M-by-M matrix and $\hat{\mathbf{r}}_{\mathbf{x}d}$ is an M-by-1 vector. Therefore, the inversion of $\hat{\mathbf{R}}_x$ and its multiplication by $\hat{\mathbf{r}}_{\mathbf{x}d}$ requires multiplications and additions on the order of M^3.